Relay Selection Based Full-Duplex Cooperative Systems under Adaptive Transmission

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Abstract—The present work analyzes multi-relay full-duplex systems with relay selection under multipath fading conditions in the context of channel capacity under: i) optimum power and rate adaptation; ii) truncated channel inversion with fixed rate. Useful analytic expressions are derived for these measures as well as for the associated optimum cut-off level. The offered results are then employed in the analysis of the corresponding end-to-end performance by also quantifying the effects of the involved relay self-interference. It is shown that high capacity levels are achieved even for a moderate number of relays and self-interference levels, at no considerably added system complexity. This is particularly useful in demanding emerging applications that are subject to transmit power constraints or fixed rate requirements.

Index Terms—Full-duplex relaying, relay selection, outage probability, channel capacity, adaptive transmission.

I. INTRODUCTION

Cooperative communications is an effective wireless technology and relay selection (RS) constitutes a widely used efficient method for mitigating inter-relay interference, whilst achieving enhanced performance without excessive transmit power levels and spectral efficiency (SE) losses [1]–[3]. Likewise, full-duplex (FD) relay systems have attracted considerable attention by both academia and industry, since the fundamental issue of induced loop interference can be resolved adequately within reasonable complexity levels [5]. To that end, the authors in [8] and [6] quantified the average channel capacity of FD systems for the case of RS in amplify-and-forward (AF) networks. Then, the authors in [2] analyzed the average capacity of opportunistic decode-and-forward (DF) RS, whereas the outage probability (OP) of FD RS in spectrum sharing networks was addressed in [8]. Likewise, the authors in [9] evaluated the performance of opportunistic RS with limited feedback, while investigations in the context of secure communications were recently reported in [11] and [12].

Nevertheless, despite the advantages of FD systems, the channel capacity in the context of multiple relays has not been fully addressed. Specifically, the authors in [12] derived upper bounds for the capacity under adaptive transmissions for conventional half-duplex (HD) systems, while the authors in [3] analyzed the ergodic capacity for single-relay HD systems. Likewise, [5] and [6] analyzed the ergodic capacity of FD systems with fixed transmit power. Motivated by this, the present contribution investigates RS based multi-relay FD systems under multipath fading conditions in the context of: 1) channel capacity under optimum power and rate adaptation (C-OPRA); 2) truncated channel inversion with fixed rate (C-TIFR). To this end, simple analytic expressions are derived for these measures that are subsequently employed to the analysis of the considered scenarios and in quantifying the effects of the involved relay self-interference (SI) and cut-off threshold. The offered results provide meaningful insights on the design and deployment of future systems with diverse quality of service (QoS) requirements. This is achieved by the resulting overall system efficiency as RS is a relatively low complexity technique that is capable of reducing the overhead and the stringent synchronization requirements among participating relays, whilst the considered adaptation policies can practically assist in meeting high QoS requirements at no considerably added complexity thanks to the existing CSI knowledge due to the adopted RS. Therefore, the considered setup can be useful in demanding and critical wireless applications of reduced complexity that are subject to transmit power constraints and/or fixed-rate requirements, such as for example, device-to-device communications and telemedicine, among others.

II. SYSTEM AND CHANNEL MODELS

We consider a two-hop FD system consisting of a source S, a destination D and K intermediate relays, denoted by R_k, where k = 1, 2, ···, K. Also, the corresponding channel coefficient between node i and j is denoted by h_{i,j}, where i, j ∈ {S, R_k, D} while, without loss of generality, additive white Gaussian noise (AWGN) is assumed in each link. Based on this, the received signal at the kth relay is represented as

\[ y_{R_k} = \sqrt{P_S} h_{S,R_k} x_S + \sqrt{P_{R_k}} h_{R_k} x_{R_k} + n \] (1)

where P_S and P_{R_k} are the transmit powers at the source and relay nodes, respectively, x_S and x_{R_k} denote the transmitted signals from the source and relay nodes with normalized unit energy, whereas h_{R_k} represents the introduced SI at the
relays. Based on (3), the instantaneous signal-to-interference plus noise ratio (SINR) for S–R_k can be expressed as
\[ \gamma_{S_k} = \gamma_{S,R_k}/(\gamma_{I_k} + 1), \]
where \( \gamma_{S,R_k} = |h_{S,R_k}|^2 P_{R_k}/N_0 \) and \( \gamma_{I_k} = |h_{I_k}|^2 P_{R_k}/N_0 \) denote the corresponding instantaneous signal-to-noise ratio (SNR) in each case, with average values of \( \bar{\gamma}_{S,R_k} \) and \( \bar{\gamma}_{I_k} \), respectively. By also assuming that signals in S–R_k links experience Rayleigh distributed multipath fading and that the SI channel is unneeded, i.e. \( \gamma_{I_k} = \bar{\gamma}_{I_k} \), the probability density function (PDF) of \( \gamma_{S_k} \) is expressed as
\[ f_{\gamma}(\gamma) = \frac{\bar{\gamma}_{I_k} + 1}{\bar{\gamma}_{S,R_k}} e^{-\frac{\bar{\gamma}_{I_k} + 1}{\bar{\gamma}_{S,R_k}}}. \] (2)

In addition, based on the max-min loop interference RS policy, the relay with the best S–R_k-D link is selected by
\[ \gamma_{R_k} = \max_{k=1,\ldots,K}\{\min(\gamma_{S_k}, \gamma_{R_k,D})\} \] (3)
where \( \gamma_{R_k,D} \) denotes the instantaneous SINR of the R_k-D link with average value of \( \bar{\gamma}_{R_k,D} \) and PDF \( f_{\gamma_{R_k,D}}(\gamma) = \exp(-\gamma/\bar{\gamma}_{R_k,D})/\bar{\gamma}_{R_k,D} \). Furthermore, we consider a direct link with instantaneous and average SNRs \( \gamma_{S,D} \) and \( \gamma_{S,D} \), respectively, and a PDF \( f_{\gamma_{S,D}}(\gamma) = \exp(-\gamma/\bar{\gamma}_{S,D})/\bar{\gamma}_{S,D} \). In the considered RS process, a single relay is selected among a set of relays, depending on which relay provides the best path between source and destination, i.e. the best S–R_k–D link, as also described in detail in [8]. To this effect and assuming maximum-ratio combining (MRC) at the destination, the output SINR is expressed as \( \gamma = \gamma_{S,D} + \gamma_{R_k,D} \), and its corresponding PDF is given by
\[ f_{\gamma}(\gamma) = \frac{K}{k} \frac{(-1)^k k \alpha_k}{1 - k \alpha_k} e^{-\frac{\gamma}{\bar{\gamma}_{S,D}}} e^{-k \bar{\gamma}_{S,D}} \] (4)
where \( \alpha_k = (\bar{\gamma}_{I_k} + 1)/\bar{\gamma}_{S,R_k} + 1/\bar{\gamma}_{R_k,D} \) [5].

III. CAPACITY UNDER ADAPTIVE TRANSMISSION
A. Optimum Power and Rate Adaptation
It is recalled that the average channel capacity is fundamentally based on fixed power transmission as CSI is available only at the receiver. Yet, when CSI is also available at the transmitter, the transmit power level can be adapted. Based on this, increasing the transmit power at favorable fading conditions and reducing it at unfavorable fading conditions increases the performance with efficient utilization of power resources. This method is also known as water-filling in time and is useful in scenarios with transmit power constraints [13].

**Theorem 1.** For \( \{\bar{\gamma}_{S,D}, \bar{\gamma}_{S,R_k}, \bar{\gamma}_{R_k,D}, \gamma_0, \gamma_1\} \in \mathbb{R}^+ \), the spectral efficiency of RS FD systems under optimum power and rate adaptation over Rayleigh fading channels is expressed as
\[ \frac{C_{\text{OPRA}}}{B} = \sum_{k=1}^{K} \left( \frac{K}{k} \frac{(-1)^k k \alpha_k}{1 - k \alpha_k} \frac{\Gamma(0, \gamma_0/\bar{\gamma}_{S,D})}{\Gamma(0, \gamma_0/\bar{\gamma}_{S,D}) \log(2)} \right) \]
\[ - \sum_{k=1}^{K} \left( \frac{K}{k} \frac{(-1)^k \Gamma(0, k \alpha_k \gamma_0)}{1 - k \alpha_k} \frac{\bar{\gamma}_{S,D}}{\bar{\gamma}_{S,D}} \log(2) \right) \] (5)
where \( \gamma_0 \) is the optimum cut-off SNR level below which data transmission is suspended, whereas \( \Gamma(\cdot, \cdot) \) denotes the upper incomplete gamma function [12], [13].

**Proof.** The channel capacity with optimum power and rate adaptation (OPRA) is defined as
\[ C_{\text{OPRA}} = \sum_{k=1}^{K} \left( \frac{K}{k} \frac{(-1)^k k \alpha_k}{1 - k \alpha_k} \frac{\Gamma(0, \gamma_0/\bar{\gamma}_{S,D})}{\Gamma(0, \gamma_0/\bar{\gamma}_{S,D}) \log(2)} \right) \]
\[ \times \left( \int_{\gamma_0}^{\infty} \log(2) e^{-\frac{\gamma}{\bar{\gamma}_{S,D}}} d\gamma - \int_{\gamma_0}^{\infty} \log(2) e^{-k \bar{\gamma}_{S,D}} d\gamma \right) \]
\[ + \sum_{k=1}^{K} \left( \frac{K}{k} \frac{(-1)^k k \alpha_k \log(\gamma_0)}{1 - k \alpha_k} \right) \]
\[ \times \left( \int_{\gamma_0}^{\infty} e^{-\frac{\gamma}{\bar{\gamma}_{S,D}}} d\gamma - \int_{\gamma_0}^{\infty} e^{-k \bar{\gamma}_{S,D}} d\gamma \right). \] (6)

By evaluating the two simple integrals in (6) and integrating by parts the integrals that involve the logarithmic term yields
\[ \frac{C_{\text{OPRA}}}{B} = \sum_{k=1}^{K} \left( \frac{K}{k} \frac{(-1)^k k \alpha_k}{1 - k \alpha_k} \frac{\Gamma(0, k \alpha_k \gamma_0)}{\Gamma(0, k \alpha_k \gamma_0) \log(2)} \right) \]
\[ \times \left( \int_{\gamma_0}^{\infty} e^{-\frac{\gamma}{\bar{\gamma}_{S,D}}} d\gamma - \int_{\gamma_0}^{\infty} e^{-k \bar{\gamma}_{S,D}} d\gamma \right). \] (7)

The above integrals can be solved using [10, eq. (3.351.4)] and [13, eq. (8.359.1)], yielding (5) and completing the proof. □

B. Optimum SNR Cut-off Level
**Lemma 1.** For \( \{\bar{\gamma}_{S,D}, \bar{\gamma}_{S,R_k}, \bar{\gamma}_{R_k,D}, \gamma_0, \gamma_1\} \in \mathbb{R}^+ \), the optimum SNR cut-off level for the considered multi-relay RS FD systems under Rayleigh fading conditions is expressed as
\[ \gamma_0 = \bar{\gamma}_{S,D} \Gamma^{-1}\left(0, \frac{\sum_{k=1}^{K} \left( \frac{K}{k} \frac{(-1)^k k \alpha_k}{1 - k \alpha_k} \frac{\Gamma(0, k \alpha_k \gamma_0)}{\Gamma(0, k \alpha_k \gamma_0) \log(2)} \right)}{\sum_{k=1}^{K} \left( \frac{K}{k} \frac{(-1)^k k \alpha_k}{1 - k \alpha_k} \right)} \right) \] (8)
where \( \Gamma^{-1}(\cdot, \cdot) \) is the inverse incomplete gamma function [12].

**Proof.** The optimum value of \( \gamma_0 \) must satisfy [14, eq. (6)], which can be equivalently expressed as follows
\[ \gamma_0 = \int_{\gamma_0}^{\infty} p(\gamma) d\gamma - \gamma_0 \int_{\gamma_0}^{\infty} p(\gamma) \gamma d\gamma. \] (9)

Substituting (6) in (2) and taking the first derivative with respect to \( \gamma_0 \) along with some algebraic manipulations yields
\[ \sum_{k=1}^{K} \left( \frac{K}{k} \frac{(-1)^k k \alpha_k}{1 - k \alpha_k} \right) \frac{\bar{\gamma}_{S,D}}{\bar{\gamma}_{S,D}} \int_{\gamma_0}^{\infty} e^{-\frac{\gamma}{\bar{\gamma}_{S,D}}} d\gamma - e^{-k \bar{\gamma}_{S,D}} = -1. \] (10)

To this effect and using [12, eq. (8.350.2)], it follows that
\[ \sum_{k=1}^{K} \left( \frac{K}{k} \frac{(-1)^k k \alpha_k}{1 - k \alpha_k} \right) \frac{\bar{\gamma}_{S,D}}{\bar{\gamma}_{S,D}} \int_{\gamma_0}^{\infty} \frac{1}{\bar{\gamma}_{S,D}} d\gamma = 1 \]
\[ + \sum_{k=1}^{K} \left( \frac{K}{k} \frac{(-1)^k k \alpha_k}{1 - k \alpha_k} \Gamma(0, k \alpha_k \gamma_0) \right) \] (11)

Evidently, by solving (11) with respect to \( \Gamma(0, \gamma_0/\bar{\gamma}_{S,D}) \) and recalling the definition of the inverse incomplete gamma function, equation (8) is deduced, which completes the proof. □
C. Truncated Channel Inversion and Fixed Rate

Fixed rate scenarios can be effectively achieved through channel inversion thanks to its low implementation complexity. The only drawback of this approach is the large transmit power requirements in case of deep fades; yet, this can be resolved by inverting the channel fading above a fixed cut-off level [13].

Theorem 2. For \( \{ \gamma_{S,D}, \gamma_{S,R_k}, \gamma_{R_k,D}, \gamma_0, \gamma_{I_k} \} \in \mathbb{R}^+ \), the spectral efficiency of RS FD systems with truncated channel inversion and fixed rate over Rayleigh fading channels is expressed by (13), at the top of the next page.

Proof. It is recalled that the C-TIFR is defined as

\[
C_{\text{TIFR}} = B \log_2 \left( 1 + \frac{1}{\int_{\gamma_0}^{\infty} \frac{1}{f_\gamma(\gamma)} d\gamma} (1 - P_{\text{out}}) \right) (13)
\]

where \( P_{\text{out}} \) is the corresponding OP. Therefore, by substituting (3) into (13), one obtains (14), at the top of the next page. By also recalling that \( F_\gamma(z) = \int_0^\infty f_\gamma(x) dx \) and substituting (3) in it yields the corresponding OP, \( P_{\text{out}}(\gamma_0) = F_\gamma(\gamma_0) \), namely

\[
P_{\text{out}}(\gamma_0) = \sum_{k=1}^{K} \binom{K}{k} \left( -1 \right)^k k \alpha_k \gamma_{S,D}^z \left( 1 - e^{-\gamma_{0,S,D}^z} \right) + \sum_{k=1}^{K} \binom{K}{k} \left( -1 \right)^k \left( e^{-k \gamma_0} - 1 \right) \frac{1}{1 - k \alpha_k \gamma_{S,D}^z}. (15)
\]

Hence, by substituting (15) in (14), evaluating in closed-form the two involved integrals with the aid of [13, eq. (8.350.2)] and after some algebraic manipulations, equation (12) is deduced, which completes the proof.

It is noted here that \( \gamma_0 \) in this case can be selected in order to either achieve a specified OP, or to maximize (12), [13].

IV. NUMERICAL RESULTS

In this section, we employ the offered results in thoroughly analyzing the performance of the considered set up for different communication scenarios. To this end, Fig. 1(a) illustrates the C-OPRA policy as a function of the average SNR for different number of relays, \( K = \{1, 2, 5\} \), cut-off level, \( \gamma_0 = \{3, 7\} \) dB and SI \( \gamma_{I_k} = \gamma_I = \{-5, 10\} \) dB, along with the ideal case of \( \gamma_I = 0 \). As expected, the channel capacity per unit bandwidth improves considerably by increasing the number of relays and/or by decreasing the SI levels and the cut-off SNR levels. For example, at average SNR of 10 dB, \( \gamma_I = -5 \) dB and \( \gamma_0 = 3 \) dB, the SE improvements are: 0.2772 bits/s/Hz and 0.365 bits/s/Hz, when \( K \) changes from 1 to 2 and from 2 to 5, respectively. It is also noticed that SE deteriorates considerably as \( \gamma_I \) increases, which verifies the core necessity for effective SI cancellation methods. In addition, a SE improvement of about 0.1223bits/s/Hz is achieved for \( K = 5 \) and \( \gamma_0 = 3 \) dB at an average SNR of 10dB when \( \gamma_I \) changes from -5dB to the case of no relay SI, \( \gamma_I = 0 \). Likewise, SE of 0. 4173bits/s/Hz and 0.6623/s/Hz are achieved for \( K = \{1, 2\} \) at \( \gamma_0 = 7 \) dB when \( \gamma_I \) reduces from 10dB to the ideal case of \( \gamma_I = 0 \).

In the same context, Fig. 1(b) demonstrates the SE as a function of the number of employed relay nodes at a moderate average SNR value of 20dB for the indicative realistic cases of \( \gamma_I = \{0, 5, 10\} \) dB along with the ideal case of \( \gamma_I = 0 \) with \( \gamma_0 = \{0, 5\} \) dB. Similar behavior is in general observed, while it is interestingly shown that the capacity improvement practically saturates as the number of relay nodes increases substantially. Specifically, at \( \gamma_I = 0 \) dB and \( \gamma_0 = 5 \) dB, the SE increments are 0.611bits/s/Hz and 0.221bits/s/Hz, when \( K \) varies from \( K = 1 \) to \( K = 5 \) and from \( K = 5 \) to \( K = 10 \), respectively. Also, no particular gains are achieved for a higher number of relays as the SE increase is rather small when the number of relays is greater than ten. In addition, it is noticed that a nearly 30% capacity increase is achieved for \( K = 5 \), when \( (\gamma_I, \gamma_0) \) changes from \( (0, 5) \) dB to \( (0, 0) \) dB, which indicates that the value of \( \gamma_0 \) becomes more crucial.
than the number of employed relays, when this is moderate or large. Also, the performance of the ideal case outperforms, as expected, that of the realistic scenarios that experience the relay SI. For example, SE improvements of 0.171 bits/s/Hz, 0.326 bits/s/Hz and 0.358 bits/s/Hz are achieved, respectively, when $\gamma_l$ changes from 0 dB to the ideal case of $\gamma_l = 0$, for $K = \{1, 5, 10\}$ at $\gamma_0 = 0$ dB.

Fig. 2(a) illustrates the C-TIFR vs. the average SNR for $K = \{1, 2, 5\}$ and realistic values of $\gamma_l = \{-10, 0\}$ dB along with the ideal scenario of $\gamma_l = 0$ at $\gamma_0 = 0$ dB. Similar to the OPRA case, the achieved spectral efficiency also increases considerably as the number of relays increases, since for an average SNR of 20 dB, SE improvements of about 0.441 bits/s/Hz and 0.326 bits/s/Hz are achieved, respectively, when $\gamma_l$ changes from 0 dB to the ideal case of $\gamma_l = 0$, for $K = \{1, 5, 10\}$ at $\gamma_0 = 0$ dB. Changes from some practical values to that of ideal, no relay $K$ improvements of 0.228 bits/s/Hz is achieved at an average SNR $\gamma_l$ has considerable effect on the achieved capacity levels, which verifies the need for careful selection according to the corresponding channel capacity vs OP trade-off, in specific practical applications. This is also clearly demonstrated in Fig. 2(b), which illustrates the corresponding SE vs. $\gamma_l$ for the realistic cases of $K = \{3, 5\}$, $\gamma_l = \{0, 5\}$ dB and different average SNR values. As in the previous cases, considerable SE improvement is observed when the relay SI changes from some practical values to that of ideal, no relay SI, case. For example, at $\gamma_0 = 5$ dB, $K = 5$ and average SNR of 20 dB a SE improvement of 0.281 bits/s/Hz is achieved when $\gamma_l$ varies from 0 dB to $\gamma_l = 0$.

V. CONCLUSION

This work quantified the channel capacity under different adaptation policies for RS based FD relaying system under Rayleigh fading conditions. Novel analytic expressions were derived for the case of optimum power and rate adaptation, its optimum cut-off level for efficient adaptation of the transmit power, and the truncated channel inversion with fixed rate. It was shown that high capacity levels can be achieved with a moderate number of relays, as a notable saturation tendency was observed as the number of relays was greater than ten. Furthermore, satisfactory capacity levels are achieved at no considerable complexity increase even at moderate levels of transmit power and the induced relay self-interference, while thorough selection of the value of the involved SNR cut-off is also of considerable impact in the overall system performance. These characteristics verify that the considered set-ups are useful in demanding, energy efficient and not highly complex wireless communication scenarios that are subject to transmit power constraints or fixed rate requirements.

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REFERENCES