Autonomous satellite orbit prediction

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Abstract

A method to predict satellite orbits in a GPS device without a network connection is presented. The motivation for this work was to improve the startup performance of a navigation device without Assisted GPS. Tests of our algorithm show that in 95% of the cases the error in satellite's predicted position stays under 21 meters for one day and under 94 meters for three days.

Introduction

In Global Positioning System (GPS) each satellite transmits its position in the form of ephemeris parameters and its clock time. The user's receiver measures the time delay it takes for the signal to travel from the satellite to the receiver, from which the apparent range to the satellite can be calculated. This range still includes an unknown bias originating from the time difference between the receiver's clock time and GPS time. When the biased range measurements and positions from four satellites are known simultaneously the unknown bias as well as the position of the user can be solved.

When a GPS navigation device is turned on, it typically takes about 30 seconds to get the first position estimate, even in an ideal environment. This delay occurs because the part of broadcast containing the ephemeris takes 12 seconds to transmit and the satellite sends it once every 30 seconds. When the receiver is without a straight view to the sky, for example blocked by trees or high buildings, the signal acquisition and demodulation slows down and it takes much longer to get the first position estimate. If the signal gets too weak, the demodulation is no longer even possible. As little time as this 30 seconds delay may seem, it is usually frustrating even from a typical user's point of view, to say nothing of emergency call cases or other special situations. Therefore some innovations are needed to provide the first position estimate in a significantly shorter time. That time is often discussed with abbreviation TTFF (time to first fix).

Because the main reason for the long TTFF is the time it takes to receive the satellites' ephemeris broadcast, alternative ways of obtaining satellite position information can be used to reduce it. If the current ephemerides were in the device already before turning it on, the connection between satellite and the receiver would be needed only for the range measurements. This is fast, because the receiver is potentially capable of getting a new pseudorange measurement at the beginning of each subframe of the broadcast or every 6 seconds. Thus it would be possible to get the first position fix often under 5 seconds after turning on the device, if we only knew the positions of the satellites.

A widely used alternative to get satellite's orbital data is to use assisted GPS (AGPS). AGPS is a positioning technique in which a GPS receiver does not only receive information from the satellites, but also from an assistance data server using a data connection (Internet or other). The assistance data servers send data to the navigation device that enable the computation of satellites' position coordinates. There are, however, problems with such assistance data: Connection to the assistance server may fail or the data connection fee may be too expensive for the user. Furthermore, many navigation devices are designed to operate without any network connection. There is therefore interest in methods that can be implemented entirely in the navigation device, without network connection.

In self-assisted GPS the aim is to attain properties similar to assisted GPS without a network connection. This presumes prediction of the information which is would have been received as assistance data. As explained above, AGPS provides satellite's ephemeris, which is needed in order to reduce the time to first fix. In addition, it provides other important information, for example satellite's current clock correction terms. This work focuses, however, prediction of the satellites' orbits only.

Satellite's equation of motion

In this work satellite's orbit is predicted by forming its equation of motion and solving it numerically. Usually, in order to predict satellite orbits with very high accuracy, a large number of different forces have to be included to the model. However, only the 4 forces shown in Figure 1 have been included in our motion model. The largest of these forces is the Earth's gravitation taking into account the asymmetrical mass distribution of the Earth. Other gravitational forces included in this model are the lunar and solar gravitation. The smallest but not the least important force is the solar radiation pressure (SRP), which arises out of the sunlight that hits the satellite and its solar panels.

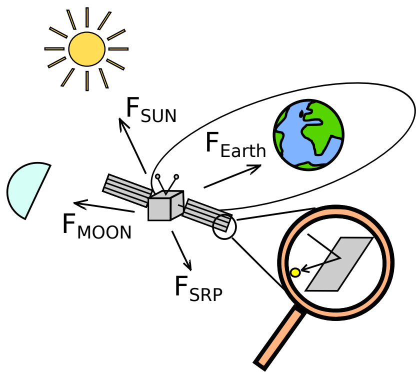

Figure 1: Forces included in the satellite's equation of motion

The equation of motion for the GPS satellite can be formed as follows: We first compute the vector sum

$$\Sigma\mathbf{F} = \mathbf{F}_{\text{Earth}} + \mathbf{F}_{\text{Moon}} + \mathbf{F}_{\text{Sun}} + \mathbf{F}_{\text{SRP}}$$

of the chosen forces. Dividing this sum by the mass of the satellite we get the acceleration according to Newton's 2nd law: $\Sigma\mathbf{F} = m\mathbf{a}$. As the forces depend on both the satellite's location $\mathbf{r}$ and time, so does the acceleration function $\mathbf{a}$. Therefore, the satellite's equation of motion is of the form

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} = \mathbf{a}(\mathbf{r}, t).$$

What we have here is a second order differential equation, and it can be solved numerically, as long as the satellite's position $\mathbf{r}$ and velocity $\mathbf{v}$ at some initial moment, say t_0, are known. Then the satellite's position at moment t is

$$\mathbf{r}(t) = \mathbf{r}_0 + \int_{t_0}^{t} \left(\mathbf{v}_0 + \int_{t_0}^{t} \mathbf{a}(t, \mathbf{r})\mathrm{d}t \right) \mathrm{d}t, \tag{1}$$

where $\mathbf{r}_0 = \mathbf{r}(t_0)$ and $\mathbf{v}_0 = \mathbf{v}(t_0)$. Furthermore, let us define for further purposes a function $\boldsymbol{\psi}$, which, given satellite's position and velocity at time instant t_1, returns the satellite's state position and velocity another time instant t_2. The definition for this function is

$$\boldsymbol{\psi}\left(t_1, t_2, \begin{bmatrix} \mathbf{r}(t_1) \\ \mathbf{v}(t_1) \end{bmatrix}\right) = \begin{bmatrix} \mathbf{r}(t_2) \\ \mathbf{v}(t_2) \end{bmatrix} = \begin{bmatrix} \mathbf{r}(t_1) + \int_{t_1}^{t_2} \left(\mathbf{v}(t_1) + \int_{t_1}^{t_2} \mathbf{a}(t, \mathbf{r})\mathrm{d}t \right) \mathrm{d}t \\ \mathbf{v}(t_1) + \int_{t_1}^{t_2} \mathbf{a}(t, \mathbf{r})\mathrm{d}t \end{bmatrix}. \tag{2}$$

Reference frames

The position and velocity coordinates computed from the satellite's ephemeris broadcast are in an Earth Centered, Earth Fixed reference frame (ECEF), which rotates with the Earth. Newton's laws, however, are valid only for an inertial or non-rotating reference frame. Therefore, we have to transform the received position and velocity coordinates into the inertial reference frame before substitution to the equation (1) or to the function (2).

In this work we do not explain the reference frame transformation in details, but introduce a function $\boldsymbol{\xi}$ to describe it. If satellite's state in ECEF is denoted with subscript ECEF and satellite's state in inertial reference frame with subscript IN, then

$$\begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix}_{\text{IN}} = \boldsymbol{\xi}\left(\begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix}_{\text{ECEF}}, t, x_p, y_p \right).$$

The transformation function $\boldsymbol{\xi}$ is time dependent, as can be seen from its arguments. In addition, the function requires so called polar motion parameters, x_p and y_p. These are two angles, which express the orientation of Earth's instantaneous rotation axis with respect to Earth's crust. The values of parameters x_p and y_p are very small: less than half an arc-second. This corresponds to around 10 meters movement on the Earth's crust as one can see from the Figure 2, which illustrates the movement of Earth's instantaneous rotation axis during the years 2005-2008.

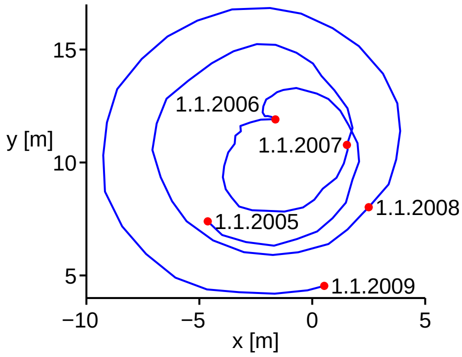

Figure 2: Earth's instantaneous rotation axis moves with respect to Earth's crust

Even though the polar motion parameters are small, it is necessary to know their values in order to predict satellite's orbit accurately. The problem is that we could not find a model which would predict these parameters in long term, say for three to five years, which is the life time of an navigation device. Therefore the values for these parameters have to be obtained other way.

The orbit prediction function (2) was defined in inertial reference frame. Now we define a function $\boldsymbol{\psi}^*$ as the ECEF version of this function, that is

$$\boldsymbol{\psi}^*(t_1, t_2, \begin{bmatrix} \mathbf{r}(t_1) \\ \mathbf{v}(t_1) \end{bmatrix}_{\text{ECEF}}, \begin{bmatrix} x_p \\ y_p \end{bmatrix}) = \begin{bmatrix} \mathbf{r}(t_2) \\ \mathbf{v}(t_2) \end{bmatrix}_{\text{ECEF}}$$

$$= \boldsymbol{\xi}^{-1}\left(\boldsymbol{\psi}\left(t_1, t_2, \boldsymbol{\xi}\left(\begin{bmatrix} \mathbf{r}(t_1) \\ \mathbf{v}(t_1) \end{bmatrix}_{\text{ECEF}}, t_1, x_p, y_p\right)\right), t_2, x_p, y_p\right). \tag{3}$$

Here $\boldsymbol{\xi}^{-1}$ is the inverse of the transformation function $\boldsymbol{\xi}$ from ECEF to the inertial reference frame.

Least squares fitting

The daily values of polar motion parameters are published in the Internet by the International Earth Rotation Service [1]. When predicting satellite orbits normally with computer, we can use these parameters as well as very accurate satellite positions and velocities delivered by the National Geospatial-Intelligence Agency [2] to predict satellite orbits with our motion model. In contrast, when doing the prediction in a navigation device without any network connection, we do not know either the values for the polar motion parameters or the exact initial position and velocity. The accuracy of the satellite's real broadcast ephemeris is significantly lower than the accuracy of precise ephemeris orbits in the Internet. To overcome these obstacles we have developed a method to solve the unknown polar motion parameters while simultaneously improving the satellites' inaccurate initial velocities.

The fitting procedure goes along as described in the following and the Figure 3 illustrates the used notations. Consider one ephemeris parameter set, which is received as a broadcast message and having a certain time of ephemeris (TOE). With these parameters we are able

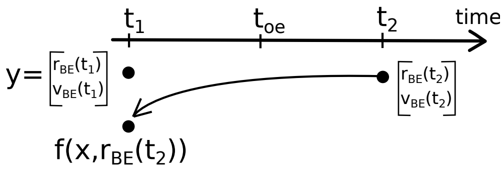

Figure 3: Nonlinear least squares fitting. The nonlinear function $\mathbf{f}$ computes the states of n satellites at t_1, given the initial states at t_2 and polar motion parameters. As measurements $\mathbf{y}$ we have satellites' broadcast states at t_1.

to calculate the satellite's state i.e. position and velocity at any time within ± 2 h from the TOE. Going outside of this range the precision of broadcast ephemeris (BE) deteriorates rapidly. We now compute satellite's state at the time instants $t_1 = \text{TOE} - 1.5\,\text{h}$ and $t_2 = \text{TOE} + 1.5$ h for n satellites, from which we have received the broadcast information. Let us denote $\mathbf{y}$ the vector containing the BE positions and the BE velocities of all these satellites at the first time instant t_1. Then we denote $\mathbf{f}$ the function which, starting from the latter time instant t_2, computes the satellites' states at t_1. This function is an extension of function $\boldsymbol{\psi}^*$ of equation (3) to many satellites. As input parameters this function requires satellites' positions and velocities at t_2 as well as the polar motion parameters x_p and y_p.

$$\mathbf{f}(\mathbf{r}^{\text{all}}, \mathbf{v}^{\text{all}}, x_p, y_p) = \begin{bmatrix} \boldsymbol{\psi}^*(t_2, t_1, \mathbf{r}^{\text{sat1}}, \mathbf{v}^{\text{sat1}}, x_p, y_p) \\ \boldsymbol{\psi}^*(t_2, t_1, \mathbf{r}^{\text{sat2}}, \mathbf{v}^{\text{sat2}}, x_p, y_p) \\ \vdots \\ \boldsymbol{\psi}^*(t_2, t_1, \mathbf{r}^{\text{satn}}, \mathbf{v}^{\text{satn}}, x_p, y_p) \end{bmatrix}$$

We fix the initial positions of the satellites on the BE positions denoted $\mathbf{r}_{\text{BE}}(t_2)$. The rest of the function inputs form the variable vector

$$\mathbf{x} = \begin{bmatrix} \mathbf{v}(t_2) \\ x_p \\ y_p \end{bmatrix},$$

where $\mathbf{v}(t_2)$ contains the velocities of all satellites at the second time instant t_2. Now we want to find the $\hat{\mathbf{x}}$, with which the value of the nonlinear function $\mathbf{f}$ is as near the measurements $\mathbf{y}$ as possible. In other words we solve the nonlinear least squares fitting problem

$$\hat{\mathbf{x}} = \arg\min_x \mathbf{p}^T(\mathbf{x})\mathbf{D}\mathbf{p}(\mathbf{x})$$

where the residual function $\mathbf{p}$ is

$$\mathbf{p}(\mathbf{x}) = \mathbf{f}(\mathbf{r}_{\text{BE}}(t_2), \mathbf{v}(t_2), x_p, y_p) - \mathbf{y} = \mathbf{f}(\mathbf{r}_{\text{BE}}(t_2), \mathbf{x}) - \mathbf{y}$$

and the diagonal matrix $\mathbf{D}$ has is a weight matrix having a value of 1000^2 in those elements corresponding to the velocity components and ones elsewhere.

The nonlinear least squares problem can be solved with Levenberg-Marquardt method. The algorithm requires an initial guess for $\mathbf{x}$. For the velocities $\mathbf{v}(t_2)$ the initial guess is taken from the broadcast velocity $\mathbf{v}_{\text{BE}}(t_2)$. For polar motion parameters we have used the values $x_{p0} = 0.05$ arcseconds and $y_{p0} = 0.35$ arcseconds, which is the approximate center of the polar motion spiral during the years 2004-2008.

The prediction results obtained with the least squares fitting method are shown in the Figure 4. The error in satellite's position was calculated as a norm of the difference between the predicted position and the reference position from NGA's precise ephemeris, i.e $||\mathbf{r}_{\text{pred}} - \mathbf{r}_{\text{ref}}||$. The prediction was done for all satellites a number of times, and the error curve of the Figure was chosen to be the 95% quantile of all prediction errors. This result can not be directly attributed to positioning error, because among other things, positioning error depends on the user's position on the Earth and on the geometry of the used satellites at that particular moment. It is, however, a good upper bound for the errors in pseudo-ranges.

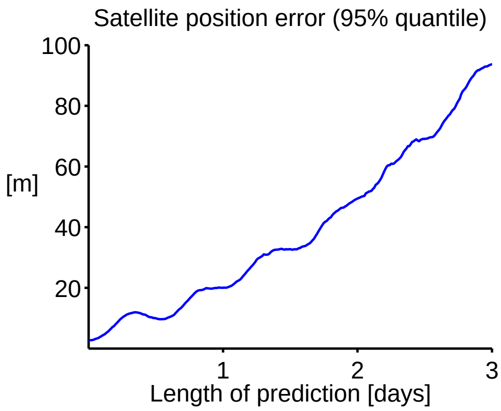

Figure 4: Error in satellite's position as a function of prediction length.

Conclusion

To summarize, a method to predict satellite orbits in a navigation without a network connection was presented. The challenge of this problem was that without Internet the precise satellite orbits computed from accurate satellite attitude measurements are not available. Nor can the recent information of the Earth's orientation be used. However, in this work it was explained how to improve the inaccurate satellite velocity computed from broadcast ephemeris and solve the unknown polar motion parameters describing the Earth's orientation. The algorithm is able to predict satellite orbits up to 3 days with an accuracy of 94 meters, which is acceptable. The method can be used to reduce time to first fix in navigation devices without assisted GPS.

References

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