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Gaussian Mixture Filter Allowing Negative Weights and its Application to Positioning Using Signal Strength Measurements

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***Abstract*—This paper proposes a novel Gaussian Mixture Filter (GMF) that allows components with negative weights. In case of a ring-shaped likelihood function, the new filter keeps the number of components low by approximating the likelihood as a Gaussian mixture (GM) of two components, one with positive and the other with negative weight. In this article, the filter is applied to positioning with received signal strength (RSS) based range measurements. The filter is tested using simulated measurements, and the tests indicate that the new GMF outperforms the Extended Kalman Filter (EKF) in both accuracy and consistency.**

I. Introduction

Hybrid navigation means navigation using measurements from different sources, such as Global Navigation Satellite Systems (e.g. GPS), Inertial Measurement Unit, or local wireless networks such as cellular networks, Bluetooth or WLAN. Range, pseudorange, deltarange, altitude, restrictive and compass measurements are examples of measurements in hybrid navigation. This paper focuses on hybrid navigation using different local wireless networks. The ranges from the network's base stations (BS) that are calculated using received signal strengths (RSS) are used as measurements.

Bayesian filtering theory is applied to improve the position and error estimates. The measurement model of range measurements is typically nonlinear. Thus, the traditional Kalman filter cannot be applied. The most popular example of navigation filters is the Extended Kalman Filter (EKF), which linearizes system and measurement models and then applies the Kalman Filter [1, 2]. This filter is commonly used in satellite positioning and has also been applied to hybrid navigation [3].

Unfortunately, the EKF has a serious consistency problem in highly nonlinear situations, which means that it does not work correctly [4]. In highly nonlinear situations sometimes multiple static position solutions exist, which means that the likelihood function has multiple peaks with significant weight. In this case, it is more reasonable to approximate the likelihood as a Gaussian Mixture (GM) and use Gaussian Mixture Filter (GMF) than to approximate it with only one Gaussian as done by the EKF.

One major challenge in using GMF efficiently is keeping the number of components as small as possible without losing significant information. Therefore, this paper introduces a GMF in which the likelihood is approximated as GM consisting of only two Gaussian components. The novel idea that allows to use this small amount of components and simultaneously approximating the multiple peaks of the likelihood function is to bypass the restriction to nonnegative component weights, which for traditional GM has to be fulfilled.

This paper is organized as follows. After problem formulation in section II, the mathematical fundamentals of the new algorithm are presented in section III. Section IV explains how to apply the new filter for navigating and how to select the model parameters. The results and analyses of tests with simulated cellular data are presented in Section V. Section VI concludes.

II. Bayesian Filtering

This section considers the discrete-time non-linear non-Gaussian system

$$x_k = f_{k-1}(x_{k-1}) + w_{k-1}, \tag{1}$$

$$y_k = h_k(x_k) + v_k, \tag{2}$$

where the vectors $x_k \in \mathbb{R}^{n_x}$ and $y_k \in \mathbb{R}^{n_{y_k}}$ represent the state of the system and the measurement at time t_k, $k \in \mathbb{N}$, respectively. The errors w_k and v_k are assumed to be white, mutually independent and independent of the initial state x_0. In the following the density functions of w_k and v_k are denoted by p_{w_k} and p_{v_k}, respectively. The aim of filtering is to find the conditional probability density function (posterior)

$$p(x_k|y_{1:k}),$$

where $y_{1:k} \triangleq \{y_1, \ldots, y_k\}$. The posterior can be determined recursively according to the following relations [5, 6].

Prediction (prior):

$$p(x_k|y_{1:k-1}) = \int p(x_k|x_{k-1}) p(x_{k-1}|y_{1:k-1}) \,\mathrm{d}\, x_{k-1}; \tag{3}$$

Update (posterior):

$$p(x_k|y_{1:k}) = \frac{p(y_k|x_k) p(x_k|y_{1:k-1})}{\int p(y_k|x_k) p(x_k|y_{1:k-1}) \,\mathrm{d}\, x_k}, \tag{4}$$

where the transition pdf is

$$p(x_k|x_{k-1}) = p_{w_{k-1}}(x_k - f_{k-1}(x_{k-1}))$$

and the likelihood

$$p(y_k|x_k) = p_{v_k}(y_k - h_k(x_k)). \tag{5}$$

The initial condition for the recursion is given by the pdf of the initial state $p(x_0|y_{1:0}) = p(x_0)$. Knowledge of the posterior distribution (4) enables one to compute an optimal state estimate with respect to any criterion. For example, the minimum mean-square error (MMSE) estimate is the conditional mean of x_k [1, 6]. In general and in the case analyzed within this paper, the conditional probability density function cannot be determined analytically. Because of this, there are many approximative solutions of conditional mean. One popular solution is the Extended Kalman Filter (EKF), which applies Kalman filtering to a linerization of system (1), (2).

III. Gaussian Mixture Filter allowing negative weights

In this section an analytic solution to a particular instance of the Bayesian filtering problem (Section II) is presented. Here the motion model (1) is assumed to be linear and Gaussian:

$$x_k = \mathrm{F}_{k-1}x_{k-1} + w_{k-1}, \tag{6}$$

where w_{k-1} is zero-mean Gaussian (Normal) with covariance Q_{k-1}, that is, $w_{k-1} \sim \mathrm{N}(0, \mathrm{Q}_{k-1})$. Moreover the likelihoods are assumed to have forms

$$p(y_k|x_k) = \mathrm{N}^{\mathrm{H}_{k,1}x_k}_{\mathrm{R}_{k,1}}(m_1(y_k))\left(1 - \bar{c}\,\mathrm{N}^{\mathrm{H}_{k,2}x_k}_{\mathrm{R}_{k,2}}(m_2(y_k))\right) \tag{7}$$

where y_k represents the measurement, $\mathrm{H}_{k,1} \in \mathbb{R}^{n_1\times n}$, $\mathrm{H}_{k,2} \in \mathbb{R}^{n_2\times n}$, parameter $\bar{c} = c\cdot(2\pi)^{\frac{n_2}{2}}\sqrt{\det(\mathrm{R}_{k,2})}$, $c \leq 1$, and m_1 and m_2 are some known functions. In the case considered in this paper measurement y is RSS-value from base station BS_i and $m_1(y) = m_2(y)$ are BS_i's position. Function $\mathrm{N}^{\mu_j}_{\Sigma_j}(x)$ denotes the Gaussian density function with vector mean μ_j and covariance matrix Σ_j

$$\mathrm{N}^{\mu_j}_{\Sigma_j}(x) = \frac{\exp\left(-\frac{1}{2}\|x-\mu_j\|^2_{\Sigma_j^{-1}}\right)}{(2\pi)^{\frac{n}{2}}\sqrt{\det(\Sigma_j)}},$$

where $\|x-\mu_j\|^2_{\Sigma_j^{-1}} = (x-\mu_j)^T\,\Sigma_j^{-1}\,(x-\mu_j)$. Because of parameter $\bar{c}$ the likelihood (7) is always nonnegative and so it is a valid likelihood. That kind of likelihoods are not entirely new, they have been used in tracking with "negative" information [7]. If $\bar{c} = 0$ and $m_1(y_k) = y_k$ then the conventional Kalman Filter computes the analytic solution. Using likelihood (7) results in a Gaussian mixture, which can also have negative weights. That kind of Gaussian mixture is defined in Definition 1.

Definition 1 (Gaussian Mixture allowing negative weights): An n-dimensional random variable x is an N-component Gaussian Mixture allowing negative weights (GMA) if its probability density function has the form

$$p(x) = \sum_{j=1}^{N}\alpha_j\,\mathrm{N}^{\mu_j}_{\Sigma_j}(x), \tag{8}$$

where $\alpha_j \in \mathbb{R}$ and $\sum_{j=1}^{N}\alpha_j = 1$. Here some weights are possibly negative but it is required that $p(x) \geq 0$ for all $x \in \mathbb{R}^n$. The abbreviation $x \sim \mathrm{M}(\alpha_j, \mu_j, \Sigma_j)_{(j,N)}$ is used.

Mean and covariance formulas of Gaussian Mixture allowing negative weights are the same as for conventional Gaussian Mixture.

Theorem 2 (Mean and covariance of GMA): Let $x \sim \mathrm{M}(\alpha_j, \mu_j, \Sigma_j)_{(j,N)}$ then

$$\mathrm{E}(x) = \sum_{j=1}^{N}\alpha_j\mu_j \stackrel{\triangle}{=} \mu \text{ and}$$
$$\mathrm{V}(x) = \sum_{j=1}^{N}\alpha_j\left(\Sigma_j + (\mu_j-\mu)(\mu_j-\mu)^T\right)$$

Proof:

$$\begin{aligned}\mathrm{E}(x) &= \int x\left(\sum_{j=1}^{N}\alpha_j\,\mathrm{N}^{\mu_j}_{\Sigma_j}(x)\right)\mathrm{d}\,x \\ &= \sum_{j=1}^{N}\alpha_j\int x\,\mathrm{N}^{\mu_j}_{\Sigma_j}(x)\,\mathrm{d}\,x = \sum_{j=1}^{N}\alpha_j\mu_j\end{aligned}$$

$$\begin{aligned}\mathrm{V}(x) &= \int xx^T\left(\sum_{j=1}^{N}\alpha_j\,\mathrm{N}^{\mu_j}_{\Sigma_j}(x)\right)\mathrm{d}\,x - \mu\mu^T \\ &= \sum_{j=1}^{N}\alpha_j\int xx^T\,\mathrm{N}^{\mu_j}_{\Sigma_j}(x)\,\mathrm{d}\,x - \mu\mu^T \\ &= \sum_{j=1}^{N}\alpha_j\left(\Sigma_j + \mu_j\mu_j^T\right) - \mu\mu^T \\ &= \sum_{j=1}^{N}\alpha_j\left(\Sigma_j + (\mu_j-\mu)(\mu_j-\mu)^T\right)\end{aligned}$$

■

In contrast to the new Gaussian Mixture Filter (GMFA) (Algorithm 1), conventional Gaussian Mixture Filter (GMF) does not allow negative weights [8, 9].

Algorithm 1 Gaussian mixture filter allowing negative weights

Initial state at time t_0: $x_0 \sim \mathrm{M}(\alpha_{i,0}, \mu_{i,0}, \Sigma_{i,0})_{(i,n_0)}$
for $k = 1$ to n_{meas} **do**

1) Prediction (see Sec. III-A):
$$x_k|y_{1:k-1} \sim \mathrm{M}(\alpha^-_{i,k}, \mu^-_{i,k}, \Sigma^-_{i,k})_{(i,n_k^-)}$$
2) Update (see Sec. III-B):
$$x_k|y_{1:k} \sim \mathrm{M}(\alpha_{i*j,k}, \mu_{i*j,k}, \Sigma_{i*j,k})_{(i*j,\bar{n}_k)}$$
3) Re-indexing and possibly reducing the number of components (see Sec. III-C):
$$x_k|y_{1:k} \sim \mathrm{M}(\alpha_{i,k}, \mu_{i,k}, \Sigma_{i,k})_{(i,n_k)}$$

end for

A. Prediction, Step (1)

Prediction is based on Eq. (3). Previous posterior distribution is

$$x_{k-1}|y_{1:k-1} \sim \mathrm{M}(\alpha_{i,k-1}, \mu_{i,k-1}, \Sigma_{i,k-1})_{(i,n_{k-1})}.$$

$$\begin{aligned}p(x_k|y_{1:k-1}) &= \int p(x_k|\xi) p_{x_{k-1}|y_{1:k-1}}(\xi)\,\mathrm{d}\,\xi \\ &= \int p(x_k|\xi)\left(\sum_{i=1}^{n_{k-1}} \alpha_{i,k-1}\,\mathrm{N}^{\mu_{i,k-1}}_{\Sigma_{i,k-1}}(\xi)\right)\mathrm{d}\,\xi \\ &= \sum_{i=1}^{n_{k-1}} \alpha_{i,k-1} \int p(x_k|\xi)\,\mathrm{N}^{\mu_{i,k-1}}_{\Sigma_{i,k-1}}(\xi)\,\mathrm{d}\,\xi \\ &= \sum_{i=1}^{n_{k-1}} \alpha_{i,k-1}\,\mathrm{N}^{\mathrm{F}_{k-1}\mu_{i,k-1}}_{\mathrm{F}_{k-1}\Sigma_{i,k-1}\mathrm{F}_{k-1}^T+\mathrm{Q}_{k-1}}(x_k)\end{aligned} \tag{9}$$

$$x_k|y_{1:k-1} \sim \mathrm{M}(\alpha^-_{i,k}, \mu^-_{i,k}, \Sigma^-_{i,k})_{(i,n^-_k)},$$

where

$$\begin{aligned}n^-_k &= n_{k-1}, \\ \alpha^-_{i,k} &= \alpha_{i,k-1}, \\ \mu^-_{i,k} &= \mathrm{F}_{k-1}\mu_{i,k-1} \text{ and} \\ \Sigma^-_{i,k} &= \mathrm{F}_{k-1}\Sigma_{i,k-1}\mathrm{F}_{k-1}{}^T + \mathrm{Q}_{k-1}.\end{aligned}$$

Since the previous posterior and $p(x_k|\xi)$ are always non-negative the prior distribution $p(x_k|y_{1:k-1})$ is always non-negative and therefore a valid GMA.

B. Update, Step 2

The update is based on Eq. (4) and likelihood is given in (7).

$$\begin{aligned}&p(x|y_{1:k}) \propto p(y_k|x)p(x|y_{1:k-1}) = p(y_k|x)\sum_{i=1}^{n_k}\alpha^-_{i,k}\,\mathrm{N}^{\mu^-_{i,k}}_{\Sigma^-_{i,k}}(x) \\ &= \sum_{i=1}^{n_k}\alpha^-_{i,k}\left(\mathrm{N}^{\mathrm{H}_1 x}_{\mathrm{R}_{k,1}}(m_1)\left(1-\bar{c}\,\mathrm{N}^{\mathrm{H}_2 x}_{\mathrm{R}_{k,2}}(m_2)\right)\mathrm{N}^{\mu^-_{i,k}}_{\Sigma^-_{i,k}}(x)\right) \\ &= \sum_{i=1}^{n_k}\alpha^-_{i,k}\,\mathrm{N}^{\mathrm{H}_1\mu^-_{i,k}}_{\bar{\mathrm{R}}_{1,i}}(m_1)\left(\mathrm{N}^{\mu_{i*1,k}}_{\Sigma_{i*1,k}}(x)\left(1-\bar{c}\,\mathrm{N}^{\mathrm{H}_2 x}_{\mathrm{R}_{k,2}}(m_2)\right)\right) \\ &= \sum_{i=1}^{n_k}\alpha^-_{i,k}\,\mathrm{N}^{\mathrm{H}_1\mu^-_{i,k}}_{\bar{\mathrm{R}}_{1,i}}(m_1)\cdot \\ &\qquad \left(\mathrm{N}^{\mu_{i*1,k}}_{\Sigma_{i*1,k}}(x) - \bar{c}\,\mathrm{N}^{\mu_{i*2,k}}_{\Sigma_{i*2,k}}(x)\,\mathrm{N}^{\mathrm{H}_2\bar{\mu}_{1,i}}_{\bar{\mathrm{R}}_{2,i}}(m_2)\right)\end{aligned} \tag{10}$$

where

$$\begin{aligned}\bar{n}_k &= 2n^-_k, \\ \alpha_{i*1,k} &\propto \alpha^-_{i,k}\,\mathrm{N}^{\mathrm{H}_1\mu^-_{i,k}}_{\bar{\mathrm{R}}_{1,i}}(m_1), \\ \alpha_{i*2,k} &\propto -\bar{c}\alpha^-_{i,k}\,\mathrm{N}^{\mathrm{H}_1\mu^-_{i,k}}_{\bar{\mathrm{R}}_{1,i}}(m_1)\,\mathrm{N}^{\mathrm{H}_2\mu_{i*1,k}}_{\bar{\mathrm{R}}_{2,i}}(m_2), \\ \mu_{i*1,k} &= \mu^-_{i,k} + \mathrm{K}_{i*1,k}(m_1 - \mathrm{H}_1\mu^-_{i,k}), \\ \mu_{i*2,k} &= \mu_{i*1,k} + \mathrm{K}_{i*2,k}(m_2 - \mathrm{H}_2\mu_{i*1,k}), \\ \Sigma_{i*1,k} &= (\mathrm{I} - \mathrm{K}_{i*1,k}\mathrm{H}_1)\Sigma^-_{i,k} \\ \Sigma_{i*2,k} &= (\mathrm{I} - \mathrm{K}_{i*2,k}\mathrm{H}_2)\Sigma_{i*1,k} \\ \mathrm{K}_{i*1,k} &= \Sigma^-_{i,k}\mathrm{H}_1^T\left(\mathrm{H}_1\Sigma^-_{i,k}\mathrm{H}_1^T + \mathrm{R}_{k,1}\right)^{-1} \\ \mathrm{K}_{i*2,k} &= \Sigma_{i*1,k}\mathrm{H}_2^T\left(\mathrm{H}_2\Sigma_{i*1,k}\mathrm{H}_2^T + \mathrm{R}_{k,2}\right)^{-1} \\ \bar{\mathrm{R}}_{1,i} &= \mathrm{H}_1\Sigma^-_{i,k}\mathrm{H}_1^T + \mathrm{R}_{k,1} \text{ and} \\ \bar{\mathrm{R}}_{2,i} &= \mathrm{H}_2\Sigma_{i*1,k}\mathrm{H}_2^T + \mathrm{R}_{k,2}.\end{aligned}$$

For simplicity, abbreviations $x = x_k$, $m_1 = m_1(y_k)$, $m_2 = m_2(y_k)$, $\mathrm{H}_1 = \mathrm{H}_{k,1}$ and $\mathrm{H}_2 = \mathrm{H}_{k,2}$ are used. Since prior and likelihood are always non-negative the posterior is always non-negative and therefore a valid GMA.

C. Re-indexing and possible reduce the number of components, Step 3

In this step the posterior is re-indexed to obtain

$$x_k|y_{1:k} \sim \mathrm{M}(\alpha_{i,k}, \mu_{i,k}, \Sigma_{i,k})_{(i,n_k)}$$

If the number of components is not reduced then $n_k = \bar{n}_k$ and the posterior is the analytic solution of the Bayesian filtering problem.

However, if the number of components should be reduced, the conventional methods are forgetting and merging [8–11]. When these methods are applied to GMFA it has to be ensured that the resulting Gaussian mixture is a valid probability density function. One possible approach would be always forgetting negative weights and/or collapsing the whole posterior to one Gaussian. The latter is used in this paper, in Section V.

IV. How to select parameters for application

In this section the GMFA is applied for navigation based on wireless radio signals. It is assumed that there is a network of base stations that transmit signals with their specific transmission power. The user is able to identify the transmitting BS and to measure the RSS level. RSS level is a base-10 logarithm of the signal intensity, so the simple path loss model

$$y = a - 10n\log_{10}(||x_{\text{bs}} - r_u||) + w, \tag{11}$$

is used to model the measurement. There y represents the RSS level, a, n and x_{bs} are model parameters and $w \sim \mathrm{N}(0, \mathrm{R})$ is the stochastic noise term. a is the RSS level in the unit range, n is the path loss exponent, and x_{bs} is the BS-position. Variable r_u represents the position of the user equipment (UE).

The model parameters are assumed to be known, and the coverage area of the BS is modeled as a Gaussian ellipse using the algorithm proposed in [12, 13]. This approach has the advantage that it requires a significantly smaller database than techniques that rely on fingerprint data for both, radio map generation and navigation.

The model (11) produces a ring-shaped likelihood with BS-position x_{bs} as the centre point. In this paper, this distribution is approximated by the difference of two Gaussian components. They are both x_{bs}-centered (i.e. $\mathrm{H}_{k,2}x_k = x_{\text{bs}}$ and $\mathrm{H}_{k,1}x_k = x_{\text{bs}}$) and have covariance matrices $\mathrm{R}_{k,1} = \sigma_{\max}^2\mathrm{I}$ and $\mathrm{R}_{k,2} = \sigma_{\min}^2\mathrm{I}$ with $\sigma_{\min} < \sigma_{\max}$. Component 1 has positive and component 2 negative weight. Parameter c in (7) is set to 1, which ensures that the likelihood is nonnegative.

The exact likelihood function $p(y|r_u)$ gets its maximum value when $w = 0$, the range being

$$r \stackrel{\triangle}{=} ||x_{\text{bs}} - r_u|| = 10^{\frac{y-a}{-10n}}, \tag{12}$$

so inequalities $\sigma_{\min} \leq r$ and $\sigma_{\max} > r$ should hold. The derivative of r with respect to y in equation (12) increases as y decreases, so the variance of the range increases as y decreases and the tails of the distribution are heavier outside the ring than inside. Therefore, the approximative model

$$\sigma_{\min} = \max\{1, 0.68r - 48\} \tag{13}$$

$$\sigma_{\max} = 0.9r + 23, \tag{14}$$

is adopted. Equations (13) and (14) are both of form $\sigma = \alpha r + \beta$, and α, β are optimized so that the Lissack-Fu distance [9], which is defined as $\int |p_{\text{true}}(y|r_u) - p_{\text{approx}}(y|r_u)| \,\mathrm{d}\, r_u$, is minimized in the off-line phase.

Figure 1 shows some examples for the performance of the likelihood function (11). The upper row contains plots of likelihoods for strong (left), medium (middle) and weak RSS-values for which exact value of $p(y_k|r_{u,k})$ is calculated in points of a grid with step width 100 and using $a = 30$, $n = 3.5$, $x_{bs} = [0, 0]^T$ and $\mathrm{R} = 36$. In the lower row there are the likelihoods derived from function (11). The errors of the likelihood, which are normalized in the grid area, are 0.2118 (strong RSS), 0.1117 (medium RSS) and 0.1002 (weak RSS).

V. SIMULATIONS

For evaluating the performance of the proposed GMFA, in comparison to EKF and a coverage area algorithm (CAF, [14]), simulations with cellular data were done using Matlab. In contrast with GMFA, the CAF uses only coverage areas for positioning, and ignores RSS values.

The state $x = \begin{bmatrix} r_u \\ v_u \end{bmatrix}$ consists of user position vector r_u and user velocity vector v_u, which are in East-North coordinate system; the Up-coordinate is neglected. Motion model matrix $\mathrm{F}_{k-1} = \begin{bmatrix} \mathrm{I} & \Delta t\mathrm{I} \\ 0 & \frac{9}{10}\mathrm{I} \end{bmatrix}$, which means that constant velocity is used (damped with constant $\frac{9}{10}$), and $w_{k-1} \sim \mathrm{N}(0, \mathrm{Q})$ with $\mathrm{Q} = 9 \begin{bmatrix} \frac{(\Delta t)^3}{3}\mathrm{I} & \frac{(\Delta t)^2}{2}\mathrm{I} \\ \frac{(\Delta t)^2}{2}\mathrm{I} & \Delta t\mathrm{I} \end{bmatrix}$. For the simulations $\Delta t = 1$.

One-hundred different test tracks are simulated. In the simulations, RSS measurements are used, calculated according to equation (12). Therefore, for each simulation (i.e. test track) 3,500 base station positions (results in table 1) or 10,000 (results in table 2) respectively are simulated as 2-dimensional vectors, uniformly distributed on a 15-by-15 km square around the simulated track. Path loss model parameter a is modeled as $a \sim \mathrm{N}(0, 18^2)$ for each base station, and path loss exponent n is modeled as $n \sim \mathrm{N}(3, 0.7^2)$ with the constraint $n \geq 2$. This choice for distribution of n is based on the fact that the path loss exponent for normal environments is in the range between 2 and 4, and $n = 2$ when the signal propagates through free space [15]. The shadowing variance is fixed to $\mathrm{R} = 36$.

In the simulation, the size of cell coverage areas is chosen based on earlier studies and our experimental knowledge. The size of a cell may range from some meters to some kilometers, for instance 0.1-1 km is reported for microcells [16, 17]. In [16], the experimental results of cell-ID location technique are presented. The average distance between cell-ID location estimate and GPS location estimate is reported to be 800 meters in USA and 500 meters in Italy. For the purpose of this paper, semi-major axis e_1 and semi-minor axis e_2 of coverage area ellipses are therefore simulated as $e_1, e_2 \sim \mathrm{N}(650, 500^2)$, centers of the ellipses are simulated as $c_e \sim \mathrm{N}(x_{\text{bs}}, 200^2\mathrm{I})$, and ellipse angles γ_e are chosen from a continuous uniform distribution on the interval $[0, \pi)$.

Table 1 contains the summary of hundred 300 second simulations using CAF [12–14], EKF and GMFA that is modified such that it approximates the posterior as one Gaussian at every time step in order to increase calculation speed. The simulations are forced to a very poor geometry, at each time step only one base station measurement at most is available. Therefore, if more than one BS are heard then all of them except for one randomly chosen are discarded. In addition, this measurement is used in the next n_{rm} steps, where n_{rm} is a discrete uniform random number between 1 and 10. This emulates real world cases, in which not always new measurements could be obtained every second. Note that it is also possible that any BS is heard outside its coverage area (limited to 1.5 times the coverage area). For the three approaches results using static positioning and filtering are presented. For filtering of CAF the basic Kalman filter is applied. In the static case for EKF, firstly the coverage areas are processed in order to get a position estimate. This estimate is then used as prior for the EKF.

The columns of table 1 are as follow: *Time* is relative computation time using Matlab and the specific implementation, so that computation time of EKF is one. This gives a rough idea of each algorithm's complexity. *Mean* is mean and *Med.* is median, and 95% *err.* is 95th percentile of the 2D position error. Column *Cons.* $\%$ displays the percentage of time steps for which the filters are consistent with respect to the Gaussian consistency test, with risk level 5%.

For the static case GMFA clearly outperforms EKF in terms of consistency, and reduces the three error types significantly. These improvements come with the cost of an approximately 70% larger calculation time. In comparison to the CAF the GMFA provides only modest improvements of mean and

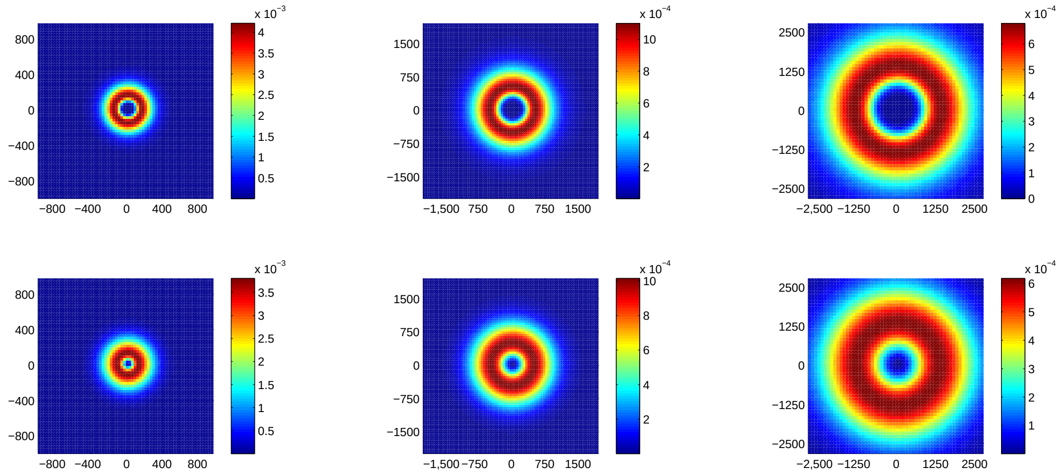

Figure 1. **Approximation of ring-shaped likelihood:** In upper row are plots of likelihoods for strong (left), medium (middle) and weak RSS-values for which exact value of $p(y_k|x_k)$ is calculated in points of a grid with step width 30 (left), 50 (middle) and 100. The lower row contains plots of the likelihoods derived from function (11).

median error, which are unable to justify the major increase in calculation time. Contrary, for the filtered case the improvements of GMFA, compared with CAF, in terms of the three error statistics are justifying the significant increase in calculation time. Compared with the EKF, the GMFA provides similar decreases in the errors as for the static case, but the advantage of significantly higher consistency does not hold.

The weak overall consistencies might be a result of using the Gaussian consistency test that assumes Gaussian distributions, which does not hold in the analyzed cases. An alternative, which should ensure higher consistency levels, is the general inconsistency test, introduced in [4].

Solver	time	Mean	Med	95% err	cons. %
CAF (static)	0.94	702	643	1445	86
EKF (static)	1	758	653	1712	33
GMFA (static)	1.69	670	597	1452	84
CAF (filtered)	0.75	625	572	1304	37
EKF (filtered)	1	477	376	1250	31
GMFA (filtered)	1.75	431	345	1087	38

Table I

Table 1: Summary of 100 different simulations with very poor geometry. Simulations use only one base station measurement at most per time step and usually measurement is excatly the same than previous time step. Time of EKF is used as reference time.

In Table 2 the summary of hundred simulations using CAF, EKF and GMFA are listed. For those tests a good geometry was assumed, at each time step up to six base station measurements are available. It becomes visible that GMFA outperforms CAF and EKF also for good geometries. However, the improvement in the filtered case compared with EKF is less significant and comes at the cost of stronger increase in calculation time than for very poor geometry.

The reason for the significantly larger calculation time of GMFA, for the good geometry case, compared with both other methods is that it uses a maximum of $2^6 = 64$ components, whereas CAF and EKF use only one. In the filtered case, the whole posterior is collapsed to one Gaussian and used as prior in the next time step. Thus, the maximum number of components is still 64, and therefore the run time of GMFA for both static and filtered case are almost similar, with respect to the EKF for the corresponding case. Note that the algorithm is nevertheless clearly faster than common Gaussian mixture filters, which require non-negative component weights. Within these GMFs the likelihood is, mainly for non-linear cases, typically approximated by a large number of Gaussians [9].

Solver	time	Mean	Med	95% err	cons. %
CAF (static)	1.01	423	368	933	61
EKF (static)	1	328	253	857	43
GMFA (static)	14.63	248	184	674	70
CAF (filtered)	0.78	246	224	508	37
EKF (filtered)	1	112	84	290	55
GMFA (filtered)	16.24	100	84	232	65

Table II

Table 2: Summary of 100 different simulations with good geometry. Simulations use up to six base station measurements per time step. Time of EKF is used as reference time.

VI. Conclusion

In this article, a Gaussian mixture filter (GMF) for hybrid positioning applications has been studied. A new way for approximating the likelihood function as a Gaussian mixture with only two components, one having negative weight, was introduced (GMFA). This allows using the GMF (more) efficiently (than conventional GMFs) by keeping the number of components small without losing significant information. In both very poor and good geometry, the new filter (GMFA) clearly outperform the EKF, which has serious consistency problems in highly nonlinear situations that were analyzed in the paper. It also provides better results than the CAF, since in addition to the coverage areas of base stations, it also uses RSS-measurements, which are generally available in the UE. For real-world applications a tuning strategy should be applied to improve the computational time. In the future, it will be analyzed if using the GMFA only in highly nonlinear cases, and simpler approaches for almost linear cases, results in insignificant accuracy loose and major drop in calculation time. In addition, it will be studied more deeply how the GMFA behaves for cases in which the likelihood displays a shape other than the ring-shape examined in this paper.

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