

A new paradigm for fatigue analysis - evolution equation based continuum approach

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Summary. A very general continuum based approach to model both low- and high cycle fatigue behaviour is described. The approach allows for both isotropic and anisotropic properties under very general random multiaxial loading histories.

Key words: high-cycle fatigue, low-cycle-fatigue, endurance surface, out-of-phase loading, multiaxial stress state, stochastic loading, gradient effects

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Introduction

In mechanical engineering design when dimensioning products, fatigue is often the most critical issue. Fatigue of materials under variable loads is a complicated physical process which is characterized by nucleation, coalescence and stable growth of cracks. Nucleation of cracks starts from stress concentrations near persistent slip bands, grain interfaces and inclusions [1, 12, 16, 17]. Depending on the intensity of loading two ranges of fatigue lives are identified, namely the low- and high-cycle regime, abbreviated as LCF and HCF, respectively. However, in recent years, it has been observed that fatigue failures can also occur at very high fatigue lives $10^9 - 10^{10}$, below the previously assumed fatigue limits for infinite life. The key difference between LCF and HCF behaviour is that in high-cycle fatigue the macroscopic behaviour of the material is primarily elastic, while in the low-cycle fatigue regime considerable macroscopic plastic deformations take place. This fact can be effectively utilized in the analysis. Transition between low- and high-cycle fatigue for metallic materials occurs between $10^3 - 10^4$ cycles.

This paper describes in general terms a unified approach to model both low- and high-cycle fatigue. The high-cycle fatigue part of the model is based on the concept of a moving endurance surface in the stress space with an associated evolving scalar damage variable. In this concept, originally proposed by Ottosen et al. [13] the movement of the endurance surface, as a function of the stress history, is tracked by an evolving back stress type of stress tensor. Therefore this model avoids the ambiguous cycle-counting techniques. This

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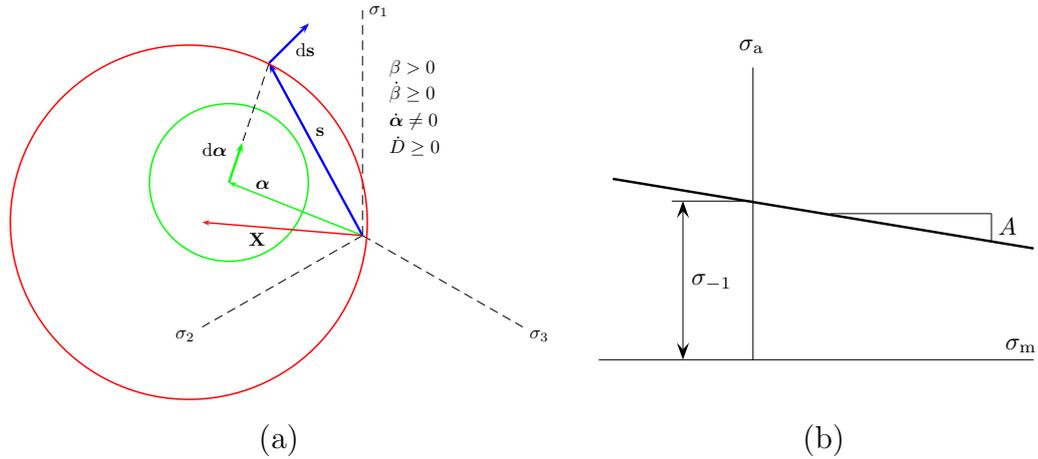


Figure 1. (a) Endurance surface (green line) inside the yield surface (red line), shown here in the deviatoric plane, moves in the stress space and fatigue damage increases when the stress increment is pointing away from the endurance surface. (b) Haigh-diagram.

approach allows consistent generalisation to anisotropic cases. A transversally isotropic version of this model was developed by Holopainen et al.[3] and an initial approach to model both LCF- and HCF-regimes is described in [10, 5], also polymer fatigue has been investigated [4].

One of the key deficiencies of most fatigue models is that under complex load histories, cycle-counting methods need to be applied. These methods require definition of a representative uniaxial cycle from which a cumulative damage theory is applied to evaluate the accumulated damage. For complex load histories and general stress states these kind of approaches become increasingly speculative. Reviews and comparisons of different high-cycle fatigue models can be found in studies by [15], [2], [9], and [11].

Basic principle

Three key ingredients of the evolution equation based fatigue model are introduced: (i) the existence of an endurance surface, (ii) evolution equations for its movement and (iii) damage accumulation. The endurance surface β is defined in the stress space

$$\beta(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\}; \text{parameters}) = 0, \quad (1)$$

where $\boldsymbol{\sigma}$ is the stress tensor and $\{\boldsymbol{\alpha}\}$ denotes the set of internal variables. Evolution of the internal variables and the damage D are described by the rate equations

$$\{\dot{\boldsymbol{\alpha}}\} = \{\mathbf{G}\}(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\})\dot{\beta}, \quad \text{and} \quad \dot{D} = g(\beta, D, \boldsymbol{\varepsilon}^p)\dot{\beta}. \quad (2)$$

In high-cycle fatigue there is no macroscopic plastic straining, thus $\boldsymbol{\varepsilon}^p$ vanish. The form of the functions \mathbf{G} and g are important for modelling the finite life durability, while the endurance surface mainly dictates the infinite life resistance. In contrast to plasticity the stress state can lie outside the endurance surface and the evolution of the internal variables and the damage take place only when $\beta \geq 0$ and $\dot{\beta} > 0$. The idea of the moving endurance surface is depicted in Fig. 1a in the deviatoric plane.

In the high-cycle fatigue regime, material damage is highly localized and its effect to the macroscopic structural response can be neglected. This fact uncouples the “fictitious”

fatigue damage from the structural constitutive equations, thus facilitating HCF-analysis as a post-processing from the structural analysis data. This is also true in the LCF/HCF regime, if we assume that the “fatigue damage” does not couple to the constitutive equation. In this case the post-processing, however, requires evaluation of elasto-plastic response. In addition, the plastic region should be small as compared to the measures of the analysed structure, to justify the fatigue post-processing.

For the endurance surface, the simplest form for isotropic HCF-modelling can be written as

$$\beta(\boldsymbol{\sigma}, \boldsymbol{\alpha}; A, \sigma_{-1}) = \frac{1}{\sigma_{-1}}(\bar{\sigma}_{\text{eff}} + AI_1 - \sigma_{-1}) \quad (3)$$

where σ_{-1} is the fatigue strength for fully reversed uniaxial normal stress loading ($R = -1$), $\bar{\sigma}_{\text{eff}} = \sqrt{3\bar{J}_2}$ is the reduced effective stress, $I_1 = \text{tr } \boldsymbol{\sigma}$ the first invariant of the stress tensor and A is a constant. The second deviatoric invariant is defined as $\bar{J}_2 = \frac{1}{2} \text{tr}[(\boldsymbol{s} - \boldsymbol{\alpha})^2]$, where $\boldsymbol{s} = \boldsymbol{\sigma} - \frac{1}{3}I_1\boldsymbol{I}$ is the deviatoric stress tensor. This form results for uniaxial cyclic loading in a linear relationship between the mean stress and the fatigue strength amplitude, see Fig. 1b and Section 4 in [13] for a detailed analysis. In addition, it allows for closed form expressions for the parameters appearing in the endurance surface as a function of experimental fatigue strengths.

For anisotropic fatigue the effective stress depend on the structural tensors describing the anisotropy. Transverse isotropic HCF model is described in [3] and extension to orthotropic symmetry in [6]. It has been successfully utilized in the design of a telescopic boom structure [7].

Concluding remarks

An evolution equation based fatigue modelling concept which can be used under general multiaxial irregular stress histories and is amenable to consistent extension to anisotropy, unification of low-cycle and high-cycle regimes and is ideally suited for stochastic analysis [8] is briefly presented. In addition, the size effect influenced by stress gradients can also be included in this model [14].

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