Finite element method incorporating coupled magneto-elastic model for magneto-mechanical energy harvester

Finite Element Method Incorporating Coupled Magneto-Elastic Model for Magneto-Mechanical Energy Harvester

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This paper presents a numerical method for modeling magneto-mechanical energy harvesting devices. Our existing energy-based single-valued (SV) magneto-mechanical material model is utilized for the first time in a 2-D finite element formulation for an energy-harvesting application. The SV material model yields the magnetic field strength as a function of strain and magnetic flux density. The proposed method can predict the voltage induced in a pickup coil due to inverse magnetostriction, when the test sample is subjected to dynamic loading. The results from the numerical method are experimentally verified using a prototype energy harvester.

***Index Terms* — Coupled problem, energy harvesting, finite element analysis, magneto-elasticity, strain, stress.**

I. Introduction

THE MAGNETIC properties such as permeability of ferromagnetic materials change while subject to mechanical loading [1]. The stress dependent magnetic characteristics of the material can be utilized to harvest energy from mechanical vibrations. Utilization of ferromagnetic construction materials would allow energy to be harvested from ambient vibrational sources. Most existing energy harvesters utilize strong magnetostrictive materials like Terfenol-D or Galfenol etc. whereas the proposed energy harvesting concept utilizes construction steel because of its practical applications in bridges, buildings and rail tracks [2]-[3].

In this paper, a previously developed energy-based single-valued (SV) constitutive law [4] for magneto-elastic materials is utilized in 2-D coupled magneto-mechanical finite element (FE) analysis of a magnetostrictive energy-harvesting prototype. Similar energy-based models have been presented in [5] and [6]. The goal is to show that this numerical approach can be used to analyze and design magnetostrictive energy harvesting devices.

II. Experimental Setup

The experimental setup and FE mesh are shown in Fig. 1. A test sample made up of solid construction steel is utilized. The sample dimensions are 20x20x100 mm. The sample is magnetized with a U-shaped core and two magnetizing coils, and it can be vertically stressed with a hydraulic press. The magnetic field is parallel to the stress in the middle part of the sample.

The measurement setup was first used with static stress and AC magnetization in order to measure the magnetization curves of the sample under static stress, which are needed for the identification of the SV model. The results of the measurement were compared with the no stress *B-H* curve to analyze the behavior of the magnetization curve under static stress.

Secondly, the setup was used for validating the FE model under dynamic stress and DC magnetization. In this case, the test sample was subjected a sinsusoidal cyclic loading of 11 Hz from zero to 25 MPa tension. The voltage induced to the pickup coil by the flux density variation due to inverse magnetostriction was measured using an oscilloscope. A low pass filter with the cutoff frequency of 80 Hz was utilized to remove the high frequency noise form the measured signal. The measured results were compared with the simulated results to validate the proposed numerical method.

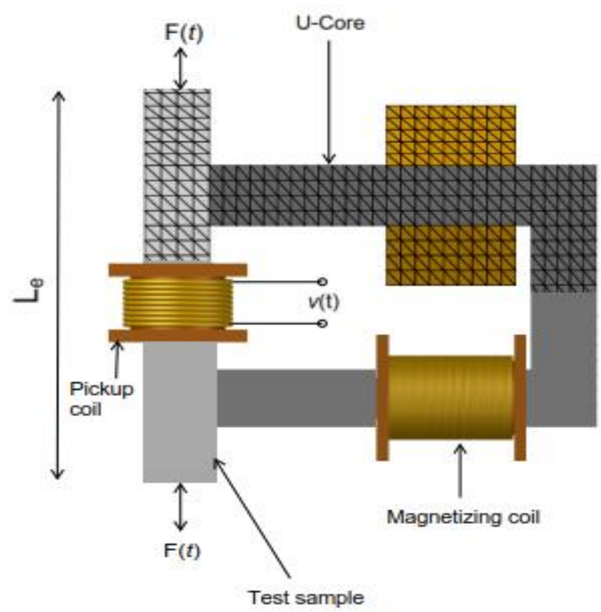

Fig. 1. Experimental setup and FE mesh of the energy harvester.

III. Numerical Model

In the numerical analysis the coupled magneto-mechanical model described in [4] is used. In this model the constitutive equations for the magnetic field strength $\boldsymbol{H}$ and stress $\boldsymbol{\sigma}$ are derived by using the Coleman-Noll procedure from the specific Helmholtz free energy density ψ as

$$\boldsymbol{H} = \rho \frac{\partial \psi}{\partial \boldsymbol{B}} \text{ and } \boldsymbol{\sigma} = \rho \frac{\partial \psi}{\partial \varepsilon}, \quad (1)$$

where ρ is the mass density. Dependency of the free energy from the state variables, i.e. the magnetic flux density vector $\boldsymbol{B}$ and strain tensor $\boldsymbol{\varepsilon}$ is established using the invariants $I_1 = \text{tr}\,\boldsymbol{\varepsilon}$, $I_2 = (\text{tr}\,\boldsymbol{\varepsilon})^2$, $I_4 = ||\boldsymbol{B}||^2$, $I_5 = \text{tr}(\boldsymbol{eB} \otimes \boldsymbol{B})$, and $I_6 = \text{tr}(\boldsymbol{eB} \otimes \boldsymbol{eB})$, where

$$\boldsymbol{e} = \boldsymbol{\varepsilon} - \frac{1}{3}\text{tr}(\boldsymbol{\varepsilon})\boldsymbol{I} \tag{1}$$

is the deviatoric strain, $\boldsymbol{I}$ the identity tensor and $\otimes$ denotes the tensor product. The cubic invariant $J_3 = \det \boldsymbol{e}$ is not used since in the absence of magnetic excitation the linear stress-strain relation should be recovered.

The free energy density ψ is expressed as

$$\psi = \frac{1}{2}\lambda I_1^2 + 2\mu I_2 + \sum_{i=1}^{n_\alpha}\alpha_i I_4^i + \sum_{i=1}^{n_\beta}\beta_i I_5^i + \sum_{i=1}^{n_\gamma}\gamma_i I_6^i\,, \tag{2}$$

where λ and μ are the Lamé parameters calculated from the Young's modulus $E = 200$ GPa and Poisson's ratio $\nu = 0.34$, and yielding the Hooke's law. The polyonomial coefficients α_i, β_i and γ_i are fitted so that the *B-H* curves predicted by (1) correspond to the ones measured at static stress and AC magnetization. Fig. 2 shows both measured and predicted magnetization curves.

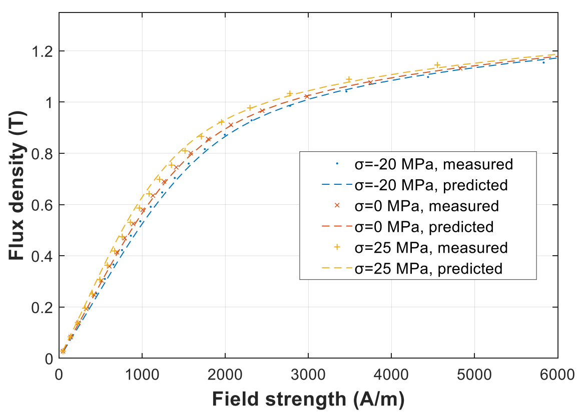

Fig. 2. Measured *B-H* curves and fitted SV model.

The 2-D finite element model is implemented in MATLAB using linear triangular elements and a magnetic vector potential $\boldsymbol{A} = A\boldsymbol{u}_z$, such that $\boldsymbol{B} = \nabla\times\boldsymbol{A}$. Fig. 1 presents the FE mesh used for the computation. The model solves Ampere's law

$$\nabla\times\boldsymbol{H}\left(\boldsymbol{B},\boldsymbol{\varepsilon}\right) = \boldsymbol{0}\,, \tag{3}$$

using the SV model for the calculation of $\boldsymbol{H}(\boldsymbol{B}, \boldsymbol{\varepsilon})$. The eddy-currents in the laminated U-core and test sample are neglected.

The time-stepping is performed with the implicit Euler method and the nonlinearity is handled with the Newton-Raphson method. A sinusoidally varying uniaxial stress is introduced in the sample and the strain $\boldsymbol{\varepsilon}$ is calculated from the stress using generalized Hooke's law. The SV model is used to calculate $\boldsymbol{H}$ from known $\boldsymbol{B}$ and $\boldsymbol{\varepsilon}$ at each integration point in elements located in test sample at each time step. The induced voltage in pickup coil is calculated from the average flux density inside test sample at the vertical location of the pickup coil using Faraday's law.

IV. Results and Discussion

The maximum root mean squared induced voltage was recorded 7.8 mV when the test sample was subjected to cyclic loading of zero to 25 MPa tension, whereas, an induced voltage of 8.2 mV was calculated by utilizing the proposed model. The maximum voltage is obtained at a magnetization current of 0.4 A, which corresponds to field-strength and flux-density values of about 2000 A/m and 0.9 T, respectively, in Fig. 2. The results at different magnetization currents are shown in Fig. 3 where the induced voltage tends to zero when the material reaches saturation. At higher field strengths the fitting error increases and the simulated voltage decreases faster than the measured one. Precise fitting of the SV model parameters is crucial in order to obtain the accurate results, since the induced voltage is determined by the difference of the *B-H* curves at different stresses. In the experimental setup, small air gaps are present between the U-core and test sample. These air gaps are difficult to determine which also affects the simulated results. In addition, the eddy-currents in the solid bar cannot be properly accounted for by the 2-D model.

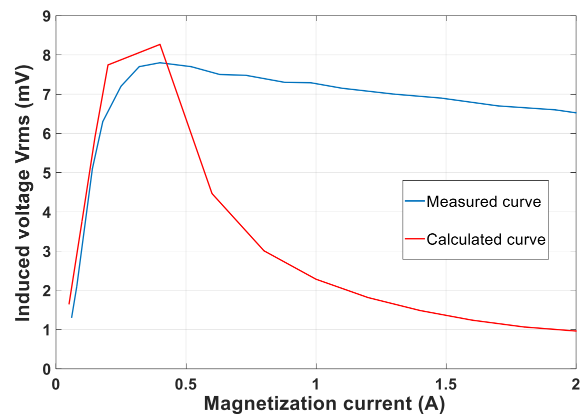

Fig. 3. Induced voltage in the pickup coil.

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