Robust Regulation of Port-Hamiltonian Systems

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1 Introduction

We will give sufficient conditions for a controller to achieve robust output regulation for boundary control systems.

A minimal implementation of such a controller will be given for impedance passive port-Hamiltonian systems.

2 Plant, Exosystem and Controller

The plant is an impedance passive port-Hamiltonian system

$$\begin{aligned} \dot{x}(t) &= \mathcal{A}x(t), \\ \mathcal{B}x(t) &= u(t) + w(t), \\ \mathcal{C}x(t) &= y(t) \end{aligned}$$

where the disturbance $w(t)$ is generated by the exosystem:

$$\dot{v}(t) = Sv(t), \quad w(t) = Ev(t), \quad y_{ref}(t) = -Fv(t)$$

which is a linear system on a finite-dimensional space $W = \mathbb{C}^q$. Here $S = \operatorname{diag}(i\omega_1, i\omega_2, \ldots, i\omega_q)$ with $\omega_i \neq \omega_j$ for $i \neq j$, and E and F are matrices.

The controller is a linear system on a Banach space Z:

$$\begin{aligned} \dot{z}(t) &= \mathcal{G}_1 z(t) + \mathcal{G}_2 e(t), \\ u(t) &= Kz(t) - \kappa e(t) \end{aligned}$$

where $e(t) = y(t) - y_{ref}(t)$ is the regulation error. The operators $(\mathcal{G}_1, \mathcal{G}_2, K, \kappa)$ are bounded and the feedthrough term $-\kappa e(t)$ acts as negative output feedback for the plant.

3 Robust Output Regulation

Robust Output Regulation Problem. Choose the controller parameters $(\mathcal{G}_1, \mathcal{G}_2, K, \kappa)$ in such a way that

1. The closed-loop system is exponentially stable.
2. The regulation error goes asymptotically to zero.
3. Items 1 and 2 hold even if $(\mathcal{A}, \mathcal{B}, \mathcal{C}, E, F)$ are perturbed to $(\tilde{\mathcal{A}}, \tilde{\mathcal{B}}, \tilde{\mathcal{C}}, \tilde{E}, \tilde{F})$ for some class $\mathcal{O}$ of perturbations.

Theorem 1 *If a controller $(\mathcal{G}_1, \mathcal{G}_2, K, \kappa)$ exponentially stabilizes the closed-loop system and satisfies the $\mathcal{G}$-conditions:*

$$\begin{aligned} \mathcal{R}(i\omega_k - \mathcal{G}_1) \cap \mathcal{R}(\mathcal{G}_2) &= \{0\} \qquad \forall k \in \{1, 2, \ldots, q\}, \\ \mathcal{N}(\mathcal{G}_2) &= \{0\}, \end{aligned}$$

then it solves the robust output regulation problem (RORP). The controller is robust with respect to all perturbations under which the closed-loop system is exponentially stable, $(\tilde{\mathcal{A}}, \tilde{\mathcal{B}}, \tilde{\mathcal{C}})$ is a boundary control system and $\tilde{E}, \tilde{F}$ are bounded.

Remark. As a standing assumption, the transfer function of the triple $(\tilde{\mathcal{A}}, \tilde{\mathcal{B}}, \tilde{\mathcal{C}})$ has to be surjective for all $\{i\omega_k\}_{k=1}^q$.

4 Port-Hamiltonian Systems

The operators $\mathcal{B}$ and $\mathcal{C}$ can be written in the form

$$\mathcal{B}x := \begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} f_\partial \\ e_\partial \end{bmatrix} \quad \text{and} \quad \mathcal{C}x := \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} f_\partial \\ e_\partial \end{bmatrix}$$

where $B_{1,2}, C_{1,2}$ are matrices and f_∂, e_∂ are the *boundary port variables* associated with the port-Hamiltonian system.

A system $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ is called *impedance passive* if

$$\operatorname{Re}\langle \mathcal{A}x, x \rangle \leq \operatorname{Re}\langle \mathcal{B}x, \mathcal{C}x \rangle \qquad x \in \mathcal{D}(\mathcal{A}).$$

An impedance passive port-Hamiltonian system satisfies $B_2B_1^* + B_1B_2^* \geq 0$, $C_2C_1^* + C_1C_2^* \geq 0$ and $B_2C_1^* + B_1C_2^* = I$. Impedance passive port-Hamiltonian systems can be exponentially stabilized using negative output feedback, which accounts for the feedthrough term $-\kappa e(t)$ in the controller.

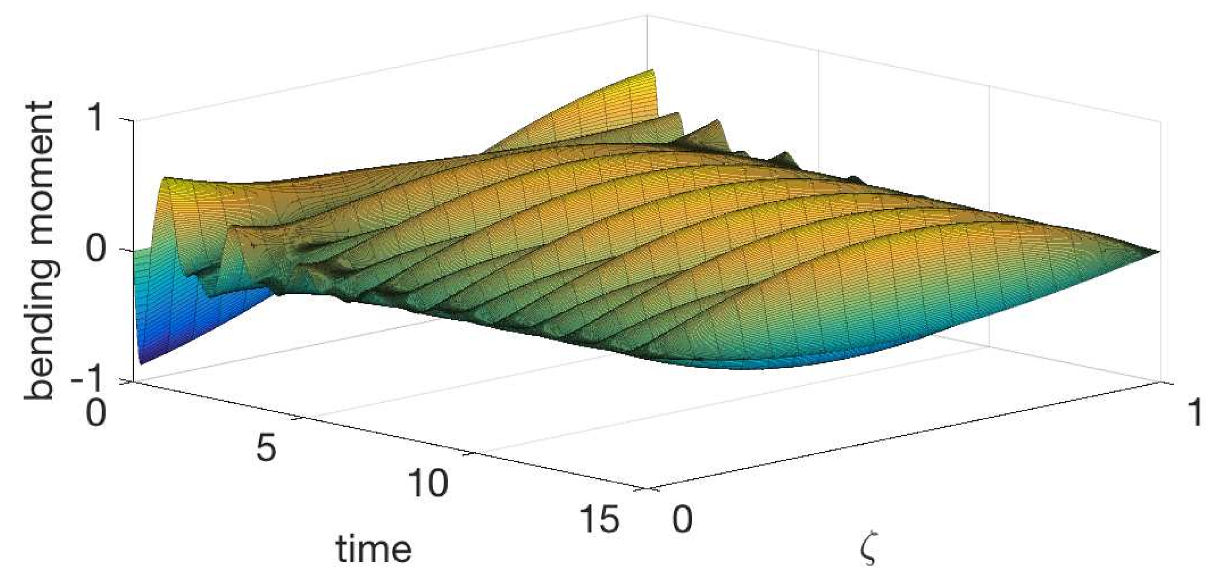
5 Construction of a Robust Controller

Theorem 2 *Choose $Z = Y^q$ and choose the controller parameters $(\mathcal{G}_1, \mathcal{G}_2, K, \kappa)$ as $\kappa > 0$ and*

$$\begin{aligned} \mathcal{G}_1 &= \operatorname{diag}(i\omega_1 I_Y, i\omega_2 I_Y, \ldots i\omega_q I_Y), \\ \mathcal{G}_2 &= (\mathcal{G}_2^k)_{k=1}^q, \qquad \mathcal{G}_2^k = -I_Y \qquad \forall k \in \{1, 2, \ldots, q\}, \\ K &= \epsilon K_0 = \epsilon[P_\kappa(i\omega_k)^\dagger, P_\kappa(i\omega_2)^\dagger, \ldots, P_\kappa(i\omega_k)^\dagger] \end{aligned}$$

where $P_\kappa(\cdot)^\dagger$ denotes the Moore-Penrose pseudoinverse of the transfer function of the triple $(\mathcal{A}, \mathcal{B} + \kappa\mathcal{C}, \mathcal{C})$. Then the controller solves the RORP for all sufficiently small $\epsilon > 0$.

Example. Robust control of Euler-Bernoulli beam on $\zeta \in [0, 1]$. The bending moment is regulated to zero at both ends. Other two boundary observations follow periodic references.



References

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