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Abstract: Raising awareness of educational games and game-based learning is an important step for large-scale adoption of these new educational methods. Digital game-based learning provides unique opportunities to engage students in learning, which is especially important for subjects that students struggle with, such as mathematics. Therefore, the aim of the current paper was to evaluate the usefulness of a math game competition for engaging, assessing, and training rational numbers in students from different schools. In particular, we investigated whether playing a digital game would improve students' magnitude understanding of rational numbers and whether playing behavior can be used for assessment purposes. Finnish fourth ($n = 59$; $M_{age} = 10.36$) and sixth-graders ($n = 105$; $M_{age} = 12.34$) participated in a math game competition relying on intra-classroom cooperation and inter-classroom competition. The students played a digital rational number game called Semideus, which is founded on number line estimation task mechanics in which players have to estimate the spatial position of a target number on a number line with only its start and endpoint specified. In previous empirical studies, this task mechanic has been successfully used to assess and foster students number magnitude understanding in conventional non-game based settings. Consequently, students were allowed to play the game as much as they wanted during a three-week period in order to improve their rational number knowledge and were able to check the status of the competition online. As expected, sixth grade students performed more accurately than fourth grade students in the game-based rational number magnitude estimation tasks. Moreover, results indicated that students benefited significantly from participating in the math game competition with respect to rational number knowledge. Importantly, the Semideus game was particularly effective with students who started with less rational number knowledge. Overall, the study demonstrated that participation in a math game competition seems to be a useful and engaging approach to assess and support the development of students' rational number knowledge.

Keywords: rational numbers; game-based learning; assessment; competition; number line

1. Introduction

Numerical knowledge has increasing importance for success in modern society (Siegler & Braithwaite 2017). In fact, Parsons and Bynner (2005) argued that, insufficient mathematical competencies might be even more detrimental to individual career prospects than reading or spelling deficiencies. Knowledge about rational numbers or fractions, respectively, seems to be particularly relevant as proficiency with fractions is strongly associated with high school students' current math achievement and is predictive of future math achievement and algebra performance (e.g., Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012). However, students often struggle to learn this challenging domain in mathematics education (Gigerenzer, 2002; Siegler, Fazio, Bailey, & Zhou, 2013 for a review).Therefore, more effective and engaging ways to teach basic numerical skills, such as rational number magnitude understanding, that form the foundation to learn more complex mathematics are needed. Devlin (2013) has argued that video games can provide new interfaces to learn mathematics that are far easier and natural to use than symbolic expressions that we have used to employ primarily. Thus, digital learning games have the potential to provide effective ways of training mathematics and can engage also persons who are anxious about mathematics. In line with this, previous research has indicated that digital learning games can be used to support mathematics instruction (e.g., Ninaus et al. 2017; Kiili & Ketamo 2017; Fazio, Kennedy, & Siegler 2016; Bakker, van den Heuvel-Panhuizen, & Robitzsch 2015; Riconscente 2013). Today there is a large number of mathematics learning games on the market (Apple's App Store lists over 20,000). However, the majority of these math games do not utilize the real power of game-based learning, but focus on traditional drill to develop mastery of basic skills and procedures. Most importantly, they often lacking empirical evaluation of their actual effects. This is to say, that in an era of digitalization of education, teachers face huge challenges to select good games from the different market places that slows down the diffusion of game-based learning and decreases the possible benefits of digitalization. According to Liu et al. (2015) game campaigns and competitions provide possibilities to raise awareness of games and such endeavours can be used to distribute games to schools. This is to say, that campaigns might be used to raise awareness of promising games and support teachers in selecting appropriate games for learning to be used in their classrooms. This is particularly relevant in teaching fractions and rational numbers as they are considered to be one of the most challenging problems in mathematics education (National Mathematics Advisory Panel, 2008). Therefore, the current paper employs a math game competition in order to to raise awareness about number line based Semideus games that have shown to be a valid assessment tool of rational number knowledge (Ninaus et al. 2017; Kiili & Ketamo 2017). In the following, we will first give a brief summary on the empirical foundations of the employed game to foster rational number knowledge. After that, we describe our hypotheses and methodology of the current study in more detail.

1.1 Number line training

Learning to deal with fractions or rational numbers, respectively, is considered to be one of the most challenging problems in early mathematics education (National Mathematics Advisory Panel, 2008). Importantly, high school students' knowledge about rational numbers is not only highly correlated with present mathematic achievement but it is also predictive of future math and algebra performance (e.g. Bailey, Hoard, Nugent, & Geary 2012; Booth & Newton, 2012). However, children often struggle to acquire sufficient knowledge in dealing with fractions or rational numbers (Gigerenzer 2002; see Siegler, Fazio, Bailey, & Zhou 2013 for a review). An often employed tool to train and assess students rational number knowledge is the so called number line estimation task (e.g. Link, Moeller, Huber, Fischer & Nuerk, 2013; Siegler & Opfer 2003). These tasks are based on the mental number line - a metaphor for number magnitude. Consequently, in the number line estimation task students have to indicate the spatial position of a target number on a number line with only its endpoints specified (e.g. where goes 4/5 on a number line ranging from 0 to 1). Performance in the number line estimation task is not only associated with actual mathematical achievement (e.g. Ninaus et al. 2017) but can also predict future mathematical achievement and algebra performance (e.g. Booth & Siegler 2006). In previous studies, game-based assessment and training utilizing the number line estimation mechanic have proven to be successful in assessing and fostering students' knowledge about rational numbers (Ninaus et al., 2017; Fazio, Kennedy, & Siegler 2016). Therefore, in the employed game we utilized the number line estimation mechanic by using it as a central game-play element ensuring optimal training conditions for the participants.

1.2 Present study

We used a math game competition (campaign) to support Finnish 4th and 6th graders rational number instruction. According to Liu et al. (2015) a campaign is a focused, widespread event where students from several schools engage in a learning activity over a short timespan. Liu et al. (2015) have distinguished four common campaign goals. In line with these goals, our math game competition aimed at 1) raising awareness about Semideus rational number games, 2) assessing students' rational number knowledge, and 3) facilitating the development of students' rational number knowledge. Moreover, we set up four hypotheses that directed the present study.

> *Hypothesis 1: In Finland rational numbers are taught in fourth, fifth, and sixth grade. Thus, we expect that 6th graders should perform better in estimating rational number magnitudes on a number line ranging from zero to one as compared to 4th graders.*
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> *Hypothesis 2: As previous research consistently demonstrated that number line based training is an effective training approach for improving number magnitude understanding (e.g., Fazio, Kennedy, & Siegler 2016; Link, Moeller, Huber, Fischer, & Nuerk, 2013; Siegler & Opfer, 2003), we expect that students' rational number estimation skills will improve significantly during the competition.*
>
> *Hypothesis 3: We expect that 4th graders gain more from the competition, because their baseline rational number understanding before the competition should be lower than 6th graders and thus they have more room for improvement.*
>
> *Hypothesis 4: Previous research has shown that in particularly low achieving students benefit from game-based math training (Fazio, Kennedy, & Siegler 2016) which is probably due to their higher possibility for improvement as compared to already high achieving students. Therefore, we expect that students that have weak prior number line estimation skills gain more from the competition than students with satisfactory and good prior number line estimation skills.*

2. Method

Participants. The current study focuses on Finnish 4th and 6th graders. We specifically selected 4th graders as they usually have just started to get familiar with rational numbers in school, and 6th graders were selected because they should already have quite a good understanding about rational numbers. Students that played at least 50 number line estimation tasks during the competition were included in the current analyses of this paper (N = 164), which ensures that those students could show at least minimal training effects in number magnitude estimation of fractions and decimals. 59 4^{th} graders and and 105 6^{th} graders participated in the study. Participants were from Finnish public primary schools around the country. Of these participants 64 were females, 100 were males. Participants were 10-13 years old with a mean of 11.63 years (SD = 1.07 years). Median math grade of 4^{th} graders (9.00) was statistically significantly higher than 6^{th} graders (9.00), U = 1915, z = 4.257, p < .001. Median math grade for females (9.00) and males (9.00) was not statistically significantly different, U = 2946.5, z = .898, p = .369.

2.1 Description of the competition

The competition was open for all Finnish 3-6 graders. In the competition, students had an opportunity to play a number line based rational number game during a three-week period. In order to recruit participants to the competition, we implemented a web-based form that allowed teachers to create game accounts online for their students. Teachers were required to provide some information of their class such as group name, number of students, municipality, grade level and an email account. The email account provided us a way to communicate with the teacher. The implemented system automatically sent students' game accounts to the teacher as well as required instructions about the game and the competition.

To be more precise, the competition was organized around one randomized game level that can be completed in a couple of minutes. However, one goal of the competition was to motivate students to play the level several times and share effective playing strategies with their peers. We tried to facilitate motivation with competitive aspects of the event. In order to support social interaction each participating class formed a team that competed against other teams. Furthermore, municipalities competed among each other. Our competition web-page included leaderboards for both teams and municipalities (Figure 1). In other words, we formed the competition around intra-classroom cooperation and inter-classroom competition in order to support social interaction and avoid negative side effects of individual competition within a classroom. To add further incentives to the competition, prizes (movie ticket sets) were awarded for several categories: two best teams, best municipality, most hard-working student, and randomly selected player who answered a questionnaire at the end of the competition. The best team and the best municipality prizes were given to one member of the group drawn at random. The hard-working (most tasks completed) prize was the only one that could be achieved by personal effort only.

Figure 1: Leaderboards for teams and municipalities. The score is the average of every team/municipality members' highest scores

The obtained game scores were used in rankings. Number line estimation accuracy formed 70% of the game score and 30% of the score was given by remaining energy when player completed the level. The player could lose energy by stepping on traps (locations shown with rational numbers), being hit by enemies, taking too long in estimation tasks (in each level there was one task that included a time limit), jumping on a mushroom at the wrong place (player could earn extra energy points by jumping on the mushroom if it's spots reflected the right place (fraction) on the number line - mushrooms were area models; see also Figure 2). The team rankings were based on the average value of each team member's highest game score.

Figure 2: A number line estimation task that includes a trap at 3/9 and a mushroom reflecting a fraction 1/4. A skill to reveal the location of traps has been used, indicated by the blue lightning symbol on the ground showing the location of the trap

The competition game had two playable levels. The first one was a tutorial level for introducing the game goals and mechanics. This level and its tasks were omitted from the competition ranking calculations. The second was the actual competition level used to evaluate the success in the competition. Both levels were playable for multiple times.

The competition level contained fourteen number line based tasks, which were selected from a larger set of different tasks including fractions as well as decimal numbers (57 different tasks in total). For each task subsamples were created from the larger pool of tasks to maintain difficulty across the playtime and to ensure that different kind of mathematical challenges were present each time players engaged in a level. For example, the first task of the level was selected randomly from seven possible tasks of the corresponding subsample, i.e. subsample task I. The second task was selected from its own pool (i.e., subsample task II) and so on. Moreover, this randomization offered some variety over multiple playthroughs. It also made it much harder to just memorize the correct answers and helped to maintain the actual learning goal of the current study. In contrast, the tutorial level had fixed tasks.

2.2 Task description

In the game the player had to estimate either fraction or decimal number magnitudes on a number line. The game also had some traps and enemies to avoid. To assist the player some of the tasks contained visual hints, such as sequently placed torches to divide the number line according to the denominator of the estimated fraction number. The game also included specific user-activatable in-game skills. Some of these skills made the task easier by removing enemies or time pressure while others provided direct assistance to the estimation itself by offering a hint towards the correct answer. For instance, one skill made an in-game animal companion to walk back and forth near the correct spatial position on the number line in order to guide the player to the correct position of the number magnitude. Because this animal provided only an approximate answer, the player did not automatically get the highest points available, but needed still do the estimation in the limits that the animal showed. Another in-game skill placed multiple birds as visual markers (similarly to the abovementioned torches). One of these markers indicated the correct location. That is, players could easily get an accurate answer as long as they understood the partitioning of the number line based on these visual markers or birds, respectively (i.e. understanding that a given fraction a/b as the quantity formed by multiple parts of size 1/b). Figure 3 illustrates a situation in which a player has activated the in-game bird skill while he or she should estimate point 3/4 on the number line ranging from 0 to 1. Depending on the task, some of the skills were not available every time as for example one of the skills was only relevant to fraction number estimations and was hidden with decimal number based tasks. Moreover, to activate these skills the player had to spend in-game currency, diamonds. The skills that were the most powerful in terms of estimation accuracy were also the most expensive. The player began each game with a set of diamonds and was also able to collect a couple of diamonds throughout the game level. Overall, this allowed players to use about 2-4 skill activations per level depending on the selected skills.

Figure 3: Player has activated in-game bird skill that divided the number line to as many parts as the denominator of the task (i.e. first bird at position 1/4, second bird on 2/4 and third bird on position 3/4)

2.3 Measures and analyses

Students playing performance was logged in order to assess students' rational number estimation accuracy. The development of students' estimation skills was assessed with an in-game pre- (first 10 fraction and 10 decimal tasks of the game) and posttest (last 10 fraction and 10 decimal tasks of the game). Instead of the game score being used as performance indicator for data analysis, we used estimation accuracy for pre- and posttest performance assessment, which is the most direct assessment of rational number magnitude understanding. A gain score was calculated from the tests (posttest score - pretest score) to identify possible improvement in rational number knowledge or magnitude estimation accuracy, respectively. The tasks that contained visual hints were excluded from the tests. Before participating in the pretest, students had to complete a tutorial level to ensure that all players mastered the user interface of the game. The pretest score was used in assessing students' rational number knowledge at the beginning of the competition. Moreover, in order to study the meaning of prior rational number estimation skills, the students were divided into three skill groups:

- Low skilled group (pretest accuracy < 82%)
- Average skilled group (pretest accuracy > 82% & < 92%)
- Good skilled group (pretest accuracy > 92%).

Non-parametric tests were used in all comparative analyses because normal distribution assumptions were violated. Cohen's d was used to evaluate the effect size of the training.

3. Results

During a three-week period, approximately 1500 students started to play the game. The students were from 35 different municipalities which indicates that the competition was an effective way to raise awareness about the game and get students to play the math game. The number of players is rather high with respect to the very limited email-based announcement of the competition. 4th and 6th graders that played at least 50 number line estimation tasks during the competition were included in the analyses of this paper (N = 164). On average students played through 210 number line estimation tasks. The first part of the results chapter focusses on assessing students' initial rational number knowledge and the second part considers learning gains achieved during the competition.

3.1 Rational number estimation accuracy

Figure 4 shows 4th and 6th graders mean of pretest estimation accuracy scores and standard deviation of fractions, decimals, and mean of both. A Mann-Whitney U test was run to determine whether there was a difference in the rational number estimation accuracy between 4th and 6th graders. As expected (Hypothesis 1), median rational number estimation accuracy of 6th graders (91.63%) was statistically significantly higher than 4th graders accuracy (87.75%), U = 4392, z = -4.435, p < .001. Moreover, a related-samples Wilcoxon signed rank test showed that students' estimation accuracy of fractions was significantly higher (Mdn = 91.85%) than estimation accuracy of decimals (Mdn = 88.93%), z = -5.244, p < .001.

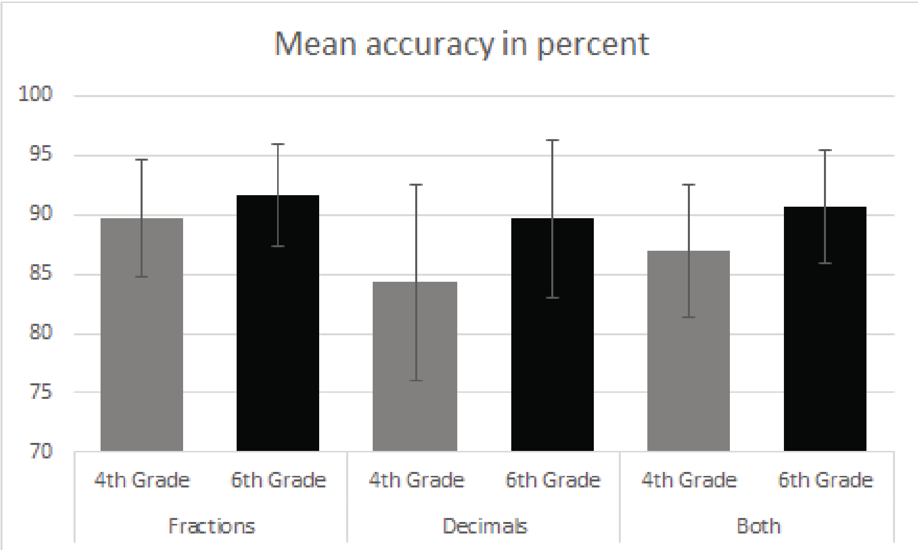

Figure 4: Means of rational number estimation accuracy by class grades at pretest for fractions, decimals and the mean of fractions and decimals (both). Error bars represent standard deviation

3.2 Learning gains

An interesting question is whether students' rational number estimation skills improved during the competition. Consistent with Hypothesis 2, a related-samples Wilcoxon signed rank test showed that students' posttest scores were significantly higher (Mdn = 93.93%) than the pretest scores (Mdn = 90.21%), z = 6.638, p < .001, d = 0.62, indicating that students clearly benefited from participating in the competition with respect to rational number knowledge.

A Mann-Whitney U test was run to determine whether the gain in rational number magnitude estimation accuracy was different for 4th and 6th graders. Contrary to our expectations (Hypothesis 3), median gain score of 4th graders (3.128) was not statistically significantly different than 6th median gain score (3.331), U = 3459, z = 1.239, p = .215.

We also investigated the meaning of the prior knowledge for the gain of participating in the competition. In fact, there was a significant negative correlation between prior rational number knowledge (pretest score) and gains from the pretest to the posttest, r(164) = -.58, p < .001. In order to study this in a more comprehensive way, a Kruskal-Wallis test was conducted to determine whether the gain in rational number magnitude estimation accuracy was different for students with low prior estimation skills (n = 19), average prior estimation skills (n = 85), and good prior estimation skills (n = 60). Median gain scores were statistically significantly different between the prior estimation skill groups, $\chi2(2)$ = 39.089, p < .001. Subsequent pairwise comparisons using Dunn's (1964) procedure and a Bonferroni correction for multiple comparisons revealed statistically significant differences in median gain scores between the low (10.12), average (4.19), and good (1.56) skilled groups, $p < .001$. In line with our Hypothesis 4, these results clearly show that students with low prior estimation skills benefited most from the number line based game competition. The effect size of the low skilled group was very large ($d = 2.01$). It is noteworthy that the number of tasks that low skilled group on average played through did not differ statistically significantly from other groups.

4. Discussion and conclusion

The present study demonstrated that a game competition was a useful and engaging approach to assess and train rational number knowledge. As expected the results showed that 6th graders estimated rational numbers more accurately than 4th graders. Moreover, current results revealed that students' number magnitude estimations for fractions were more accurate than for decimal numbers. This might originate from the fact that some teachers as well as some learning materials tend to emphasize training of fractions more than training of decimals. Sixth graders average fraction estimation accuracy was similar to a previous study (Ketamo & Kiili, 2017) suggesting that the used game provided similar results in competition context and when a similar type of game was used as an exam or assessment, respectively. One of the most important implications of the present study is that it demonstrated that math game competitions can be used to support the development of rational number knowledge. Importantly, the Semideus game played in the competition was particularly effective with students who started with less rational number knowledge which is in line with previous findings (e.g. Fazio, Kennedy, & Siegler 2016). These results highlight the potential of not only conventional game-based trainings being effective for fostering rational number knowledge but also that is achievable in a game competition, in which students were engaged in intra-classroom cooperation and inter-classroom competition. Thus, current results provide insight into the potential of game competitions in order to reach a rather larger number of students for educational purposes, which seem to be particularly promising for struggling students. Even better results might be achieved by including more scaffolding methods to the game. However, such an approach is more challenging to balance with respect to competitive aspects. Next we share some general lessons that we learned about organizing a math game competition.

First of all, we want to emphasize that it is very important that joining the competition is made easy for teachers and students. In our case, teachers appreciated our web-based game account creation system that made participation easy and facilitated teachers' sense of control over the competition. It was very important that the main communication with teachers was automated and teachers got all the instructions and materials instantly. It is crucial that teachers understand and believe that collected data is anonymous and students can not be identified based on the data. In our case, the prizes of the competition were delivered to winners with the help of the teachers.

Second, since the competition had prizes for the best performing teams and municipalities, the results needs to be checked for possible manipulations. The game logs of the best teams were checked for possible irregularities. It became apparent that the two best scoring teams had indeed used illegal means to achieve their scores. For several players of these teams the highest score was not in line with his/her other scores. This could indicate that the highest score was achieved by someone else. When the timestamps of the highest scoring games were compared, it showed that those games were played in a quick succession meaning that one skilled player had played the game for multiple accounts, usually after school hours. Moreover, we removed game scores if the same user account was simultaneously used to play several games. For the other of the highest scoring teams, the teacher had actually contacted the organizers and confirmed this kind of behaviour. These highest scores were omitted from the records and a new average was calculated for the affected teams and municipalities. It was a bit problematic that in the end the prizes were not awarded in a way that the leaderboard suggested. While the leaderboard was updated to match the omitted records, this was done only after the competition. The score to beat, which was obtained illegally, might have seemed impossible to reach and that might have had a negative influence for those players aiming for the prizes. The actual winner's score was lower and therefore more reachable and could have faced more competition from those willing to make a last push for the first rank. Thus, for the future game competitions we will implement an automated and real time cheating detection system. Furthermore, in order to more validly study the learning effectiveness of the competition we will implement in-game pre- and posttests in more controllable way.

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