Flexible Spectrum Management in a Smart City
Within Licensed Shared Access Framework

EKATERINA MARKOVA1, IRINA GUDKOVA1,2, ALEKSANDR OMETOV1,3, ILYA DZANTIEV1, SERGEY ANDREEV1, YEVGENI KOUCHERYAVY3, AND KONSTANTIN SAMOUYLOV1,2

1Peoples’ Friendship University of Russia, 117198 Moscow, Russia
2Institute of Informatics Problems, Federal Research Center “Computer Science and Control” of the Russian Academy of Sciences, 119333 Moscow, Russia
3Tampere University of Technology, FI-33720 Tampere, Finland

Corresponding author: Aleksandr Ometov (aleksandr.ometov@tut.fi)

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ABSTRACT The new generation of communication technologies, named 5G, brings along a variety of emerging applications and services from both human and machine perspectives. The growing demand for bandwidth in 5G may therefore lead to massive deficiency in wireless spectrum availability despite its underutilization in urban areas. The Smart City paradigm assumes a multitude of communicating machines at high density, which requires improved spectrum management flexibility. The novel licensed shared access (LSA) framework that has attracted recent industrial and academic attention may become a feasible solution to leverage such underutilized spectrum more efficiently. This paper analyzes the effects of applying LSA in the Smart City context by proposing an appropriate mathematical model. Particularly, we focus on the vehicle-to-everything 5G use case where connected devices attempt to distribute their sensed data including occasional video information. The proposed analytical framework allows to capture the probabilities of rare events during such operation by providing with a high level of precision in the resulting performance estimates.

INDEX TERMS Licensed Shared Access, Smart City, Admission control, Quality of service, 5G mobile communications.

I. INTRODUCTION AND BACKGROUND
The advent of next generation of wireless networks (5G) gives rise to new challenges, particularly, in spectrum capacity. It has been estimated that spectrum limit will be reached by 2025 [1], [2]; hence, new strategies of efficient wireless spectrum utilization are essential [3]. At the same time, the number of interconnected and autonomously operated low-cost devices is growing tremendously [4] as the vision of Smart City is taking shape [5]. Communication is envisioned to become less human oriented and more leans toward Machine-to-Machine (M2M) communication [6].

An example of M2M communication paradigm in 5G networks is an automated Smart City, where a high number of interconnected and remotely controlled machines form an extensive urban-scale machine-type communication (MTC) cluster within high-density environment [7], [8]. Despite the connectivity enablers, a large number of applications are envisioned in the Smart City context, including e.g., wireless sensor networks [9], industrial automation [10], smart grid [11], public safety [12], smart metering [13], smart parking [14], e-healthcare [15], smart house and office automation [16], and green energy [17], among others.

Since most of these applications are machine-oriented [18], the interconnected sensors may become an integral part of the environment, especially in the Vehicle-to-Everything (V2X) scenario [19]. The surge in the number of communicating machines is expected to influence spectrum utilization even more, as present-day wireless networks were developed to be primarily utilized by humans [20]. The management techniques were therefore designed to satisfy the need for bandwidth with predictable request and access policies [21], while machines are expected to communicate in a more unpredictable manner.

One of the enablers for dynamic wireless spectrum management is the emerging Licensed Shared Access (LSA) framework, which permits more flexible spectrum control for highly demanding applications of tomorrow [22], [23]. At the same time, LSA allows for improved radio resource utilization by relocating inactive or unused frequency bands.

Considered from the communication point of view, LSA is a highly promising concept due to the possibility of providing predictable shared-spectrum operation with certain
Quality-of-Service (QoS) guarantees. For example, the vertical sharing structure of LSA permits multiple spectrum users in the same area to operate according to the priority tiers, see in Fig. 1.

![Conceptual operation of dynamic LSA framework.](Image)

More precisely, LSA allows for controlled spectrum sharing between two parties: (i) current owner of the spectrum, named the incumbent, and (ii) a temporary user, named the LSA licensee. Both parties obtain access to the same frequency band in a mutually agreed manner [24]. The envisaged LSA framework enables flexible infrastructure operation [25]. The spectrum is provided depending on a set of constraints i.e., in terms of time, frequency, and bandwidth. After the spectrum sharing request is confirmed, it is further forwarded to the corresponding spectrum owner and next translated into the Radio Access Network (RAN) instructions i.e., transmit power, interference- and frequency-related configuration, LSA policy, etc.

If the commands are successfully received and accepted, RAN executes the required actions according to the predefined instructions. Recently, a number of policies for efficient interference coordination between an incumbent and the LSA licensees were described in literature. The authors in [26] consider different approaches: (i) ignore policy and (ii) limit power policy (used for aeronautical telemetry). This work was further extended with respect to shutdown policy in [27].

For the V2X scenario, LSA may act as a powerful enabler leveraging the available secondary spectrum in the areas with low population densities as well as allowing for spectrum subutilization in dense areas during off-peak hours i.e., when the conventional users do not require most of the spectrum. A large number of characteristic LSA scenarios are being considered from the business perspective: Programme making and special events (PMSE) [3]; Public Safety [28]; Smart City [29], [30], and many others [31].

In what follows, we study a typical LSA use case, where the spectrum license owner in an urban area requires its frequency resources only occasionally, in small and localized portions. Based on the assumption that cellular network availability in the city is high, the incumbent (the network operator) has an opportunity to request underutilized spectrum for the rest of the time.

The remainder of this text is organized as follows. In Section II, a detailed summary of our proposed LSA-aware system model suitable for Smart City operation is discussed. Next, in Section III the corresponding analytical model based on a continuous Markov Chain formulation is provided. Further, we propose a recursive algorithm that employs the needed calculations in Section IV. Section V provides selected numerical results supporting the LSA utilization possibilities. The last section concludes this work.

II. BACKGROUND AND SYSTEM MODEL

In this paper, we focus on the scenario that considers a cooperative intelligent transportation system (ITS). The main objective of the ITS is to reach enhanced safety on the roads by using V2X communication in a Smart City [32] by video and telemetry exchange between the connected entities. The data are sent from each node involved into the system operation and further periodically delivered to the cloud control framework for further storage and analysis. At the same time, the spectrum owner may temporarily allocate it to V2X nodes except for the infrequent cases of its own data collecting periods.

Since the topic of LSA is only attracting the attention of the community recently, the number of works on the corresponding simulation-based evaluations is rather large while the analytical side remains underrepresented. The authors in [33] consider various network-management scenarios. Particular attention is paid to allocating bandwidth for M2M devices, which is expected to be one of the key drivers for shared resource access. The main idea of that work is that, unlike in the currently allocated spectrum, most of new frequencies (about 80%) are expected to remain in common use i.e., either under the LSA rules or with opportunistic access spectrum (OSA) model. This research proposes conceptual reasoning behind the types of scenarios, while the actual analytical models are not provided.

The authors of [34] elaborate on the LSA system operation with two base stations (BSs), primary and secondary. The users of the secondary BS can be served only when transmission does not degrade the QoS at the primary BS below a certain level; otherwise, the secondary BS is kept in idle mode. To achieve this goal, both BSs are assumed to be perfectly synchronized. The analytical contribution proposed in that work enables joint scheduling for two BSs. Further, the authors of [35], [36] propose a mechanism for distributing the LSA spectrum between several LSA licensees using a joint auction with mixed graph mechanism. This scheme allows for unhindered access to common spectrum by different (unknown to each other) commercial operators, whose BSs are coordinated with a dedicated management entity.

An optimized belief-based decision-making framework is proposed in [37]. That work develops a solution,
which exploits Cognitive Radio technology to mitigate the spectrum scarcity problem by enabling Dynamic Spectrum Access (DSA). The provided model is evaluated with both simulations and prototyping. The authors in [38] focus on capturing spatial user locations, which become a crucial factor with respect to the system performance. This approach combines queuing theory and stochastic geometry, while the actual analysis is conducted for the 3GPP LTE cellular system. However, the LSA framework is not taken into consideration in that work.

In contrast to the models available in the literature, our proposed framework utilizes Markov chain based analysis, since this approach is powerful, relatively simple to use, could be understood by a broad audience, and at the same time remains rather precise. However, other alternative techniques could also be applied in this context [39]. We have already completed a preliminary evaluation of the relevant QoS and QoE aspects by utilizing methods based on a similar approach in [38]. Indeed, the shortage of available spectrum has become a major barrier in developing today’s wireless systems. This leads to insufficient radio resources and may compromise the required levels of QoS and QoE for the users.

In our current model, we analyze one urban wireless cell of radius $R$ with the BS located in its geometrical center, and $k$ uniformly distributed vehicles located within the area of interest, see Fig. 2. When the considered devices are active, they operate in the full-buffer mode i.e., if the device enters the active mode, it attempts a transmission immediately. The idle-to-active rate is $\lambda$, while the user data transmission follows an exponential distribution with the parameter $\mu$.

Each node has its unique Channel Quality Indicator (CQI) value $c$ in the range $[1..15]$ (higher value of $c$ corresponds to better possible throughput). Further, we assume that all of the users with the same CQI are grouped into the corresponding coalition with similar maximum distance to the BS and certain throughput. The maximum distance between the node and the BS is defined as $\xi_d(\eta) = RL^{-1}\eta$, where $\eta = 16 - c$.

Due to the uniform distribution of devices across the area of interest, the density function is $f_{\xi_d(\eta)}(d) = \frac{2d}{R^2}$, $0 \leq d \leq R$. Then, the CDF $F_l(l)$ of the random variable $l = 1, \ldots, L = 15$, is: (i) $F_l(l) = 0$ if $l < 0$; (ii) $F_l(l) = \left(\frac{L}{L}ight)^2$ if $0 < l < L$; or (iii) $F_l(l) = 1$ if $l > L$.

In this paper, we focus on the LSA frequency bands utilized by e.g., law enforcement units for communication with vehicles. The operators that own the frequencies are seldom renting them to the command center via the BS in order to enhance the network performance. Further in this work, we assume that the average time of the vehicle’s travel over the connectivity area $\beta^{-1}$, and $\alpha^{-1}$ is the average idle spectrum duration. To increase the LSA spectrum usage efficiency, we utilize the Full Power (FP) policy i.e., the bandwidth is constant $\omega$ and the data transmissions are executed at the maximum transmit power $p^{\max}$ in case the spectrum owner does not utilize it. Otherwise, the transmit power is limited to $p^0_{\max}$. $p^0_{\max} < p^{\max}$ for the communicating devices.

Clearly, the transmit power variations cause changes in the data throughput as $r(\xi_{d(\eta)}), p^{\max}_s$, $s = 0, 1$, which depends on the distance between the BS and the device. According to the Shannon’s formula, the corresponding dependency is captured as

$$r(\xi_{d(\eta)}), p^{\max}_s = \omega \ln\left(1 + \frac{Gp^{\max}_s}{\left(\frac{\xi_d(\eta)}{L}ight)^s N_0}\right),$$  

$\omega$ where $s = 0, 1$, $N_0$ is the noise power, $G$ is the propagation constant, and $\kappa$ is the propagation exponent.

One of the system requirements is the guaranteed lower limit $r_0$ on the delivered throughput for each device. In case $r_0$ cannot be provided, the request is being blocked. If the device is in “infinitely” close proximity to the BS, its throughput theoretically tends to infinity. To limit this effect, we introduce the minimum distance between the BS and the device $\xi_{d(\eta)}$ as $d_0$, and constrain the maximum throughput as $r^{\max}_{s} = r(d_0, p^{\max}_s), s = 0, 1$. Therefore, the maximum throughput can be represented as $r(\xi_{d(\eta)}), p^{\max}_s = r^{\max}_s$ with $\eta = 1$ or by applying (1) with $\eta = 2, \ldots, L$. The core system modeling parameters used in this work are summarized in Table 1.

### III. ANALYTICAL METHODOLOGY

To analyze the scenario described above, in this section we introduce a Markovian process based analytical model. The behavior of the system states can be described by a continuous Markov chain (CMC) $(\xi(t), \eta_1(t), \ldots, \eta_{\xi(t)}(t), \zeta(t), t \geq 0)$, where $\xi(t)$ is the number of active devices, $\eta_i(t)$, $i = 1, \ldots, \xi(t)$ is the value of the random variable $\eta$ that defines the CQI level reported by the $i^{th}$ device, and $\zeta(t)$ is the state of the multi-tenant band at $t \geq 0$. Therefore, the system operation could be defined as $(k, l_1, \ldots, l_k, s); l_j = 1, \ldots, L; k = 0, 1, \ldots$ and $s = 0, 1$.

Before continuing with the general scenario, we consider a particular case with unlimited transmit power. Then, the system states are described as in (2), as shown at the bottom of
TABLE 1. Main modeling parameters.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>Cell radius</td>
<td>m</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Bandwidth of uplink channel</td>
<td>MHz</td>
</tr>
<tr>
<td>$L$</td>
<td>Number of CQI levels</td>
<td>-</td>
</tr>
<tr>
<td>$c = 16 - \eta$</td>
<td>Value of reported CQI level (random variable)</td>
<td>-</td>
</tr>
<tr>
<td>$q_l = \frac{2L-2l-1}{L^2}$</td>
<td>Probability that the reported CQI level is equal to $l$</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha^{-1}$</td>
<td>Average time when multi-tenant band is available</td>
<td>s</td>
</tr>
<tr>
<td>$\beta^{-1}$</td>
<td>Average time when multi-tenant band is unavailable</td>
<td>s</td>
</tr>
<tr>
<td>$s$</td>
<td>State of the multi-tenant band, $s = 1$ if the band is available and $s = 0$ if the band is unavailable</td>
<td>-</td>
</tr>
<tr>
<td>$k$</td>
<td>Number of active devices</td>
<td>-</td>
</tr>
<tr>
<td>$K_0$</td>
<td>Maximum number of devices when multi-tenant band is unavailable</td>
<td>-</td>
</tr>
<tr>
<td>$K_1$</td>
<td>Maximum number of devices when multi-tenant band is available</td>
<td>-</td>
</tr>
<tr>
<td>$P_0^{\text{max}}$</td>
<td>Threshold value of maximum uplink power of devices when multi-tenant band is unavailable</td>
<td>W</td>
</tr>
<tr>
<td>$P_1^{\text{max}}$</td>
<td>Threshold value of maximum uplink power of devices when multi-tenant band is available</td>
<td>W</td>
</tr>
<tr>
<td>$d_0$</td>
<td>Minimum distance to the BS (random variable)</td>
<td>m</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>Guaranteed bit rate</td>
<td>bps</td>
</tr>
<tr>
<td>$P_0^{\text{max}}$</td>
<td>Maximum bit rate when multi-tenant band is available</td>
<td>bps</td>
</tr>
<tr>
<td>$P_1^{\text{max}}$</td>
<td>Maximum bit rate when multi-tenant band is available</td>
<td>bps</td>
</tr>
<tr>
<td>$\tau(d) = \frac{\xi(d)}{P_2^{\text{max}}}$, $s = 0, 1$</td>
<td>Achievable bit rate</td>
<td>bps</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Arrival rate of sessions to transmit data</td>
<td>1/s</td>
</tr>
<tr>
<td>$\mu^{-1}$</td>
<td>Average service time of one device</td>
<td>s</td>
</tr>
<tr>
<td>$\rho = \lambda/\mu$</td>
<td>Offered load</td>
<td>-</td>
</tr>
</tbody>
</table>

Assume that the random variable $\eta$ would be equal to $l$ with the probability $q_l$, $l \in \{1, \ldots, L\}$. Consider our assumption regarding the uniform distribution of users across the network coverage zone, $q_l = \frac{2L-2l-1}{L^2}$, $l = 1, \ldots, L$. Next, if there are $k$ users with some CQI ($c = 16 - l_i, i = 1, \ldots, k$) value assigned to each of them and the shared bandwidth is $s$, then the transition state machine can be considered as illustrated in Fig. 3.

In order to simplify the calculation process, we aggregate the states based on the user number $k$. Therefore, $L \triangleq \bigcup_{k=0}^{\infty} (\tilde{L}(k, 1) \cup \tilde{L}(k, 0)), \tilde{L}(K) = \{(k, l_1, l_2, \ldots, l_k, s) \in L \}$, and the updated state machine is given in Fig. 4.

Realistically, the transmit power is not infinite and thus the maximum number of served devices is limited. Therefore, the system description should be updated as $\tilde{\xi}(t), \eta_1(t), \ldots, \eta_{\xi(t)}(t), \xi(t), t \geq 0$ in $L \subset \tilde{L}$. We introduce the access function according to the Full Power policy as

$$g_{\xi(d)}(k, l_1, l_2, \ldots, l_k, s) = \begin{cases} 1, & \text{if } \sum_{i=1}^{k} \alpha_i \leq \frac{r_0}{\tau(d)}, \frac{r_0}{P_2^{\text{max}}} \leq 1; \\ 0, & \text{otherwise}. \end{cases}$$  \ (3)

Next, we split the states $L$ by the number of active devices $k$ as $L = \bigcup_{k=0}^{\infty} (L(k, 1) \cup L(k, 0)), L(k, s) = \{(k, l_1, \ldots, l_k, s) \in L \}$. Considering (3), $L(k, s)$ is further...
represented as

\[
L(k, s) = \begin{cases} 
0 \leq d_1 \leq R, \ldots, 0 \leq d_k \leq R: \\
\times \sum_{i=1}^{k} \frac{r_0}{\text{ln}(1 + \frac{Gp_{s}^{\max}}{d_{i} N_0})} \leq 1 \end{cases}.
\] (4)

Let us define \( P_s(k), s = 0, 1 \) as a conditional probability of \( k + 1 \)th to be served with \( l_{k+1} \in \{1, \ldots, L\} \) when there are already \( k \) served devices with the same CQI as \( P_s(1, s) \in L(k + 1, s) \). In case the number of served devices is zero, the probability of a new device to be served is

\[
P_s(0) = F_{\xi_{i0}}\left(\min\left\{R, \left(\frac{Gp_{s}^{\max}}{d_0} - (e^{r_0/\omega} - 1)N_0\right)^{1/k}\right\}\right).
\] (5)

The remaining conditional probabilities \( P_s(k), k > 0 \) are further estimated based on (3) as

\[
P_s(k) = \frac{P\left(\sum_{i=1}^{k+1} \frac{1}{(l_{i}, p_{s}^{\max})} \leq \frac{1}{r_0}, \sum_{i=1}^{k} \frac{1}{(l_{i}, p_{s}^{\max})} \leq \frac{1}{r_0}\right)}{P\left(\sum_{i=1}^{k} \frac{1}{(l_{i}, p_{s}^{\max})} \leq \frac{1}{r_0}\right)}.
\] (6)

According to the central limit theorem, the sum of the random variables \( 1/r(l_i, p_{s}^{\max}) \) may be approximated as

\[
\sum_{i=1}^{k} \frac{1}{r(l_i, p_{s}^{\max})} \approx N(k\theta, k\sigma^2),
\] (7)

where \( \theta \) and \( \sigma \) are the expected value and the variance of the independent and identically distributed random variables \( 1/r(l_i, p_{s}^{\max}) \), correspondingly.

Due to the continuity feature of the standard normal distribution function, the convergence to this distribution is equivalent to the point-wise convergence of the distribution functions to the distribution function of the standard normal distribution. Hence, \( Z_k = S_k - k\theta/\sigma\sqrt{k} \), where \( S_k = \sum_{i=1}^{k} \frac{1}{r(l_i, p_{s}^{\max})} \leq \frac{1}{r_0} \). Next, we obtain \( F_{Z_k}(x) \to \Phi(x), \forall x \in R \), where \( \Phi(x) \) is the standard normal distribution function. Therefore,

\[
P\left(\sum_{i=1}^{k} \frac{1}{r(l_i, p_{s}^{\max})} \leq \frac{1}{r_0}\right) = \Phi\left(\frac{1 - k\theta r_0}{r_0 \sigma \sqrt{k}}\right),
\] (8)

where \( \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt \).

To further simplify the evaluation, we introduce

\[
m_k = k r_0 E\left[\frac{1}{r(l_i, p_{s}^{\max})}\right],
\]

\[
t_k^2 = k r_0 \left( E\left[\left(\frac{1}{r(l_i, p_{s}^{\max})}\right)^2\right] - E\left[\frac{1}{r(l_i, p_{s}^{\max})}\right]^2\right).
\]

Then, \( P_s(k), k > 0 \) could be obtained with

\[
P_s(k) = \frac{\Phi\left(\frac{1 - m_k}{n_k}\right)}{\Phi\left(\frac{1 - m_k}{n_k}\right)},
\] (9)

Next, the expected value of the random variable \( 1/r(l_i, p_{s}^{\max}) \) is

\[
E\left(\frac{1}{r(l_i, p_{s}^{\max})}\right) = \left(\frac{r_0}{r_0^{\max}}\right) F_{K_{i0}}(d_0)
\]

\[
+ \int_{d_0}^{R} \frac{1}{\text{ln}(1 + \frac{Gp_{s}^{\max}}{\lambda x N_0})} F_{K_{i0}}(x)dx,
\] (10)

and for \( \left(\frac{1}{r(l_i, p_{s}^{\max})}\right)^2 \) is

\[
E\left(\frac{1}{r(l_i, p_{s}^{\max})}\right)^2 = \left(\frac{r_0}{r_0^{\max}}\right)^2 F_{K_{i0}}(d_0)
\]

\[
+ \int_{d_0}^{R} \frac{1}{\omega^2 \text{ln}^2(1 + \frac{Gp_{s}^{\max}}{\lambda x N_0})} F_{K_{i0}}(x)dx.
\] (11)

The maximum number of users is \( K_s = r(d_0, p_{s}^{\max})/r_0 \), with \( s = 1 \) in case where the multi-tenant band is available or \( s = 0 \) otherwise. Then, \( L \) could be represented as

\[
L = \{ (k, s) \in \{0, 1, \ldots, K_s\} \times \{0, 1\} \}.
\] (12)

Further, the rules of the serving procedure for the active nodes for the LSA scenario are listed as follows:

1. The device will be served on the multi-tenant band with the maximum downlink power \( p_{s}^{\max} \), which is possible if the multi-tenant band is available i.e., \( s = 1 \) and the number of devices serviced on the multi-tenant band is less than \( K_1 \).
2. The device will be served on the multi-tenant band with the maximum downlink power \( p_{s}^{\max} < p_{s}^{\max} \), which is possible if the multi-tenant band is unavailable i.e., \( s = 0 \) and the number of devices serviced on the multi-tenant band is less than \( K_0 \).
3. Otherwise, the device’s request will be blocked without any after-effect for the corresponding Poisson process.

\[
L = \{ (0, 1), (1, 1, 1), \ldots, (1, L, 1), (0, 1), (1, 1, 0), \ldots, (1, L, 0), (2, 1, 1, 0), \ldots, (2, L, L, 0), (2, 1, 1, 1), \ldots, \\
\ldots (2, L, L, 1), \ldots, (k, \ldots, k), (k, L, \ldots, L) \ldots, (k, \ldots, 0) \ldots, (k, L, \ldots, L, 0) \}
\]

\[
= \{ (0, s), (k, l_1, l_2, \ldots, l_k, s), k = 1, 2, \ldots, l_i \in \{1, \ldots, L\}, i = 1, \ldots, k, s = 0, 1 \}.
\] (2)
Importantly, when the multi-tenant LSA band is deactivated, the maximum link power also changes from $p^{\text{max}}_1$ to $p^{\text{max}}_0$. Therefore, the serving rate also decreases down to $k = K_0$, $k > K_0$. In the opposite case, the transmit power increases, and the steady state diagram is shown in Fig. 5.

![Figure 5. Markov chain for LSA system operation.](image)

The corresponding Markov process representing the system states is described with the system of equilibrium equations as

\begin{align}
\begin{align}
 p(0,0)(\lambda p_0(0)+\beta) &= p(1,0)\mu + p(0,1)\alpha; \\
p(0,1)(\lambda p_1(0)+\alpha) &= p(1,1)\mu + p(0,0)\beta; \\
p(k,0)p(0,k+\mu + \beta) &= p(k+1,0)(k+1)\mu + p(k-1,0)\lambda p_0(k-1) + p(k,1)\alpha; \\
k = 1, \ldots, K_0 - 1, s = 0 \\
p(k,1)(\lambda p_1(k)+k\mu + \alpha) &= p(k+1,1)(k+1)\mu + p(k-1,1)\lambda p_1(k-1) + p(k,0)\beta; \\
p(0,0)(K_0\mu + \beta) &= p(K_0-1,0)\lambda p_0(K_0-1) + \alpha \sum_{k=K_0}^{K_1} p(K_0,1); \\
n = K_0 + 1, \ldots, K_1 - 1, s = 1 \\
p(k,1)(\lambda p_1(k)+k\mu + \alpha) &= p(k-1,1)\lambda p_1(k-1) + p(k+1,1)(k+1)\mu; \\
p(K_1,1)(K_1\mu + \alpha) &= p(K_1-1,1)\lambda p_1(K_1-1),
\end{align}
\end{align}

where $p(k,s)$, $(k,s) \in L$ is the stationary probability distribution.

### IV. RECURSIVE ALGORITHM

The process representing the system states is not a reversible Markov process. Hence, we propose a recursive algorithm for calculating the stationary probability distribution $p(k,s)$ for the system. We further consider the derivation of non-normalized probabilities $q(k,s)$, $(k,s) \in L$.

**Lemma 1:**

1. The values of non-normalized probabilities $q(k,s)$ are calculated with

$$q(0,0) = 1,$$

$$q(k,s) = \delta_{k,s} + \gamma_{k,s} \cdot x, (k,s) \in L : k > 0,$$

$$x = \frac{\lambda p_1(K_1 - 1)\gamma_{K_1-1,1} - \lambda p_1(K_1 - 1)}{\lambda p_1(K_1 - 1)\gamma_{K_1-1,1} - (K_1\mu + \lambda)\gamma_{K_1,1}}.$$

2. The coefficients $\delta_{k,s}$ and $\lambda_{k,s}$ are obtained by recursive equations as

$$\delta_{0,0} = 1, \quad \gamma_{0,0} = 0;$$

$$\delta_{0,1} = 0, \quad \gamma_{0,1} = 1;$$

$$\gamma_{10} = \frac{\lambda p_0(0) + \beta}{\mu}, \quad \gamma_{01} = \frac{\lambda p_1(0) + \alpha}{\mu};$$

$$k = 2, \ldots, K_0$$

$$\delta_{k,0} = \frac{\lambda p_0(k-1) + (k-1)\mu + \beta}{k \mu} \delta_{k-1,0} - \frac{\lambda p_0(k-2)}{k \mu} \delta_{k-2,0} - \frac{\alpha}{k \mu} \delta_{k-1,1};$$

$$k = 2, \ldots, K_0 + 1$$

$$\gamma_{k,0} = \frac{\lambda p_0(k-2)}{k \mu} \gamma_{k-0,0} - \frac{\alpha}{k \mu} \gamma_{k-1,0};$$

$$k = K_0 + 2, \ldots, K_1$$

$$\gamma_{k,1} = \frac{\lambda p_1(k-1) + (k-1)\mu + \alpha}{k \mu} \gamma_{k-1,1} - \frac{\lambda p_1(k-2)}{k \mu} \gamma_{k-2,1} - \frac{\beta}{k \mu} \gamma_{k-1,0};$$

Here, we note that the probability distribution $p(k,s)$ is produced with the following equation

$$p(k,s) = \frac{q(k,s)}{\sum_{(i,j) \in L} q(i,j)}, (k,s) \in L.$$

**Lemma 2:**

1. The performance metrics of our LSA model with one multi-tenant band and the unavailability probability $B$, preemption probability $\Pi$, and the average number of users $K$...
may be established as follows:

\[ B = \sum_{k=0}^{K_0-1} (1 - P_0(k))p(k, 0) + \sum_{k=0}^{K_1-1} (1 - P_1(k))p(k, 1); \]

\[ \Pi = \sum_{k=K_0+1}^{K_1-1} \frac{\alpha}{k + \mu + \lambda P_1(k)} + \frac{\alpha}{\mu K_1} \cdot \frac{C_{k_1-K_0-1}}{C_{k_1-K_0}} p(k, 1); \]

\[ K = \sum_{k=0}^{K_0} k p(k, 0) + \sum_{k=0}^{K_1} k p(k, 1). \]  

(17)

V. NUMERICAL RESULTS

In this section, we focus on the scenario discussed in Section II. Particularly, we target to characterize the fine-grained probabilities of the LSA system operation with our analytical methodology e.g., the rare system events that are hard to capture by utilizing the conventional simulation-based approaches [40]. Being more specific, for the envisioned Smart City requirement of 99.999% communication reliability, the number of the events to be modeled may vary from \(10^7\) to \(10^9\) for a point estimation, which results in prohibitive simulation times. Therefore, our analytical approach becomes a useful tool.

Below, most of the parameters related to the channel characterization were adopted from [41]. In the V2X transmit mode, vehicles generate data as: (i) 3; (ii) 5; or (iii) 10 packets/second per vehicle [42]. The number of cars per cell is estimated based on the typical urban density\(^1\) and the cell size, as \(\lambda = p_n \cdot \Pi \cdot R^2 \cdot c\), where \(p_n\) is the number of packets generated by one device per time slot, \(c\) is the device density per square km based on the selected city. Here, \(c = \text{Cars/CitySquare}\), where \(\text{CitySquare} = 8.382\) ml km\(^2\) and \(\text{Cars} = 2635.9\) ml. The data rate is set to 1 Mbps per device, which is suitable for the telemetry and low-quality video streaming [42].

In this work, we assume that the LSA band unavailability is approximately once per 20 minutes. We also summarize the initial data related to our example in Table 2. We further focus on the following set of metrics of interest: (i) the average number of active users in the system \(K\) and (ii) the probability for the user to not be served \(\Pi\) based on the cell radius \(R\) and/or the BS transmit power \(P\).

The dependence of the ‘service unavailable’ probability based on the cellular transmit power is offered in Fig. 6. Here, the probability in question lowers as the transmit power decreases. This behavior could be explained by the higher per-user throughput delivered when a higher transmit power is utilized. The second factor that influences this probability is as follows: when the spectrum is less accessible in general, the interruption probability is lower as well. Similar behavior may be observed in Fig. 7, where we analyze the cell radius. Considering a fairly small cell with the radius of 200 meters, the expected average number of users generally increases. If the cell radius grows to 400 meters, the observed peak could be explained by the higher distance between the user and the BS. In other words, if the transmitter and the receiver are sufficiently far apart from each other, the offered transmit power may be insufficient to provide the guaranteed throughput.

Further, the number of served users increases with respect to the cell radius, and the corresponding results are depicted in Fig. 8. However, in case where the transmit power is higher, such an increase would be less notable. At the same time, the service unavailable probability grows proportionally to the cell size, as it is shown in Fig. 9.

The bottom plots in Figs. 6–8 report on the rare events that are possible to capture with our analytical model. Indeed, in terms of characterizing the mean number of serviced users, the obtained results indicate only marginal fluctuations.

\[ \text{TABLE 2. Key parameter settings.} \]

<table>
<thead>
<tr>
<th>Notation</th>
<th>Dynamic value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R)</td>
<td>200 → 400 m</td>
</tr>
<tr>
<td>(P_1)</td>
<td>23 dBm (0.2 W), 42 dBm (15.58 W)</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>10</td>
</tr>
<tr>
<td>(\nu)</td>
<td>10</td>
</tr>
<tr>
<td>(\Sigma)</td>
<td>2 → 10</td>
</tr>
</tbody>
</table>

\[ \text{FIGURE 6. Session interruption probability as function of downlink transmit (Tx) power.} \]

failures on the order of $10^{-20}$ is essential to confirm sufficient system-level reliability i.e., when targeting the levels of ‘5 nines’ (99.999) or higher.

VI. CONCLUSIONS
The emerging paradigm of the Smart City as a 5G application allows for interconnected and remotely controlled machines from a large urban-scale MTC cluster to form a highly-dense environment. In this context, one of the key enablers for dynamic radio spectrum management is becoming the LSA framework that permits more flexible control of demanding V2X services, which are addressed in this work. In this paper, we studied a characteristic urban LSA use case, where the spectrum license owner in a Smart City uses its frequency resources only occasionally, for small and localized portions. Based on the premise that the cellular network availability in a city is high, the LSA licensee (cellular network operator) has an opportunity to reuse the incumbent’s underutilized spectrum for the significant portions of time.

The analytical framework developed in this work enables to capture rare events in the considered scenario, thus providing the much needed detailed assessment, while simulation may fail to offer the desired precision. We thus studied the service unavailability probability and the average number of serviced users per cell in the urban LSA use case. We also demonstrated that while the average number of users in the LSA bands may not require high precision, the service interruption probability is of more interest for the technology operators due to its intricate fine-grained behavior.

REFERENCES
E. Markova et al.: Flexible Spectrum Management in a Smart City Within LSA Framework


EKTATERINA MARKOVA received the B.Sc. and M.Sc. degrees in applied mathematics and the Ph.D. degree in applied mathematics and computer sciences from the Peoples’ Friendship University of Russia (RUDN University) in 2009, 2011, and 2015, respectively. Since 2012, she has been with the Telecommunication Systems Department, RUDN University, where she is currently an Associate Professor with the Department of Applied Probability and Informatics. Her current research interests lie in the area of performance analysis of radio resource management techniques in LTE networks.

IRINA GUDKova received the M.Sc. degree in applied mathematics and the Cand.Sc. degree in applied mathematics and computer sciences from the Peoples’ Friendship University of Russia (RUDN University) in 2009 and 2011, respectively. She is currently an Associate Professor with the Applied Probability and Informatics Department, RUDN University. She has co-authored multiple research works. Her current research interests include mathematical modeling and performance analysis of 4G/5G networks, smart cities, spectrum sharing, multicast services, radio access, teletraffic theory, and queuing theory.

ALEKSANDR OMETOV received the M.Sc. degree (Hons.) in telecommunications from the Department of Electronics and Communications Engineering, Tampere University of Technology (TUT), Finland, in 2016, and the Specialist degree in information security from the Saint Petersburg State University of Aerospace Instrumentation, Saint Petersburg, Russia, in 2013. He has been a Research Assistant with TUT since 2013. His major research interests are wireless communications, information security, heterogeneous networking, cooperative communications, and machine-to-machine applications.
ILYA DZANTIEV received the M.Sc. degree in applied mathematics from the Peoples’ Friendship University of Russia in 2015, where he is currently pursuing the Ph.D. degree with the Applied Probability and Informatics Department. His research interests include mathematical modeling and performance analysis of spectrum sharing techniques in 4G/5G networks, teletraffic theory, and queuing theory.

SERGEY ANDREEV received the Specialist and Cand.Sc. degrees from Saint Petersburg State University of Aerospace Instrumentation, Saint Petersburg, Russia, in 2006 and 2009, respectively, and the Ph.D. degree from Tampere University of Technology in 2012. He is currently a Senior Research Scientist with the Laboratory of Electronics and Communications Engineering, Tampere University of Technology, Finland. He has co-authored over 120 published research works on wireless communications, energy efficiency, heterogeneous networking, cooperative communications, and machine-to-machine applications.

YEVGENI KOUCHERYAVY received the Ph.D. degree from Tampere University of Technology (TUT), Finland, in 2004. He is currently a Full Professor and the Laboratory Director with the Department of Electronics and Communications Engineering, TUT. He has authored numerous publications in the field of advanced wired and wireless networking and communications. His current research interests include various aspects in heterogeneous wireless communication networks and systems, the Internet of Things and its standardization, and nanocommunications. He is an Associate Technical Editor of the IEEE Communications Magazine and an Editor of the IEEE COMMUNICATIONS SURVEYS AND TUTORIALS.

KONSTANTIN SAMOUYLOV received the Ph.D. degree from Moscow State University and the D.Sc. degree from the Moscow Technical University of Communications and Informatics. From 1985 to 1996, he held several positions at the Faculty of Sciences, Peoples’ Friendship University of Russia (RUDN University), where he became the Head of the Telecommunication Systems Department in 1996. Since 2014, he has been the Head of the Department of Applied Informatics and Probability Theory, RUDN University. During last two decades, he has been conducting research projects for the Helsinki and Lappeenranta Universities of Technology, Moscow Central Science Research Telecommunication Institute, several Institutes of Russian Academy of Sciences and a number of Russian network operators. He has authored over 150 scientific and technical papers and three books. His current research interests are performance analysis of 4G networks (LTE and WiMAX), teletraffic of triple play networks, signaling network planning, and cloud computing.

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