



- Author(s)** Ala-Luhtala, Juha; Seppänen, Mari; Piché, Robert
- Title** An empirical solar radiation pressure model for autonomous GNSS orbit prediction
- Citation** Ala-Luhtala, Juha; Seppänen, Mari; Piché, Robert 2012. An empirical solar radiation pressure model for autonomous GNSS orbit prediction. 2012 IEEE/ION Position Location and Navigation Symposium PLANS, 23-26 April 2012, Myrtle Beach, SC, USA . IEEE/ION Position Location and Navigation Symposium Piscataway, NJ, 568-575.
- Year** 2012
- DOI** <http://dx.doi.org/10.1109/PLANS.2012.6236929>
- Version** Post-print
- URN** <http://URN.fi/URN:NBN:fi:ty-201311011414>
- Copyright** © 2012 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.

All material supplied via TUT DPub is protected by copyright and other intellectual property rights, and duplication or sale of all or part of any of the repository collections is not permitted, except that material may be duplicated by you for your research use or educational purposes in electronic or print form. You must obtain permission for any other use. Electronic or print copies may not be offered, whether for sale or otherwise to anyone who is not an authorized user.

An empirical solar radiation pressure model for autonomous GNSS orbit prediction

Juha Ala-Luhtala, Mari Seppänen and Robert Piché
Tampere University of Technology, Finland

Abstract—GNSS satellite orbits can be predicted by integrating the satellites’ equation of motion. If the prediction is done in a consumer grade positioning device, a simplified version of the equation of motion is required. The forces due to Earth’s gravitation, solar gravitation and lunar gravitation should be included, but the models for the smaller non-gravitational forces can be fairly simple. This paper presents a simple empirical two parameter solar radiation pressure model for an orbit prediction application in a navigation device that does not have a network connection. The model is tested by predicting the orbits of GPS and GLONASS satellites up to 5 days into the future, using position and improved velocity from broadcast ephemerides as an initial state. The predicted orbits are compared to the precise orbits from International GNSS Service (IGS).

I. INTRODUCTION

When a stand-alone GNSS receiver is turned on it takes some time before the first position estimate is available to the user. This delay, called Time To First Fix (TTFF), is at least 30 seconds but can be even several minutes if there are obstacles blocking the direct view to the sky. The delay is frustrating for the user and can also be a serious problem in an emergency or other special situation. The reasons for this delay are the time needed for signal acquisition and tracking, and the time needed to receive the navigation data sent by the GNSS satellite.

One method to reduce the TTFF is to provide the ephemeris data to the device from an alternative source. Then the device needs to receive only the time of the satellite and less time is spent in receiving the navigation data. The position and velocity information of the satellite can also be used to speed up the signal acquisition, because the satellite’s position coordinates can be used to identify the visible satellites and with the information about satellite’s velocity the range of possible Doppler frequency shifts can be reduced. A widely used method is to provide the needed ephemeris information using assisting data servers. However, since many navigation devices do not have a network connection, there is interest in developing methods that can provide the ephemeris information autonomously and without the need of a network connection.

“Self-assistance” techniques for computing the ephemeris data have been implemented in commercial products and are outlined in the literature [1; 2], but these publications do not give a detailed description of the algorithms. In this paper we use a method that we presented first for GPS satellites in [3; 4] and recently extended to GLONASS satellites in the paper [5]. In this method the satellites’ position and velocity are predicted by integrating the satellite’s equation of motion several days

forward using initial conditions computed from the satellites’ broadcast ephemeris.

The satellites’ equation of motion used in our method includes gravitational forces of the Earth, the Sun and the Moon and a simple empirical model to account for the effects of the solar radiation pressure. When the prediction algorithm was tested with GLONASS satellites we noticed that the orbit prediction errors were significantly smaller than for GPS satellites. We found out that the main reason for the difference was our solar radiation pressure model, which included only the effects in the direction perpendicular to the plane containing the satellite’s solar panels. Studies conducted for GPS satellites report that there is also a small acceleration, often called y-bias, along the satellite’s solar panel axis, that has a significant effect to the orbit of the satellite [6; 7]. However for GLONASS satellites the magnitude of this y-bias acceleration has been found to be on average very close to zero [8].

In this paper we present an improved empirical solar radiation pressure model that takes into account also the y-bias acceleration along the satellites’ solar panel axis. The model includes two satellite-specific parameters: a scale parameter for the direct solar radiation pressure and a magnitude for the y-bias acceleration. To estimate the parameters we need to know the satellite’s true orbit over several days. Since the navigation device has access to only a few broadcast ephemerides that have limited accuracy and span a short time interval, the estimation is done offline using the precise ephemerides from International GNSS Service (IGS).

II. FORCE MODEL

The orbit prediction algorithm used in this paper is based on the satellite’s equation of motion. For a very high precision orbit prediction algorithm a large number of different forces need to be included in the equation of motion, but since our model is intended to be used in a consumer grade positioning device that has modest computational resources, we have included only the four forces that have greatest effect to the satellites orbit.

The acceleration of the satellite is described by equation

$$\ddot{\mathbf{r}}_{\text{Sat}} = \mathbf{a}_{\text{Earth}} + \mathbf{a}_{\text{Sun}} + \mathbf{a}_{\text{Moon}} + \mathbf{a}_{\text{SRP}}, \quad (1)$$

where $\mathbf{a}_{\text{Earth}}$, \mathbf{a}_{Sun} and \mathbf{a}_{Moon} are the accelerations caused by the gravitation of the Earth, the Sun and the Moon respectively. The fourth term \mathbf{a}_{SRP} is the acceleration caused by the solar radiation pressure.

The acceleration caused by Earth gravitation can be computed by taking the gradient of the gravity potential U . Taking the unsymmetrical mass distribution of the Earth into account, the gravity potential can be written using a spherical harmonics expansion of the form [9]

$$U(r, \lambda, \varphi) = \frac{GM_E}{r} \sum_{n=0}^{\infty} \sum_{m=0}^n \left[\left(\frac{R_E}{r} \right)^n P_{nm}(\sin \varphi) \right. \\ \left. \left(C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda) \right) \right], \quad (2)$$

where r is the satellites distance to the center of the Earth, λ is the longitude and φ is the latitude. The constant R_E in this formula is the Earth's radius and the terms P_{nm} are the associated Legendre polynomials of degree n and order m . The coefficients S_{nm} and C_{nm} in the formula are experimentally determined constants whose magnitude decreases very fast with increasing n and m . At GNSS satellite altitude, the terms up to the degree and order 4 are significant [3], but we use terms up to the degree and order 8. The values for the geopotential coefficients C_{nm} and S_{nm} are obtained from the EGM2008 model [10].

The gradient of the gravity potential is computed by using a recursive algorithm introduced by Cunningham [11] and later extended by Métris et al.[12], which gives the gradient in the Earth centered Earth fixed (ECEF) coordinates. If \mathbf{R} is the transformation matrix from inertial reference frame to ECEF-coordinates, the acceleration in the inertial reference frame is given by [9]

$$\mathbf{a}_{\text{Earth}}(\mathbf{r}) = \mathbf{R}^{-1} \nabla U(\mathbf{R}\mathbf{r}). \quad (3)$$

After the Earth's gravitation the second biggest acceleration components in the satellite's equation of motion are caused by the gravitational forces of the Moon and the Sun. To compute the acceleration acting on the satellite because of the gravitational force of any celestial body, one can use the form

$$\mathbf{a}_{\text{cb}} = GM \left(\frac{\mathbf{r}_{\text{cb}} - \mathbf{r}}{\|\mathbf{r}_{\text{cb}} - \mathbf{r}\|^3} - \frac{\mathbf{r}_{\text{cb}}}{\|\mathbf{r}_{\text{cb}}\|^3} \right), \quad (4)$$

where M is the mass of the celestial body, \mathbf{r}_{cb} is its position in Earth centered inertial reference frame and \mathbf{r} is the position of the satellite in the same reference frame. The orbits of the Sun and the Moon are computed using simple models presented in [9]. The coordinates are accurate to about 0.1–1% [9].

The fourth component in the equation of motion (1) is the acceleration due to the solar radiation pressure. The model used in this paper is of the form

$$\mathbf{a}_{\text{SRP}} = \lambda \left(-\alpha_1 P_0 (1 + \epsilon) \frac{AU^2}{r_{\text{sun}}^2} \frac{A}{m} \mathbf{e}_s + \alpha_2 \mathbf{e}_y \right). \quad (5)$$

In formula (5) the factor r_{sun} is the distance from the satellite to Sun and \mathbf{e}_s is a unit vector from the satellite to the Sun. The vector \mathbf{e}_y points along the satellite's solar panel axis and is calculated using the equation

$$\mathbf{e}_y = \frac{\mathbf{r}_{\text{Sat}} \times (\mathbf{r}_{\text{Sun}} - \mathbf{r}_{\text{Sat}})}{\|\mathbf{r}_{\text{Sat}} \times (\mathbf{r}_{\text{Sun}} - \mathbf{r}_{\text{Sat}})\|}. \quad (6)$$

The constants in the formula are as follows: AU is the astronomical unit, P_0 is the solar radiation pressure at a distance of $1AU$ from the Sun, ϵ is the reflectivity coefficient of the satellite, m is the mass of the satellite and A is the satellite's surface area. The values for the constants are given in Table I. The factor λ is a shadow function, whose value equals one when satellite is in sunlight, zero when it is in umbra and something between when it is in penumbra. We have used the conical shadow model described in the book [9].

The parameters α_1 and α_2 are satellite-specific parameters that are estimated separately for each satellite using precise ephemeris data from International GNSS Service (IGS) [13]. The parameter α_1 is a scaling parameter that is used to account for the uncertainty in the mass, surface area and reflectivity of the satellite. The parameter α_2 represents the magnitude of the y-bias acceleration. The estimation is done using an extended Kalman filter, in which the measurement model is discrete-time and the state model is continuous-time. The application of extended Kalman filter to orbit determination is covered in detail for example in the book [14]. Details of the state and measurement models are presented in [3].

The model presented in equation (5) is not meant to exactly describe the solar radiation pressure acceleration experienced by the GNSS satellite. The intended use is in an autonomous orbit prediction algorithm, where no high precision modeling of the solar radiation pressure is needed. Instead, we want to model the average effect of the solar radiation pressure during a long time interval of for example 7 days.

In the earlier work by Seppänen [3] and Seppänen et al. [4; 5] only the scale parameter for the direct solar radiation pressure was estimated. Using this 1-parameter model the prediction errors for GLONASS were significantly smaller than for GPS satellites. The results presented in this paper show that including the y-bias acceleration to the model, we attain similar prediction accuracy for both GPS and GLONASS satellites.

TABLE I
CONSTANTS IN THE SOLAR RADIATION PRESSURE FORMULA

P_0 [Nm ⁻²]	ϵ	AU [km]	A [m ²]	m [kg]
$4.56 \cdot 10^{-6}$	0.21	149 597 870.691	13.4	1075

III. REFERENCE FRAMES

The position and velocity we get from broadcast ephemeris are in an Earth-fixed, Earth-centered (ECEF) reference frame. An ECEF system has its origin at the mass center of the Earth and its axes are fixed with respect to the Earth's surface. In GPS the reference frame is WGS84, which can be considered equal to the Terrestrial Reference System (TRS) maintained by the International Earth Rotation and Reference Systems Service (IERS). The origin of the TRS system is the Earth's centre of mass and its z -axis is the mean rotational axis of the Earth. This mean pole of rotation was defined, because the Earth's instantaneous rotation pole moves with respect to Earth's crust whereas in Earth fixed reference frame the axes must be pointing at a fixed point on the Earth's surface.

In GLONASS the received ephemeris is in Earth fixed PZ90.02 reference frame [15] but it can be transformed to WGS84 or TRS by a translation of origin [16]

$$\mathbf{r}_{\text{WGS84}} = \mathbf{r}_{\text{PZ90.02}} + \begin{bmatrix} -0.36 \\ 0.08 \\ 0.18 \end{bmatrix} \text{ m.} \quad (7)$$

An inertial reference system maintained by the IERS is the Celestial Reference System (CRS). CRS is a reference system whose coordinate axes maintain their orientation with respect to distant stars. The origin of this reference frame is in the center of the Earth and Earth is in an accelerated motion while orbiting around the sun. Therefore CRS is not precisely inertial, but is an adequate approximation of an inertial reference frame for our purposes. The transformation from the TRS system at epoch t to the CRS system is

$$\mathbf{r}_{\text{TRS}}(t) = \mathbf{W}(t)\mathbf{G}(t)\mathbf{N}(t)\mathbf{P}(t)\mathbf{r}_{\text{CRS}}, \quad (8)$$

where the matrices \mathbf{W} , \mathbf{G} , \mathbf{N} and \mathbf{P} describe polar motion, Earth rotation, nutation and precession, respectively. The transformation matrices are time dependent and thus the vector \mathbf{r}_{CRS} , being constant in CRS, is time dependent after transformation to TRS. We follow the book [9] and use IAU76 theory when computing the precession matrix \mathbf{P} and nutation theory IAU80 for matrix \mathbf{N} , although more recent models IAU2000A and IAU2000B are available and can be found from [17]. The matrix \mathbf{G} describes the rotation of the Earth, details for computing this matrix can be found from [4; 5]. The rotation matrix describing the polar motion is

$$\mathbf{W}(t) = \mathbf{R}_y(-x_p)\mathbf{R}_x(-y_p). \quad (9)$$

where x_p and y_p are the polar motion parameters and \mathbf{R}_x and \mathbf{R}_y are simple rotation matrices around the x - and y -axes. Together with dUT1 they are called also Earth Orientation Parameters (EOP). The daily values for these parameters can be found from the homepage of IERS [18].

The integration of the equation of motion must be done in an inertial reference frame and we choose to use the Terrestrial Intermediate Reference System (TIRS) at epoch t_0 , where t_0 is the initial time of prediction. The TIRS system at an epoch t is connected to the CRS system by the equation

$$\mathbf{r}_{\text{TIRS}(t)} = \mathbf{R}_{\text{CRS}}^{\text{TIRS}(t)} \mathbf{r}_{\text{CRS}}, \quad (10)$$

where the transformation matrix $\mathbf{R}_{\text{CRS}}^{\text{TIRS}(t)}$ is defined by equation

$$\mathbf{R}_{\text{CRS}}^{\text{TIRS}(t)} = \mathbf{G}(t)\mathbf{N}(t)\mathbf{P}(t). \quad (11)$$

Finally the transformation from TRS at an arbitrary time t to the TIRS system at epoch t_0 is

$$\mathbf{r}_{\text{TIRS}(t_0)} = \mathbf{R}_{\text{CRS}}^{\text{TIRS}(t_0)} \left(\mathbf{R}_{\text{CRS}}^{\text{TIRS}(t)} \right)^T \mathbf{W}^T(t) \mathbf{r}_{\text{TRS}}, \quad (12)$$

where we have used the result that $\mathbf{R}_{\text{TIRS}}^{\text{CRS}} = (\mathbf{R}_{\text{CRS}}^{\text{TIRS}})^T$.

For velocity the transformation from TRS to TIRS(t_0) can be computed by differentiating equation (12) with respect to time. When differentiating, the other matrices are treated as

constants and the time dependence of the Earth rotation matrix $\mathbf{G}^T(t)$ is taken into account [4]. The equation for the velocity transformation is [4]

$$\mathbf{v}_{\text{TIRS}(t_0)} = \mathbf{R}_{\text{CRS}}^{\text{TIRS}(t_0)} \mathbf{R}_{\text{TIRS}(t)}^{\text{CRS}} \left(\mathbf{W}^T(t) \mathbf{v}_{\text{TRS}} + \boldsymbol{\omega} \times (\mathbf{W}^T(t) \mathbf{r}_{\text{TRS}}) \right), \quad (13)$$

where $\boldsymbol{\omega} = [0 \ 0 \ \omega]^T$ is the angular velocity vector of the Earth's rotation.

After predicting the satellite's position and velocity at time t in the future, we have to apply the inverse transformations. Solving \mathbf{r}_{TRS} and \mathbf{v}_{TRS} from equations (12) and (13) we get

$$\mathbf{r}_{\text{TRS}} = \mathbf{W}(t) \mathbf{R}_{\text{CRS}}^{\text{TIRS}(t)} \mathbf{R}_{\text{TIRS}(t_0)}^{\text{CRS}} \mathbf{r}_{\text{TIRS}(t_0)} \quad (14)$$

$$\mathbf{v}_{\text{TRS}} = \mathbf{W}(t) \left(\mathbf{R}_{\text{CRS}}^{\text{TIRS}(t)} \mathbf{R}_{\text{TIRS}(t_0)}^{\text{CRS}} \mathbf{v}_{\text{TIRS}(t_0)} - \boldsymbol{\omega} \times \left(\mathbf{R}_{\text{CRS}}^{\text{TIRS}(t)} \mathbf{R}_{\text{TIRS}(t_0)}^{\text{CRS}} \mathbf{r}_{\text{TIRS}(t_0)} \right) \right). \quad (15)$$

When the satellite's orbit is predicted, we actually do not know the exact matrix $\mathbf{W}(t)$. However using the algorithm described in the next section we can solve the polar motion parameters x_p and y_p at the initial time t_0 . Because they do not change much in a few days long prediction, we can make the approximation

$$\mathbf{W}(t) \approx \mathbf{W}(t_0). \quad (16)$$

Similarly the third Earth orientation parameter dUT1 is needed to compute the rotation matrices $\mathbf{R}_{\text{CRS}}^{\text{TIRS}(t_0)}$ and $\mathbf{R}_{\text{CRS}}^{\text{TIRS}(t)}$ [4]. In this paper we set dUT1 = 0 when computing these matrices. This approximation is made since dUT1 may, in general, be unknown to the device and its evolution is very difficult to predict. We have found that making this approximation results in median error of about 4 meters in the satellite's position in a prediction over 4 days.

The satellite's position has to be represented in an Earth fixed reference frame to calculate the acceleration due to the Earth's gravitation. We use the TRS reference frame, so the transformation matrix \mathbf{R} used in the equation (3) is $\mathbf{W}(t) \mathbf{R}_{\text{CRS}}^{\text{TIRS}(t)} \mathbf{R}_{\text{TIRS}(t_0)}^{\text{CRS}}$. Some approximations to the calculation of this matrix can be made to speed up the numerical integration. If the length of the prediction is only few days, the precession and nutation matrices can be assumed to stay constant, and we can make the approximations $\mathbf{P}(t)\mathbf{P}(t_0)^T \approx \mathbf{I}$ and $\mathbf{N}(t)\mathbf{N}(t_0)^T \approx \mathbf{I}$. Using these approximations we have

$$\mathbf{W}(t) \mathbf{R}_{\text{CRS}}^{\text{TIRS}(t)} \mathbf{R}_{\text{TIRS}(t_0)}^{\text{CRS}} \approx \mathbf{W}(t) \mathbf{G}(t) \mathbf{G}^T(t_0). \quad (17)$$

Multiplication with the matrix \mathbf{G} rotates the reference frame according to the Earth's rotation. Mainly it is a simple rotation around the z -axis, with the angular speed of the Earth. If the x -axis points to a certain meridian at the initial time t_0 , then at the time t it points to the direction we get by rotating the x -axis around the z -axis with an angle of $(t - t_0)\omega$. Thus

$$\mathbf{W}(t) \mathbf{G}(t) \mathbf{G}^T(t_0) = \mathbf{W}(t) \mathbf{R}_z((t - t_0)\omega) \quad (18)$$

is the matrix \mathbf{R} used to transform the inertial vector to an Earth fixed reference frame when computing the Earth gravitational

acceleration. Its transpose is then used to transform the acceleration vector to the TIRS(t_0) reference frame. Again, the matrix $\mathbf{W}(t)$ is replaced with $\mathbf{W}(t_0)$ while predicting.

IV. INITIAL STATE IMPROVEMENT

In GPS the satellite's position is calculated using the 16 ephemeris parameters that are broadcast by the satellite. One of these parameters is called the time of ephemeris (TOE). With one received navigation message i.e. one set of ephemeris parameters the satellite's position can be calculated at any instant within ± 2 h from the TOE. Going outside of this range the accuracy of the ephemeris deteriorates rapidly. The GPS satellite's velocity can be computed by differentiating the ephemeris parameters with respect to time. The equations for computing GPS satellite's broadcast velocity are presented for instance in [19].

In GLONASS the broadcast ephemeris is given in the form of satellite's position and velocity at the TOE instant, which is denoted t_b in the GLONASS ICD [20]. In addition, the current value for the acceleration originating from the gravitational interactions with the Sun and the Moon is given. This acceleration is part of a simple motion model that can be used to solve the satellite's ephemeris at any other time instant within about 15 minutes from t_b . A new navigation message is broadcast every half hour.

The position and velocity computed directly from the broadcast ephemeris are not accurate enough for orbit prediction over several days. For this reason we use a fitting method to improve the initial state of the satellite. Also we assume that the earth orientation parameters are unknown at the start of the prediction, so we need a method to solve these from the broadcast ephemeris.

In this paper we use the fitting method presented by Seppänen et al. [4; 5] where we first apply an antenna correction to the position obtained from the broadcast ephemeris and then solve for the initial velocity and the two earth orientation parameters x_p and y_p using multiple positions and velocities from the broadcast ephemeris. The antenna offsets for GPS satellites are obtained from NGA [21], and for GLONASS we use the values from [22].

After applying the antenna correction we solve for the initial velocities and earth orientation parameters by fitting our model to the broadcast data. A short overview of the method is described here, for more details see [4] and [5]. If we have n satellites the vector of unknowns is

$$\mathbf{x} = \begin{bmatrix} \mathbf{v}_0^1 \\ \vdots \\ \mathbf{v}_0^n \\ \mathbf{p}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{v}_0^{\text{all}} \\ \mathbf{p}_0 \end{bmatrix}, \quad (19)$$

where \mathbf{v}_0^i is the velocity of the i :th satellite and $\mathbf{p}_0 = [x_p(t_0) \ y_p(t_0)]^T$ are the polar motion parameters. As measurements we have the satellites' states \mathbf{y}_k^i at time instants t_1, \dots, t_m which can be described by equation

$$\mathbf{y}_k^i = \mathbf{f}_k^i(\mathbf{x}) + \boldsymbol{\varepsilon}_k^i, \quad i = 1, \dots, n \quad k = 1, \dots, m. \quad (20)$$

Here the vectors $\boldsymbol{\varepsilon}_k^i$ are the differences between the measured state at instant t_k and the state that was predicted using the nonlinear function \mathbf{f}_k^i defined by equation

$$\begin{bmatrix} \mathbf{r}^i(t_k) \\ \mathbf{v}^i(t_k) \end{bmatrix} = \mathbf{f}_k^i(\mathbf{x}) = \boldsymbol{\vartheta}^{\text{TRS}}(t_0, t_k, \mathbf{r}_0^i, \mathbf{v}_0^i, \mathbf{p}_0), \quad i = 1, \dots, n, \quad (21)$$

where $\boldsymbol{\vartheta}^{\text{TRS}}$ is the function that predicts satellite states by integrating a nonlinear differential equation and carrying out the required transformations between the TRS and the inertial reference frame. Collecting the differences $\boldsymbol{\varepsilon}_k^i$ into a single vector $\boldsymbol{\varepsilon}$, the cost function to be minimized is now given by equation

$$J(\mathbf{x}) = \boldsymbol{\varepsilon}^T \mathbf{D} \boldsymbol{\varepsilon}, \quad (22)$$

where the diagonal weight matrix \mathbf{D} has the value $(1000)^2$ in those elements corresponding to velocity components and ones corresponding to position components. If only satellites' positions are used as measurements, the weights corresponding to the velocity components can be set to zero. The cost function is minimized using the Levenberg-Marquardt method. As initial values for the polar motion parameters we use $x_p = 0.05$ arcseconds and $y_p = 0.35$ arcseconds, which is the approximate center of the polar motion spiral during the years 2004-2008. In practice the initial iterate for polar motion could be taken from the results of a previous solution.

For GPS satellites the initial time instant is chosen to be $t_0 = \text{TOE} + 1.5$ h and the measurement instants are $t_1 = \text{TOE}$ and $t_2 = \text{TOE} - 1.5$ h. For GLONASS the position and velocity are obtained only at one time instant, which can be used to integrate the orbit with the simple motion model from GLONASS ICD [20] about 15 minutes to each direction. In order to achieve good prediction results also with GLONASS we have to use more than one received ephemeris parameter set [5]. For this work we decided to use the position and velocity obtained without integration from two broadcast ephemerides, with time difference of 12 hours.

The tests done in this paper use $n = 5$ satellites to solve for the earth orientation parameters and initial velocities. We have observed that in order to get sufficiently accurate values for the earth orientation parameters we need preferably at least 3 satellites in the initial state fitting algorithm.

V. RESULTS

In the estimation of the solar radiation pressure parameters we used IGS precise ephemerides from GPS weeks 1545-1618 for GPS satellites. For GLONASS satellites we use IGS precise ephemerides from GPS weeks 1570-1618. A time series of parameter estimates were obtained by taking separate consecutive 7-day arcs and estimating the parameters for each arc using the extended Kalman filter algorithm. For each arc the final output of the estimator was taken as the estimated value.

The time series of parameter estimates for two GPS satellites in the same orbit slot are shown in Figures 1 and 2. The effect of eclipse seasons to the estimates is clearly visible, especially in the estimated y-bias acceleration where sharp spikes in the

estimates can be seen during eclipse seasons. The eclipse season is a time period where the satellite enters the Earth's shadow during each orbit revolution. The time spent in the shadow varies, but lasts a maximum of about 56 minutes [7]. Similar behavior is observed for all GPS and GLONASS satellites.

Since our model uses constant values for scale factor and y-bias acceleration we take the median of the time series to be used in the model. The resulting values are shown in Tables II and III. For GPS satellites the y-bias acceleration varies from -10^{-9}m/s^2 to 10^{-9}m/s^2 , being clearly different from zero for most satellites. For GLONASS satellites the estimated y-bias accelerations are much smaller and near zero for most satellites. This is in agreement with the results of Ineichen et al. [8], where it was found out that for GLONASS satellites the y-bias accelerations were on average very close to zero.

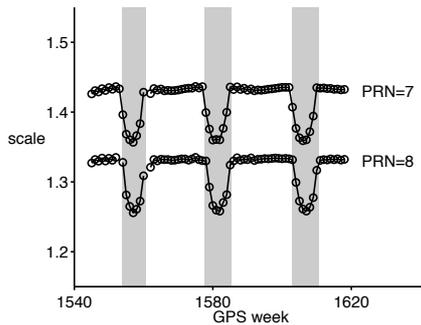


Fig. 1. Time series of scale parameter α_1 estimates for GPS satellites PRN 7 and PRN 8. Shaded areas represent eclipse seasons.

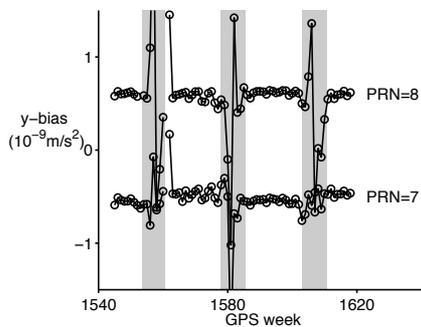


Fig. 2. Time series of y-bias acceleration α_2 estimates for GPS satellites PRN 7 and PRN 8. Shaded areas represent eclipse seasons.

The method was tested by predicting the orbits of GPS and GLONASS satellites using broadcast ephemeris data from GPS weeks 1618-1632. The initial velocity and earth orientation parameters were computed by the method described in the previous section. For GLONASS we used two broadcast ephemerides with time separation of 12 hours. The Runge-Kutta-Nyström -method [9; 23] was used to solve the equation of motion. The algorithm is an efficient integration method for second order differential equations in which the second derivative (the acceleration \mathbf{a}) is independent on the first derivative (the velocity).

TABLE II
SOLAR RADIATION PRESSURE PARAMETERS FOR GPS SATELLITES

PRN	1	2	3	4	5	6
α_1	1.424	1.473	1.326	1.331	1.423	1.328
α_2 (10^{-9}m/s^2)	-0.548	0.101	0.409	0.784	-0.572	0.376
	7	8	9	10	11	12
	1.432	1.331	1.333	1.319	1.479	1.438
	-0.523	0.599	0.636	0.572	0.089	-0.607
	13	14	15	16	17	18
	1.469	1.470	1.445	1.477	1.432	1.475
	-0.411	-0.399	-0.715	0.129	-0.521	-0.286
	19	20	21	22	23	24
	1.463	1.465	1.450	1.459	1.489	1.350
	-0.429	-0.815	-0.124	-0.516	-0.153	0.941
	25	26	27	28	29	30
	1.551	1.333	1.333	1.460	1.426	1.329
	-0.022	0.765	0.887	-0.112	-0.343	0.784
	31	32				
	1.440	1.345				
	-0.604	1.034				

TABLE III
SOLAR RADIATION PRESSURE PARAMETERS FOR GLONASS SATELLITES

PRN	1	2	3	4	5	6
α_1	2.162	2.175	2.135	2.160	2.164	2.166
α_2 (10^{-9}m/s^2)	0.101	0.235	0.021	0.116	0.041	0.101
	7	8	9	10	11	12
	2.152	2.151	2.141	2.151	2.139	2.143
	-0.021	0.047	0.165	0.125	0.060	0.053
	13	14	15	16	17	18
	2.141	2.145	2.144	2.141	2.146	2.150
	-0.077	-0.081	0.031	0.106	-0.051	0.091
	19	20	21	22	23	24
	2.131	2.155	2.147	2.166	2.110	2.173
	-0.119	0.033	0.001	0.065	0.013	0.095

The predicted orbit was compared to the precise positions obtained from the IGS. A box plot of the norms of the prediction errors are shown in figures 3 and 4. The boxes show the 25%-, 50%- and 75%-quantiles and the whiskers extending from the boxes show the 5%- and 95%-quantiles. From the results it can be seen that for GPS satellites a significant improvement is obtained by including the y-bias acceleration to the solar radiation pressure model. For a 5 day long prediction the median error is reduced from 87 meters to about 22 meters and the 95%-quantile is reduced from 250 meters to about 73 meters. For GLONASS satellites the improvement in orbit prediction accuracy is much smaller. For a 5 day long prediction the median error is reduced from 28 meters to 25 meters and the 95%-quantile is reduced from 98 meters to 86 meters. However this was expected since the estimated y-bias accelerations were much smaller for the GLONASS satellites.

Figures 5 and 6 show the orbit prediction errors in the RTN (Radial, Tangential, Normal) coordinate system using the 2-parameter solar radiation pressure model. The transformation from ECI to RTN is calculated using equation [14]

$$\mathbf{r}_{\text{RTN}} = [\mathbf{e}_R \quad \mathbf{e}_T \quad \mathbf{e}_N] \mathbf{r}_{\text{ECI}}, \quad (23)$$

where \mathbf{r}_{ECI} and \mathbf{v}_{ECI} are the position and velocity of the satellite in ECI coordinate system and the unit vectors \mathbf{e}_R , \mathbf{e}_T and \mathbf{e}_N

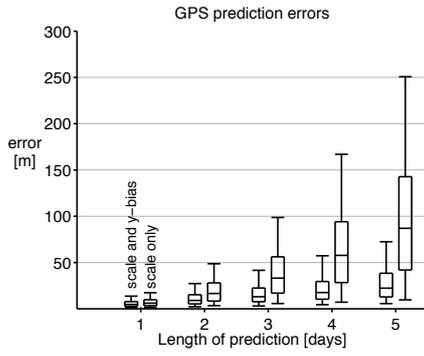


Fig. 3. Prediction errors for GPS satellites with the different solar radiation pressure models.

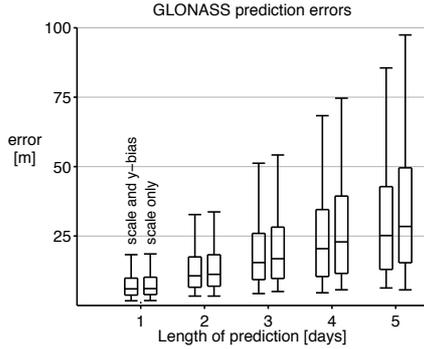


Fig. 4. Prediction errors for GLONASS satellites with the different solar radiation pressure models.

are calculated using equations

$$\mathbf{e}_R = \frac{\mathbf{r}_{\text{ECI}}}{\|\mathbf{r}_{\text{ECI}}\|} \quad (24)$$

$$\mathbf{e}_T = \mathbf{e}_N \times \mathbf{e}_R \quad (25)$$

$$\mathbf{e}_N = \frac{\mathbf{r}_{\text{ECI}} \times \mathbf{v}_{\text{ECI}}}{\|\mathbf{r}_{\text{ECI}} \times \mathbf{v}_{\text{ECI}}\|} \quad (26)$$

Here the directions \mathbf{e}_T and \mathbf{e}_N may be referred also to as along-track direction and cross-track direction respectively. From the results it is seen that the errors in the orbit prediction are mainly in the tangential or along-track direction and the errors in the radial and normal directions are relatively small. For positioning purposes this is a favorable result because the radial error tends to have the largest effect to the pseudorange error of the satellite [5].

95%-quantiles of the RTN-errors in a 5 day long prediction for the two solar radiation pressure models are compared in Tables IV and V. For GPS satellites the tangential error reduces significantly by including the y-bias acceleration to the model. The relative reduction in the radial error is also quite large. For GLONASS satellites including the y-bias acceleration reduces somewhat the tangential error, but the accuracy in the radial and normal directions show no clear improvement.

VI. CONCLUSION

The TTFF of a GNSS positioning device can be reduced if the satellites position and velocity can be obtained before

TABLE IV
95%-QUANTILE OF THE RTN ERRORS FOR GPS SATELLITES (5 DAY PREDICTION)

	Radial	Tangential	Normal
scale only	6.7 m	250.1 m	11.5 m
scale and y-bias	3.4 m	72.3 m	11.1 m

TABLE V
95%-QUANTILE OF THE RTN ERRORS FOR GLONASS SATELLITES (5 DAY PREDICTION)

	Radial	Tangential	Normal
scale only	4.1 m	97.2 m	15.0 m
scale and y-bias	4.0 m	85.4 m	15.0 m

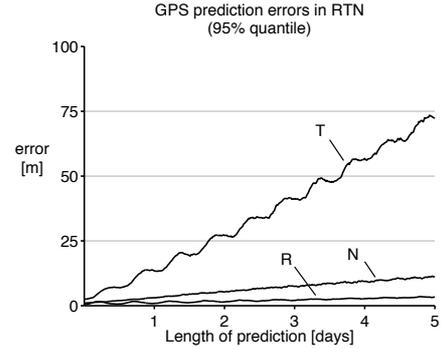


Fig. 5. 95%-quantile of the prediction errors in the RTN reference frame for GPS satellites.

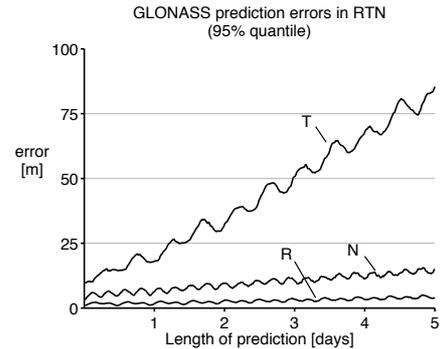


Fig. 6. 95%-quantile of the prediction errors in the RTN reference frame for GLONASS satellites.

reading the navigation message send by the satellite. For devices without a network connection the position and velocity can be predicted by integrating the satellites equation of motion. The initial conditions for the integration can be computed from a previously received broadcast message. In this paper the autonomous orbit prediction algorithm presented in [3–5] was improved by including the y-bias acceleration to the empirical solar radiation pressure model.

The model uses two parameters that have to be estimated separately for each satellite. The results presented in this paper show that for this kind of simple model the parameters can be approximated by constant values, that can be estimated offline using for example the precise orbits from IGS. The parameter

values need to be updated only when the satellite constellation is changed. This can be handled by updating the parameters a few times per year, for instance, as a part of a software update.

The model was tested with GPS and GLONASS satellites by predicting the satellites orbits several days into the future using improved initial conditions obtained from broadcast ephemerides. The results show that for GPS satellites we get a significant improvement in the orbit prediction accuracy by using the 2-parameter solar radiation pressure model. For GLONASS satellites the estimated y-bias accelerations were small and for this reason only a small reduction can be seen in the orbit prediction errors.

Looking at the errors in a satellite centered RTN coordinate system, we see that the errors are mainly in the tangential or along-track direction. The errors in the radial direction are found to be small: in a 5 day long prediction 95% of the radial errors are below 4 meters for GPS satellites and about 4 meters for GLONASS satellites. This is a favorable result because the radial error has the greatest effect on the pseudorange error of the satellite [5].

This paper was concerned only in the prediction of the satellites' position and velocity. In a completely autonomous orbit prediction algorithm we need also a method to predict the satellites' clock offsets. Our recent paper [5] discusses also the clock offset prediction and considers how the errors in the orbit and clock offset prediction are combined in order to get the total error in the range measurement.

ACKNOWLEDGMENTS

This research was funded by Nokia Inc. and Tampere Doctoral Programme in Information Science and Engineering (TISE).

REFERENCES

- [1] P. G. Mattos, "Hotstart every time - compute the ephemeris on the mobile," in *Proceedings of the 21st International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2008)*, Savannah, GA, September 2008, pp. 204–211.
- [2] W. Zhang, V. Venkatasubramanian, H. Liu, M. Phatak, and S. Han, "SiRF InstantFix II Technology," in *Proceedings of the 21st International Technical Meeting of the Satellite Division of the Institute of Navigation (ION GNSS 2008)*, Savannah, GA, September 2008, pp. 1840 – 1847.
- [3] M. Seppänen, "GPS-satelliitin radan ennustaminen," M.Sc. Thesis, Tampere University of Technology, March 2010. [Online]. Available: http://math.tut.fi/posgroup/seppanen_mscth.pdf
- [4] M. Seppänen, T. Perälä, and R. Piché, "Autonomous satellite orbit prediction," in *2011 International Technical Meeting*, San Diego, California, January 2011. [Online]. Available: <http://www.ion.org/meetings/abstractSearch/showAbstract.cfm?key=72&t=C&s=2&collection=ITM2011Abstracts>
- [5] M. Seppänen, J. Ala-Luhtala, R. Piché, S. Martikainen, and S. Ali-Löyty, "Autonomous prediction of GPS and GLONASS satellite orbits," *NAVIGATION*, in press.
- [6] T. Springer, G. Beutler, and M. Rothacher, "A new solar radiation pressure model for GPS satellites," *GPS Solutions*, vol. 2, pp. 50–62, 1999.
- [7] L. O. Froidevall, "A study of solar radiation pressure acting on GPS satellites," Ph.D. dissertation, The University of Texas at Austin, Austin TX USA, August 2009. [Online]. Available: <http://repositories.lib.utexas.edu/bitstream/handle/2152/6623/froidevall45515.pdf>
- [8] D. Ineichen, T. Springer, and G. Beutler, "Combined processing of the IGS and the IGEX network," *Journal of Geodesy*, vol. 75, pp. 575–586, 2001, 10.1007/s001900000152. [Online]. Available: <http://dx.doi.org/10.1007/s001900000152>
- [9] O. Montenbruck and E. Gill, *Satellite Orbits*, 3rd ed. Berlin Heidelberg New York: Springer, 2005, 369 p.
- [10] An Earth Gravitational Model to Degree 2160: EGM2008, presented at the 2008 General Assembly of the European Geosciences Union, Vienna, Austria, April 13-18, 2008. [Online]. Available: http://earth-info.nima.mil/GandG/wgs84/gravitymod/egm2008/first_release.html
- [11] L. Cunningham, "On the computation of the spherical harmonic terms needed during the numerical integration of the orbital motion of an artificial satellite," *Celestial Mechanics*, vol. 2, pp. 207–216, 1970.
- [12] G. Métris, J. Xu, and I. Wytrzyszczak, "Derivatives of the gravity potential with respect to rectangular coordinates," *Celestial Mechanics and Dynamical Astronomy*, vol. 71, no. 2, pp. 137–151, 1999.
- [13] J. Dow, R. Neilan, and C. Rizos, "The international GNSS service in a changing landscape of Global Navigation Satellite Systems," *Journal of Geodesy*, vol. 83, pp. 689–689, 2009, 10.1007/s00190-009-0315-4. [Online]. Available: <http://dx.doi.org/10.1007/s00190-009-0315-4>
- [14] B. D. Tapley, B. E. Schutz, and G. H. Born, *Statistical Orbit Determination*. Elsevier Academic Press, 2004.
- [15] Global navigation satellite system GLONASS: Interface Control Document (GLONASS ICD), version 5.1, (2008), Russian Institute of Space Device Engineering/Research. [Online]. Available: [http://facility.unavco.org/data/docs/ICD_GLONASS_5.1_\(2008\)_en.pdf](http://facility.unavco.org/data/docs/ICD_GLONASS_5.1_(2008)_en.pdf)
- [16] Global navigation satellite system and global positioning system. Coordinate systems. Methods of transformations for determined points coordinate, 2008. [Online]. Available: <http://protect.gost.ru/document.aspx?control=7&baseC=6&page=0&month=9&year=2009&search=&id=174517>
- [17] D. D. McCarthy and G. Petit., *iERS Conventions (2003)*. (IERS Technical Note ; 32) Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie, 2004. 127 pp., paperback, ISBN 3-89888-884-3.
- [18] IERS C04 series of the Earth orientation parameters. [Online]. Available: http://data.iers.org/products/214/14443/orig/eopc04_08_IAU2000.62-now

- [19] P. Korvenoja and R. Piché, “Efficient satellite orbit approximation,” in *Proceedings of ION GPS 2000, September 19-22, 2000, Salt Lake City, USA*, 2000, pp. 1930–1937. [Online]. Available: http://math.tut.fi/posgroup/korvenoja_piche_ion2000a.pdf
- [20] Global navigation satellite system GLONASS: Interface Control Document (GLONASS ICD), version 5.0, (2002), Russian Institute of Space Device Engineering/Research.
- [21] NGA GPS Ephemeris/Station/Antenna Offset Documentation. [Online]. Available: http://earth-info.nga.mil/GandG/sathtml/gpsdoc2010_09a.html
- [22] C. Cai, “Precise point positioning using dual-frequency GPS and GLONASS measurements,” Master’s thesis, University of Calgary, Calgary, Alberta, August 2009. [Online]. Available: http://www.ucalgary.ca/engo_webdocs/YG/09.20291_ChangshengCai.pdf
- [23] E. Hairer, S. P. Norsett, and G. Wanner, *Solving Ordinary Differential Equations I: Nonstiff Problems*, 2nd ed. Berlin: Springer-Verlag, 1993, 528 p.