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Local Oscillator Phase Noise Effects on Phase Angle Component of GNSS Code Correlation

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Abstract— **This paper demonstrates the effect of radio frequency (RF) front-end (FE) free-running local oscillator (FRO) phase noise (PN) on the phase component of the Global Navigation Satellite System (GNSS) code correlation product. It is observed that as FE PN increases, it adversely affects the stability of the phase component of the code correlation. The tracking loops in baseband processing of a GNSS receiver attempt to lock on to the frequency, delay and phase of the correlation product. Until these parameters are varying within acceptable bounds, set by the dynamics handling capability of the tracking loops, the tracking loops are able to successfully track the satellite signal. However, PN increases the variation in phase of the correlation product calculated over consecutive epochs and may also cause loss of tracking lock if these variations go beyond phase locked loop (PLL) pull-in range thresholds. This paper studies the relation between FRO PN and phase component of correlation through numerical analysis, and software simulations by artificially contaminating GNSS signal stream with PN of increasing variance and checking the result on the standard deviation (SD) of the phase component of correlation product. Based on these results, this paper recommends certain maximum limits on the FE PN in order to keep the SD of phase component below the one-sigma phase error limits of the PLL used in typical GNSS tracking loops.**

Keywords-Phase noise, phase, correlation, navigation, local oscillator, Phase locked loop

I. INTRODUCTION

In [1], a relation between the FRO PN and code correlation properties was presented. Specifically, the effects on correlation magnitude losses, signal to noise ratio (SNR) and variance were considered. These effects were studied under different pre-detection integrations times (PIT). However, the effect of PN on phase component of correlation product was not discussed. It is now clear that along with magnitude, the phase information of the correlation product is also significant to estimate probability of tracking loop loss of lock. [2] gives a detailed explanation on the different tracking loop measurement errors and specifically 1-sigma tracking errors in the PLL tracking loops. It also provides rule-of-thumb tracking thresholds for these errors. However, it does not give any performance estimates of the phase component of correlation at different levels of input phase noise. Overall it has been observed that there is not much literature available on exact quantification of the harmful effects of FE phase noise on the phase information of GNSS correlation product. This paper aims to fill exactly this void. Fig. 1 shows the block diagram of a typical GNSS receiver considered for this study.

The layout of the paper is as follows: in Section II the phase noise model used for this study has been briefly described. Then a theoretical/numerical relation is made for the phase angle of correlation product in terms of the phase noise variance of a FRO. In Section III, the Matlab model that performs the correlation between phase contaminated and pure pseudorandom (PRN) codes is explained. In Section IV, the negative effects of FE PN on phase component of correlation product are described in more detail. Also, a short description of the tracking loop measurement errors in the PLL and their relation with FE phase jitter is included. In Section V, the results of the numerical and Matlab-based simulations are presented. This section also presents certain recommendations on the maximum limit of allowable front-end PN in order to maintain phase error in code correlation within acceptable threshold of the PLL. Finally, in the conclusion section, the results and key findings are summarized.

II. PHASE NOISE MODEL AND THEORETICAL ANALYSIS

This study uses the same free running oscillator phase noise model as that used in [1]. The oscillator can be represented as in (1).

$$w(t) = A(t)\cos(2\pi f_0 t + \varphi(t)) \tag{1}$$

Where phase is (φ), amplitude is (A) and frequency is (f_0). In a general case there is phase and amplitude noise, as well as distortion, which makes both A and φ functions of time. For a FRO, the overall phase noise in dBc/Hz at a certain frequency offset f_m in terms of the phase noise variance per unit time (σ_φ^2) is given by (2) [1].

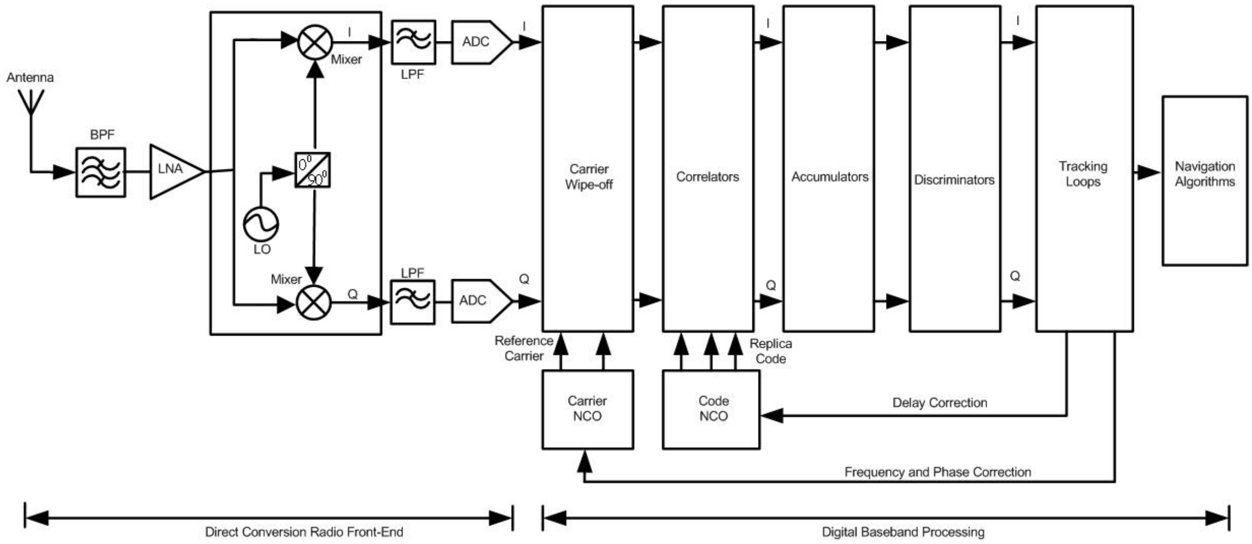

Fig. 1 Block-diagram of a GNSS direct-conversion receiver

$$L(f_m) \approx 10log\left(\frac{\sigma_\varphi^2}{(2\pi f_m)^2}\right) dBc/Hz \quad (2)$$

Where the units of σ_φ^2 are radian^2 per second (Rad^2/sec).

[1] also gives a relation for the maximum value of correlation peak in the frequency domain, which is obtained at zero time lag, as show in (3).

$$R(\tau = 0) = \frac{1}{T}\int_0^T e^{j\Phi(t)}\, dt \quad (3)$$

Where R(τ) is the correlation of the baseband version of the incoming signal with the locally-generated PRN code signal when the local oscillator is affected by phase noise. The value $\tau = 0$ corresponds to a perfect time-match between the codes of the incoming signal and the locally-generated version. T is the PIT and phase noise is represented as a complex exponential of φ(t). As mentioned in [1], the model for phase noise of a free-running oscillator is a cumulative sum of uncorrelated Gaussian random variables over the whole past history (in other words, integral of white Gaussian noise). Such cumulative sum or integral gives a process with linearly-increasing variance over time, and is thus non-stationary. However, the complex exponential of phase noise ($e^{j\varphi(t)}$), in turn is a stationary random variable and hence can be used instead of just phase noise (φ(t)) [3], [4], [5]. Equation (3) can be represented diagrammatically as in Fig. 2 meaning that code correlation peak in the presence of phase noise can be modeled as a filtered random variable ($e^{j\varphi(t)}$), passed through an integrator filter.

The goal is to find a relation for the phase angle of the correlation product. Since R(0) in (3) already represents a complex quantity, it should be enough to represent its angular component as an arctangent of the ratio of its imaginary and real components. Therefore, solving (3) is necessary.

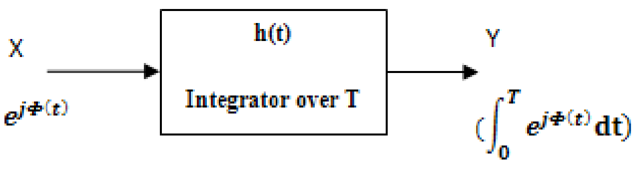

Fig. 2 Equivalent model of phase noise effect

Equation (3) can be simplified by using Euler's theorem for complex numbers.

$$R(0) = \frac{1}{T}\int_0^T (\cos(\varphi(t)) + j\sin(\varphi(t))\, dt \quad (4)$$

$$= \frac{1}{T}\int_0^T (\cos(\varphi(t))\, dt + j\,\frac{1}{T}\int_0^T (\sin(\varphi(t))\, dt \quad (5)$$

Now that R(0) is represented as a complex number in the form ($I + jQ$), the angular component can be represented as the arctangent of the ratio of imaginary component over real component.

$$angle\,[R(0)] = arctan[\frac{\frac{1}{T}\int_0^T (\sin(\varphi(t))dt}{\frac{1}{T}\int_0^T (\cos(\varphi(t))dt}] \quad (6)$$

Cancelling the common multiplier (1/T) and substituting sine and cosine terms by their Taylor series expansions (in terms of φ(t) upto seven co-efficients) gives (7):

$$= arctan[\frac{\int_0^T (\frac{\varphi(t)^1}{1!} - \frac{\varphi(t)^3}{3!} + \frac{\varphi(t)^5}{5!} - \frac{\varphi(t)^7}{7!} + \frac{\varphi(t)^9}{9!} - \frac{\varphi(t)^{11}}{11!} + \frac{\varphi(t)^{13}}{13!})dt}{\int_0^T (1 - \frac{\varphi(t)^2}{2!} + \frac{\varphi(t)^4}{4!} - \frac{\varphi(t)^6}{6!} + \frac{\varphi(t)^8}{8!} - \frac{\varphi(t)^{10}}{10!} + \frac{\varphi(t)^{12}}{12!} - \frac{\varphi(t)^{14}}{14!})dt}] \quad (7)$$

φ(t) being a Gaussian random variable (grv) (since phase noise of a free-running oscillator is a grv), it is not trivial to solve (7) further to obtain a closed-form theoretical solution. Another possibility is to plot the results for angle[R(0)] using numerical analysis of (7). For this, the time domain has to be discretized so that the continuous integral will transform into a summation over the integration interval (T) as shown in (8). The numerical analysis of (8) is further described in Section III.

$$= arctan[\frac{\sum_0^T (\frac{\varphi(t)^1}{1!} - \frac{\varphi(t)^3}{3!} + \frac{\varphi(t)^5}{5!} - \frac{\varphi(t)^7}{7!} + \frac{\varphi(t)^9}{9!} - \frac{\varphi(t)^{11}}{11!} + \frac{\varphi(t)^{13}}{13!})}{\sum_0^T (1 - \frac{\varphi(t)^2}{2!} + \frac{\varphi(t)^4}{4!} - \frac{\varphi(t)^6}{6!} + \frac{\varphi(t)^8}{8!} - \frac{\varphi(t)^{10}}{10!} + \frac{\varphi(t)^{12}}{12!} - \frac{\varphi(t)^{14}}{14!})}] \quad (8)$$

It is possible to represent the relation using numerical and software simulations since the accuracy of (3) has already been proven theoretically in [1] while studying the effect of PN on magnitude of the correlation peak.

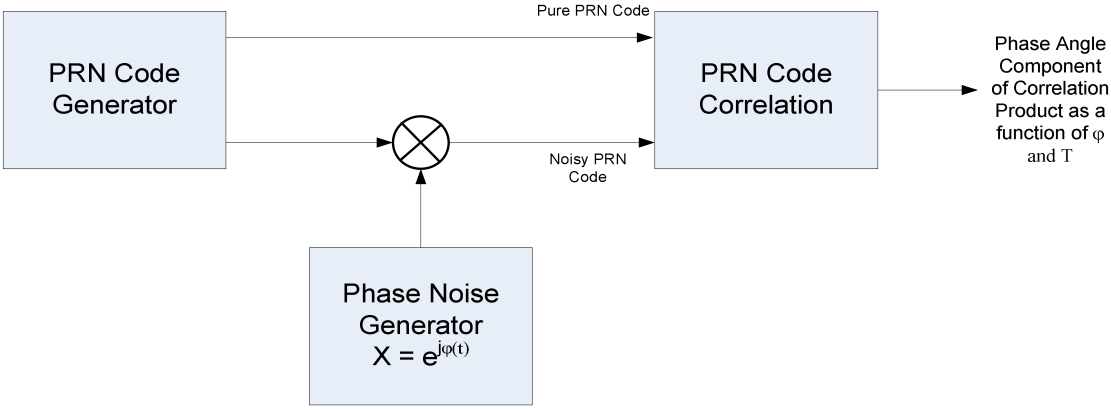

Fig. 3 Matlab model for PRN code correlation

III. Simulation Stage

A Matlab based program was generated which performed correlation between two versions of the same GPS PRN code. The first version was contaminated with different amounts of FRO phase noise in order to replicate a real world PRN code received from the RF front-end and after the carrier strip-off process. The other version of the PRN code was kept 'pure' to mimic the local replica code as generated in every GNSS receiver. The Matlab model is diagrammatically represented in Fig. 3.

The user may select the satellite vehicle number whose PRN code is to be generated. The free running oscillator phase noise was defined in terms of unit time variance (σ_{Φ}^2). Therefore, the total phase noise variance per chip of the PRN code over the coherent integration period (T) is given by (9):

$$PN_{Variance} = \frac{T\sigma_{\Phi}^2}{L_{PRN}} \quad (9)$$

Where, L_{PRN} equals to the length of the PRN code. The multiplication of the phase noise with incoming PRN code can be simulated in Matlab as a multiplication of phase noise vector and PRN code vector. A random number 'noise' vector (φ(t)) with zero mean, unity standard deviation and length of 1023 bits was created and its variance was changed to the required phase noise variance using $PN_{Variance}$ calculated in (9). The overall phase (φ_{k+1}) can be generated as $\varphi_k + w_k$, where k=0,1,2,..., and w_k is a white Gaussian zero-mean sequence with variance σ^2, which is given by $\sigma_{\varphi}^2.T_c$, where T_c is one chip duration.

Now that we have the phase noise vector of the desired length and noise variance, it can be multiplied with a PRN code vector in order to produce a noisy PRN code similar to the one obtained from the RF front-end in a real world GNSS receiver. Correlation of this noisy code is performed with a 'pure' PRN code over multiple iterations and the angular component of correlation result is stored. Once all iterations for one value of phase noise variance are complete, it is possible to calculate the SD of the angular component for the present value of σ_{Φ}^2. This SD represents the 1-sigma error due to phase jitter in the GNSS baseband tracking loops. σ_{Φ}^2 is varied from 0 Rad^2/s to 10^4 Rad^2/s on a logarithmic scale and the (SD) of angle of correlation peak is plotted over this range.

Numerical analysis using (8) is performed by using the same phase noise vector φ(t) as that used in the simulation set-up described above. Using this vector the Taylor series components are created as in (8) and the numerator and denominator summation terms are generated. Since the length of φ(t) vector is already scaled by integration time T, summation over T is equivalent to performing a cumulative sum of all elements of the resultant vectors inside the numerator and denominator summations. Next, the arctangent of the ratio of these summation terms gives the angular component of the correlation peak for that epoch. After calculating the angular component over multiple epochs, the standard deviation is calculated and plotted for every value of input phase noise.

IV. Phase noise effects on phase of correlation

Figs. 4, 5, and 6 help demonstrate the negative effect of FE PN on phase of correlation peak. In Fig. 4 the instantaneous phase value of correlation peak is plotted over 200 consecutive epochs of code correlation for small and large phase noise variance per unit time values of 10 $rad^{2/}$sec and 10000 $rad^{2/}$sec, respectively. The phase variations between consecutive epochs are much lower when PN from front-end is low. The limited variation in phase can be better observed in the polar I/Q plot of Fig. 5. Therefore, if such a correlation product is fed to the tracking loop PLL, it will be able to maintain lock as the phase variations between consecutive epochs may be within its pull-in range. When the FE phase noise is increased, the correlation peak phase variations are distinctly higher as shown in Fig. 6. In case such a signal is fed to the tracking loop PLL, it would not be able to track such random and huge changes in phase over consecutive epochs. Comparing polar plots of Figs. 5 and 6, it is noticed that the amplitude of the correlation results is degraded when the phase noise is higher. It proves that the front-end phase noise has an adverse effect to the correlation peak magnitude [1].

The real parameter of interest is the 1-sigma tracking loop measurement error, in degrees, for the PLL. Equations (10) and (11) give the rule-of-thumb threshold for this error for a PLL considering navigation data-less & with-data signals respectively [2].

$$\sigma_{PLL(data-less)} = \sqrt{\sigma_{tPLL}^2 + \sigma_v^2 + \theta_A^2} + \frac{\theta_e}{3} \leq 30° \quad (10)$$

$$\sigma_{PLL(with-data)} = \sqrt{\sigma_{tPLL}^2 + \sigma_v^2 + \theta_A^2} + \frac{\theta_e}{3} \leq 15° \quad (11)$$

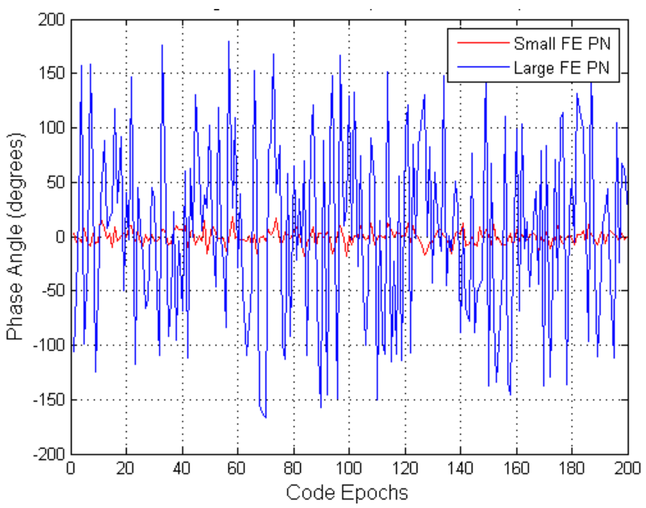

Fig. 4 Phase angle of correlation peak over 200 consecutive epochs for small and large FE PN

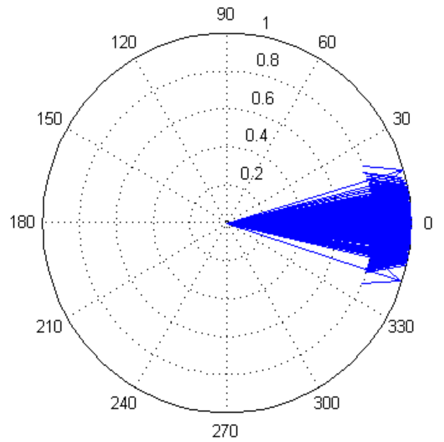

Fig. 5 Polar I/Q plot of the correlation peak over 200 consecutive epochs for small FE PN

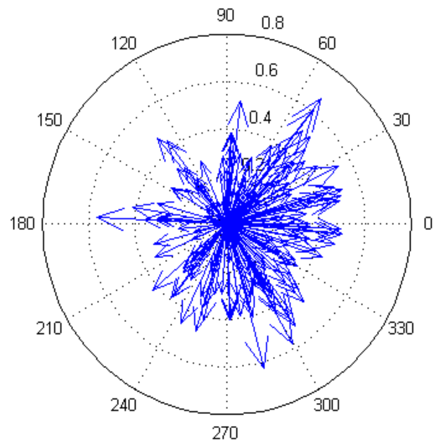

Fig. 6 Polar I/Q plot of the correlation peak over 200 consecutive epochs for high FE PN

Where, σ_{tPLL} is the 1-sigma PLL thermal noise in degrees, σ_v is the 1-sigma vibration induced oscillator jitter in degrees, θ_A= Allan variance induced oscillator jitter and $\theta_e$is dynamic stress error. For simplicity if we assume typical values for σ_v = 1.4 degrees and θ_A = 1.4 degrees and θ_e = 15 degrees, we can derive rule-of-thumb thresholds for 1-sigma PLL thermal noise in degrees so that overall PLL noise remains below 30

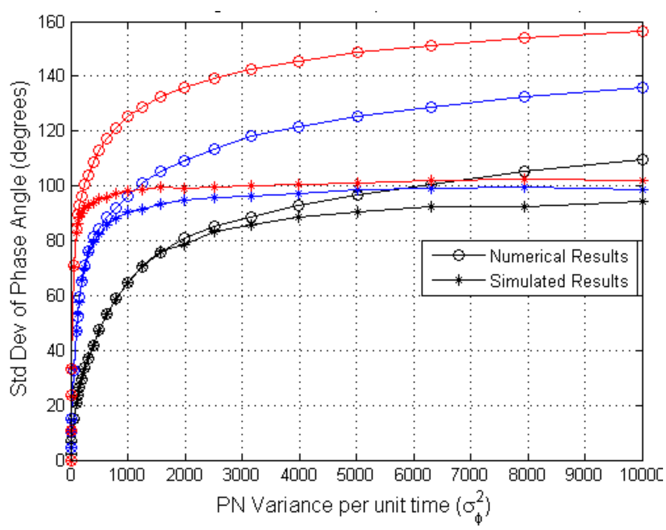

Fig. 7 Linear plot of standard deviation of phase component of correlation peak versus PN variance per unit time for different PIT values

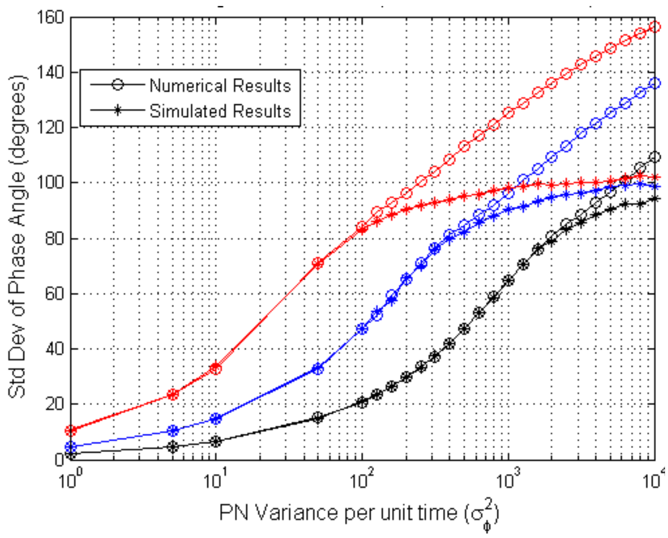

Fig. 8 Logarithmic plot of standard deviation of phase component of correlation peak versus PN variance per unit time

degrees (data-less) as in (12) and below 15 degrees (with-data) as in (13).

$$\sigma_{tPLL(data-less)} \leq 10° \quad (12)$$

$$\sigma_{tPLL(with-data)} \leq 25° \quad (13)$$

Therefore, now that we know the maximum allowed phase error for a PLL to be able to keep track, we can easily locate the front-end PN at which the SD of phase component of correlation (which is equivalent to the 1-sigma PLL thermal noise phase error σ_{tPLL}) increases beyond 10 degrees.

V. RESULTS AND MATHEMATICAL ANALYSIS

This section gives details of the results obtained from software simulation of code correlation by correlating two versions of the same PRN code: one contaminated with phase noise and the other in its original uncontaminated state. The phase angle of correlation product is of focus and more specifically, the SD of this angular component over multiple consecutive code epochs. Fig. 7 shows the SD of phase component versus phase noise variance per unit time for three

TABLE 1. Maximum FE PN to maintain Std. Dev. of phase angle below 10°

(Navigation data is present)

PIT (msec)	Maximum Phase Noise (dBc/Hz)	
	f_m = 10KHz	f_m = 1MHz
4	-83	-123
20	-89	-129
100	-96	-136

TABLE 2. Maximum FE PN to maintain Std. Dev. of phase angle below 25°

(Navigation data is absent)

PIT (msec)	Maximum Phase Noise (dBc/Hz)	
	f_m = 10KHz	f_m = 1MHz
4	-74	-114
20	-82	-122
100	-88	-128

different PIT values: 4 msec, 20 msec and 100 msec. Fig. 8 shows the same curves but now the noise variances (x-axis) is plotted on a logarithmic scale to enlarge the effects at lower values of phase noise. This is because the phase angle deviations already increase well beyond 10 degrees at quite low values of phase noise. The figures show that the phase SD increases with increasing phase noise, until it saturates at around 100 degrees. Further increase in phase noise has no effect on the phase variations. The figures also show the effect of increasing PIT on the phase errors. Greater the value of PIT, greater is the SD of phase for the same amount of FE PN.

Tables 1 and 2, show the maximum front-end phase noise (in dBc/Hz at frequency offsets of 10 KHz and 1 MHz), in order to maintain SD of phase angle below 10 degrees and 25 degrees respectively. The results show that, for 4 msec PIT, to maintain the phase angle SD of 10 degrees (navigation data present), maximum FE PN allowed (at 1 MHz offset) is -123 dBc/Hz. But if the PIT is increased to 20 msec, maximum PN requirements become more stringent by around 6 dB. If the phase noise were measured at 10 KHz offset, maximum PN requirements are scaled by around 40 dBs. If the maximum allowed PLL phase error is 25 degrees (navigation data absent), the phase noise requirements can be relaxed by 7-9 dB for each of the PIT values respectively. One can also see that the results for maximum FE PN obtained in this study are comparable to the values obtained in the initial study performed in [1], where effect of PN on magnitude, SNR and variance of correlation product were studied.

The results obtained from the numerical analysis match very closely to the simulated curves for most values of phase noise. At higher noise levels however, the numerical results continue to degrade, thus diverging from the simulated results as they saturate around 100 degrees (after all, the Taylor series expansion is an approximation of sine and cosine terms). This also proves that the results obtained are theoretically, numerically, and technically sound.

One point to remember is that the PN source used for this study is a free-running local oscillator. Noise performance of such a device is quite poor. In practical receiver front-ends there may be a free-running local oscillator to heterodyne the receiver signal from RF to baseband and the resulting complex signal will be tracked by a phase-locked loop which tries to follow the phase noise. This tracking is successful at least for the noise components inside the PLL bandwidth. But the noise components with higher frequencies cannot be tracked conveniently and may lead to loss of phase lock. Nevertheless, the overall noise performance of such a device is far superior and hence the threshold for maximum FE PN to maintain thermal noise PLL error below 10 degrees or 25 degrees would be much higher. Further studies on comparing the effects of free-running oscillator and realistic frequency sources on correlation parameters are currently being performed in our Department.

CONCLUSION

The standard deviation of phase angle component of correlation is the same as the 1-sigma thermal phase error of the PLL in the GNSS baseband tracking loops. The current paper presented an initial approximation of the effects of FE PN on this phase error of the PLL. Simulated correlation results are supported by theory and numerical methods. The results of this study can be used by designers of RF front-end local oscillators as it establishes a conservative upper bound on the phase noise originating from these devices in order to maintain the phase error in the baseband tracking loops below a certain threshold under specific conditions of coherent integration periods. The results obtained are comparable to those obtained in an earlier study on the effects of FE PN on magnitude losses and SNR of correlation peak, and hence can be considered an alternative method of determining the maximum front-end phase noise in a GNSS receiver. Future work could include the study of PN originating from more realistic frequency sources, for example, PLL in the RF FE.

ACKNOWLEDGMENT

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