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Parametrization and Prediction of EGNOS GIVD values

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Abstract—Global Navigation Satellite Systems (GNSS) enable positioning almost everywhere in the world under open-sky conditions. However, the GNSS-based position is not exact due to various error sources. Errors in range measurements caused by the ionospheric delay are the largest error source in positioning with consumer grade receivers. In this paper we propose an approach for predicting the ionospheric delay for such receivers. Our method models the ionosphere by a predefined set of trigonometric basis functions, whose coefficient are predicted using a Kalman filter (KF). For the KF we introduce a new, Klobuchar-based state model, which outperforms a standard, random walk state model in our tests with real-world data for various filter update intervals. Our tests show, furthermore, that European Geostationary Navigation Overlay Service (EGNOS) transmissions can be parameterized and predicted without significant information loss, which reduces the amount of data that has to be transmitted to the receiver.

Index Terms—Ionospheric delay, SBAS, EGNOS, GIVD, GNSS

I. INTRODUCTION

Location-aware applications have become an essential part of our everyday lives. Global Navigation Satellite Systems (GNSS), such as the Global Positioning System (GPS), have been available for the public since the start of this century. Standard commercial GNSS receivers enable location estimates that are accurate within a few meters, which is sufficient for applications such as geotagging one’s holiday pictures. However, for other applications such as surveying or air traffic control, centimetre level accuracy of location estimates is required. This accuracy can be achieved by professional GNSS receivers, but they are too expensive for usage in, for example, mobile phones and car navigation devices.

Professional receivers are more accurate mainly because they account for more GNSS error sources than standard (single-frequency) commercial receivers. The error sources include but are not limited to estimation errors in satellite positions, instrumental errors caused by the equipment, multipath errors in, for example, urban environments, and errors caused by the atmosphere. For example, dual-frequency GNSS receivers can remove the location error caused by ionospheric delay (see e.g. [1, p. 165]).

The ionosphere is located in the Earth’s upper atmosphere stretching from around 60 to 1000 km altitude and contains electrons and electrically charged atoms and molecules [2, p. 82]. Because the radio signal transmitted by GNSS satellites is delayed by traversing the ionosphere, the resulting error in the measurement of range, which is the distance between satellite and user equipment (UE), is called ionospheric delay. It is, in general, the largest error source in standard positioning [3, p. 322]. In challenging environments such as urban canyons multipath can cause larger errors than the ionospheric delay [3, pp. 285 ff.].

If the delay is known it can be subtracted from the range measurement, resulting in a significantly improved location estimate. The delay can be estimated using a mathematical model. For example, the parameters of the Klobuchar model [4] are transmitted in the GPS satellite navigation message. The model can remove approximately 50% of the ionospheric error in range measurements [4]. The satellites of the European Galileo navigation system transmit parameters for the NeQuick G model (see [5] for details), which is an electron density model that can remove about 70% of the ionospheric error.

Another approach to correct for ionospheric delays is the use of Satellite Based Augmentation Systems (SBAS), such as the European Geostationary Navigation Overlay Service (EGNOS). These systems observe the ionosphere over its whole height (from 60 to 1000 km) but approximate it as a thin shell at 350 km altitude in a set of grid points. In each grid point the Grid Ionospheric Vertical Delay (GIVD) and the Grid Ionospheric Vertical Error (GIVE) are calculated, which are transmitted in the navigation message of the systems’ geostationary satellites. Alternatively EGNOS transmissions are available over Internet via EGNOS Data Access Service (EDAS), and are sent every few seconds. SBAS transmissions enable precise estimation of the ionospheric delay, but require the UE to have either a special receiver [3, p. 439] or a high speed Internet connection.

If the UE is a smartphone with a standard commercial GNSS receiver and data transfer over the internet is limited (in speed or in amount of data per month) then other ways of estimating the ionospheric delay have to be used. In this paper we propose an approach in which we use EGNOS transmissions that are received only every few minutes to predict the ionospheric delay. Thereby significantly less data...
has to be transmitted to the UE. In addition, the approach’s computational requirements are lower compared with NeQuick G.

We model the ionosphere’s state by a predefined set of basis functions whose coefficients, the so-called parametrization of the ionosphere, we predict using a standard Kalman filter (KF). For the KF we develop a new, Klobuchar-based state model, which we test and compare to a random walk state model. We then compare our results with the results obtained for the Klobuchar and the NeQuick G models.

This paper is organized as follows. In Section II the theoretical background of our work is presented. Section III describes the trigonometric basis functions used for parameterizing EGNOS GIVD values, and the used Kalman filters and their state models. Test results for our parameterization approach using real-world EGNOS data and various filter update intervals are presented in Section IV. Section V draws conclusions and gives an outlook.

Notation: \( x \) and \( x_{1:d} \) denote column vectors and \( H \) denotes a matrix.

II. THEORETICAL BACKGROUND

In this section we briefly describe the theoretical background of this work.

A. Total Electron Content of the Ionosphere

A signal from a GNSS satellite is delayed in the ionosphere, as the propagation speed depends on the electron density along the ray path. The delay makes the measured distance from the receiver to the satellite appear longer than it actually is. The electron density along the ray path is referred to as Total Electron Content (TEC) and is defined as the number of electrons/m\(^2\) along the ray path. The unit of TEC is electrons/m\(^2\), although it is often expressed as TEC units (TECu), where 1 TECu is \( 10^{16} \) electrons/m\(^2\) [3, p. 312].

If the TEC along the ray path is known, the delay in meters can be computed as

\[
\Delta s = \frac{40.3}{f^2} \text{TEC},
\]

where \( f \) is the frequency of the transmitted signal in Hz. See [3, p. 310-312] for a derivation of (1).

B. Satellite Based Augmentation Systems

Satellite Based Augmentation Systems (SBAS), such as EGNOS, transmit ionospheric correction information GIVD (in meters) and GIVE for certain predefined locations. The SBAS message of type 18 contains the locations of the Ionospheric Grid Points (IGP), whereas the message of type 26 contains the GIVD and GIVE. The IGPs are located at a \( 5^\circ \times 5^\circ \) grid for most parts of the Earth, although in the polar regions the grid is not as dense because the meridians are spaced more densely than in regions closer to the Equator [6]. IVD values between IGPs are computed using a weighted interpolation scheme described for example in [3, p. 439-441]. Fig. 1 shows the IGPs covered in EGNOS transmissions. The solid black dots denote the IGPs in central European area within which we do the comparisons. This is the area where the corrections are available most of the time.

C. Kalman filter

In this work we use a standard Kalman filter for prediction. The Kalman filter is the closed form solution to the Bayesian filtering equations where the dynamic model and measurement model

\[
x_k = A_{k-1}x_{k-1} + q_{k-1} \quad (2a)
\]

\[
y_k = H_kx_k + r_k \quad (2b)
\]

are linear Gaussian. Here \( x_k \) and \( y_k \) are respectively the state and the measurement vectors at time instant \( t_k \) and \( q_{k-1} \sim N(0, Q_{k-1}) \) and \( r_k \sim N(0, R_k) \) are normally distributed process and measurement noise, respectively. Matrices \( A_{k-1} \) and \( H_k \) are respectively the transition matrix of the dynamic model and the measurement model matrix. The pseudocode and derivation of the Kalman filter algorithm are given in [7, p. 56-58]. The initial state and state covariance are estimated using the Expectation Maximization (EM) algorithm described in [7, p. 191], which uses a Kalman filter and a Rauch-Tung-Striebel smoother in an iterative manner to find the maximum likelihood estimates of the model parameters.

III. PREDICTION OF THE GIVD PARAMETRIZATION

In this section we describe the trigonometric basis functions that we use for parameterizing EGNOS GIVD values, and our Kalman filter implementation, for which we use a random walk state model and a Klobuchar-based state model.

A. Description of basis functions

We use trigonometric basis functions for the EGNOS GIVD value parametrization. More detailed description of this approach can be found in [8]. The trigonometric basis functions are defined as a Cartesian product of two sets of functions. The first set is defined as

\[
X = \{1, \sin(\pi x_{\text{lon}}), \cos(\pi x_{\text{lon}}), \sin(2\pi x_{\text{lon}}), \cos(2\pi x_{\text{lon}}), \sin(3\pi x_{\text{lon}}), \cos(3\pi x_{\text{lon}})\}. \tag{3}
\]

Fig. 1. IGPs covered in EGNOS transmissions. Solid black dots denote the IGPs in central European area, within which we do the comparisons.
where \( x_{\text{lon}} = \frac{\lambda \times 30}{360} \) with \( \lambda \) being the longitude value in degrees of an IGP, \(-30 \leq \lambda \leq 50\). The IGPs outside these limits are excluded from the computations.

The second set is defined as
\[
\mathcal{Y} = \{ 1, \sin(\pi y_{\text{lat}}), \cos(\pi y_{\text{lat}}), \sin(2\pi y_{\text{lat}}), \cos(2\pi y_{\text{lat}}) \},
\]
where \( y_{\text{lat}} = \frac{\varphi \times 30}{360} \) with \( \varphi \) being the latitude value in degrees of an IGP, \(30 \leq \varphi \leq 80\). The scaling factors are defined by the limits of an area in which most EGNOS GIVD values are available most of the time.

Now, the set of 35 trigonometric basis functions used in the parametrization of the EGNOS GIVD values is
\[
\phi = \mathcal{X} \times \mathcal{Y} = \{ fg : f \in \mathcal{X}, \; g \in \mathcal{Y} \}.
\]

B. Kalman filter implementation

1) Measurement model: The measurement model in the Kalman filter depends on the chosen set of basis functions, which are here referred to as \( \phi_i(\lambda_{j,k}, \varphi_{j,k}) \), where \( \phi_i \) is the \( i \)th basis function. Variables \( \lambda_{j,k} \) and \( \varphi_{j,k} \) are respectively the longitude and the latitude of the location of the \( j \)th measurement at time instant \( t_k \).

Now the measurement model matrix \( \mathbf{H}_k \) is
\[
\mathbf{H}_k = \begin{bmatrix}
\phi_1(\lambda_{1,k}, \varphi_{1,k}) & \cdots & \phi_m(\lambda_{1,k}, \varphi_{1,k}) \\
\vdots & \ddots & \vdots \\
\phi_1(\lambda_{n_k,k}, \varphi_{n_k,k}) & \cdots & \phi_m(\lambda_{n_k,k}, \varphi_{n_k,k}) \end{bmatrix},
\]
where \( m \) is the number of basis functions and \( n_k \) is the number of measurements at \( t_k \). Based on (2) the measurement equation at time instant \( t_k \) is now
\[
y_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{r}_k, \tag{7}
\]
where \( y_k \) is a vector containing the EGNOS GIVD values transmitted at \( t_k \) and state \( \mathbf{x}_k \) is the parametrization of the EGNOS GIVD values at \( t_k \). The last component of the state \( \mathbf{x}_k \) is the parameter (the speed of the parameters) of the state transition model.

The EGNOS transmissions contain error estimates, GIVE, for each of the GIVD values. Therefore, determining the measurement noise matrix \( \mathbf{R}_k \) in (2) is straightforward. For each measurement \( y_k = [y_{1,k} \ldots y_{n_k,k}]^T \) containing GIVD values at \( n_k \) IGPs there are \( n_k \) variance values \( \sigma_{1,k}^2 \ldots \sigma_{n_k,k}^2 \). Thus, assuming the measurements at different grid points are uncorrelated,
\[
\mathbf{R}_k = \text{diag}(\sigma_{1,k}^2, \ldots, \sigma_{n_k,k}^2)
\]
\[
= \begin{bmatrix}
\sigma_{1,k}^2 & 0 & \cdots & 0 \\
0 & \sigma_{2,k}^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_{n_k,k}^2
\end{bmatrix},
\tag{8}
\]
and the measurement error in (7) is \( \mathbf{r}_k \sim \mathcal{N}(0, \mathbf{R}_k) \).

2) State transition models: In this paper we use two different state transition models. One of them is a standard random walk state model, which we will compare against a novel Klobuchar-based state model, which was introduced in [8].

The random walk state transition matrix \( \mathbf{A}_{\text{RW},k} \) at time instant \( t_k \) for estimating the \( m + 1 \) parameters is
\[
\mathbf{A}_{\text{RW},k} = \begin{bmatrix}
I_{m \times m} & \Delta_k \\
0 & 1
\end{bmatrix}, \tag{9}
\]
where \( \Delta_k = \begin{bmatrix} \Delta_k[1 \ldots 1]^T \end{bmatrix}_m \) and \( \Delta_k = t_k - t_{k-1} \) is the difference of two consecutive time instants.

For the Klobuchar-based state model a quantity called the Klobuchar difference, \( \tilde{\Delta}_k(t_k, \Delta_k, \lambda, \varphi) \), has been derived in [8]. It defines the change in the vertical time delay at a certain location defined by longitude \( \lambda \) and latitude \( \varphi \), when the time changes from \( t_k \) to \( t_k + \Delta_k \). By multiplying the time delay with the speed of light \( c \) we obtain the vertical delay in meters. The Klobuchar difference is used to adjust the parameters estimated in the Kalman filter into the direction of the prediction given by the Klobuchar model. This way we can add a rough, computationally simple estimate of the physical phenomena in the ionosphere into the state transition matrix. This state model requires the knowledge of the Klobuchar parameters \( \alpha = [\alpha_0 \ldots \alpha_3] \) and \( \beta = [\beta_0 \ldots \beta_3] \), which are included in the GPS satellites’ navigation message and are therefore available in GPS-based positioning.

The Klobuchar-based state transition matrix \( \mathbf{A}_{\text{KB},k} \) is
\[
\mathbf{A}_{\text{KB},k} = \begin{bmatrix}
I_{m \times m} & \kappa_k \\
0 & 1
\end{bmatrix}, \tag{10}
\]
where
\[
\kappa_k = \begin{bmatrix}
\kappa_{1,k} & \cdots & \kappa_{m,k}
\end{bmatrix}^T = \left( \mathbf{H}_k^T \mathbf{H}_k \right)^{-1} \mathbf{H}_k^T \begin{bmatrix}
\tilde{c}_{\text{vert}}(t_k, \Delta_k, \lambda_{1,k}, \varphi_{1,k}) \\
\vdots \\
\tilde{c}_{\text{vert}}(t_k, \Delta_k, \lambda_{n_k,k}, \varphi_{n_k,k})
\end{bmatrix}, \tag{11}
\]
is the Klobuchar difference at the time instant \( t_k \) converted to the parameter space.

Based on (2) the state at the time instant \( t_k \) is
\[
\mathbf{x}_k = \mathbf{A}_{\text{XX},k-1} \mathbf{x}_{k-1} + \mathbf{q}_{k-1}, \tag{12}
\]
where \( \mathbf{A}_{\text{XX}} \) is either the random walk state transition matrix \( \mathbf{A}_{\text{RW}} \) or the Klobuchar-based state transition matrix \( \mathbf{A}_{\text{KB}} \). The state noise \( \mathbf{q}_k \sim \mathcal{N}(0, \mathbf{Q}_k) \).

We estimate the initial state mean and covariance, as well as the state noise matrix \( \mathbf{Q}_k \) with an EM algorithm that uses a Kalman filter and a Rauch-Tung-Striebel (RTS) smoother. The algorithm is described in [7, p. 191]. For the initial state estimation we use the 5 minutes of EGNOS data that are transmitted just before the prediction starts.
IV. RESULTS

We test our prediction of the parametrization by trigonometric basis functions using both the random walk and the Klobuchar-based state models. The tests were implemented using MATLAB, and all tests were done off-line. For our test we use EGNOS transmissions from 1st to 7th of March 2015. This period contains days ranging from low to high solar activity, with emphasis on low to moderate activity [9]. We test the predictions using 5, 10 and 30 minute filter update intervals. The predictions are started at a 5 minute interval such that the EGNOS transmissions used in the initial state estimation for each prediction test run do not overlap. The IVD values are computed at each IGP within the predefined central European area based on the predicted parameters. The obtained IVD values of each time instant are compared to the GIVD values in EGNOS transmissions valid at that time, and the prediction error is the absolute difference of the predicted and transmitted IVD value. The errors are computed at a 1 minute interval for the prediction tests using a 5 minute filter update interval and at a 2 minute interval for the prediction tests using 10 and 30 minute filter update intervals. The predictions are continued for a period of three hours each. The presented error quantiles are obtained by combining the results of all predictions done using the data from the given time period.

Fig. 2 displays the 68% error quantiles for the prediction results for both Klobuchar-based and random walk state models using a 5 minute update interval in the filter. It can be seen that after an initialization period of a few filter updates the prediction of the parametrization using the Klobuchar-based state model is yielding better prediction results than the standard random walk state model. Similar results are obtained for predictions with a 10 minute update interval (see Fig. 3), although the difference between the two state models is less significant. In addition, the overall filtering error grows as the update interval lengthens. Fig. 4 shows that for a 30 minute update interval there is no significant difference between Klobuchar-based and random walk state models anymore, except for the previously mentioned initialization period.

For comparison the 68% error quantiles for IVD values based on the Klobuchar and the NeQuick G models with respect to the IVD values transmitted in the EGNOS messages over the same period are 0.8740 m and 1.1719 m respectively. Thus, predicted IVD values, even when based on large update intervals of up to 30 minutes, outperform values of other
similar or smaller 68% error quantiles. Filter update intervals using the old parameter values yields update intervals of 5 and 10 minutes. However, for the longer results are superior to using the old parameterization for filtering update intervals. From the figure it can be seen that the Klobuchar-based state model and 5, 10 and 30 minute filter outdated parameter values. In this comparison we use the to the 68% error quantile that is obtained when using the initialization period of a few filter updates in comparison filtering results after shows the 68% error quantiles of the it to EGNOS GIVD every minute up to 30 minutes. Fig. 5 predicts 5, 10 and 30 minute prediction lengths, and old fitted parameters.

V. CONCLUSIONS AND FUTURE WORK
In this paper we parametrize EGNOS GIVD values using trigonometric basis functions in order to represent the ionospheric corrections in a more compact manner. We predict these parameters using a standard Kalman filter with both a standard random walk state model and a Klobuchar-based state model introduced in [8].

Our tests show that it is possible to parametrize EGNOS GIVD values and predict them without significant information loss. This allows us to significantly reduce the amount of data transferred to an user equipment in comparison to EGNOS transmissions. In addition, our approach outperforms both Klobuchar and NeQuick G models. We furthermore show that the Klobuchar-based state model yields smaller or at least similar IVD 68% error quantiles than a standard random walk state model for filter update intervals between 5 and 30 minutes. Finally, our test shows that using the Klobuchar-based model yields smaller 68% IVD error quantiles than using outdated parameters from EGNOS transmissions for update intervals of 5 and 10 minutes.

The tests were done using data from a period with mainly low to moderate solar activity. The applicability of the Klobuchar-based state model for prediction during high solar activity periods still remains to be tested. During milder conditions, the proposed model is still shown to be superior to the standard random walk state model when predicting ionospheric corrections using locally available data. The use of the Klobuchar-based state model does not have to limit only to SBAS transmissions. The model could also be used to predict IVD in situations where other ionospheric delay data, such as dual frequency GNSS measurements, are available only locally. Our results suggest that in such situations the Klobuchar-based state model derived from the Klobuchar difference might yield better prediction results than the commonly used random walk state model.

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