

# On OFDM Link Performance under Receiver Phase Noise with Arbitrary Spectral Shape

Ville Syrjälä, Mikko Valkama, Yaning Zou, Nikolay N. Tchamov, and Jukka Rinne

**Abstract**—This article addresses the signal distortion caused by receiver phase noise (PN) on OFDM waveforms in direct-conversion radio receivers. A closed-form solution for the observed signal-to-interference-plus-noise ratio (SINR) is derived, describing the level of intercarrier interference (ICI) stemming from PN. Compared to existing literature, the analysis is valid for arbitrary oscillator spectral shape, the only assumption being that reasonably small phase noise values are observed. The analysis results can be used to derive practical circuit-level oscillator design criteria in terms of the allowable PN spectral density. The applicability and validity of the derived analysis are verified with extensive computer simulations.

**Index Terms**—Inter-carrier interference, OFDM, performance analysis, phase noise

## I. INTRODUCTION

COMMUNICATIONS systems using orthogonal frequency division multiplexing (OFDM) are highly vulnerable to non-idealities of radio device components [1]. Performance-wise one of the most remarkable of these non-idealities is oscillator phase noise [2], [3]. Phase noise spreads the spectral content of the used subcarriers destroying the orthogonality between them. This causes inter-carrier interference (ICI) in addition to well known effect of symbol constellation rotation called common phase error (CPE) [2]. Due to the relatively complex nature of the phase noise induced waveform distortion, and its dependency on the used oscillator characteristics, a closed-form tool describing the level of ICI as a function of the used oscillator spectral density is developed in this article. Such tool can be used already in the design process of the oscillator, e.g., to find out the properties of oscillators that are the most significant from the applied waveform point of view.

The effects of phase noise on the performance of OFDM communications systems are widely studied in the literature,

This work was supported by TUT Graduate School, Finnish Foundation for Technology Promotion, HPY Research Foundation, the Finnish Funding Agency for Technology and Innovation (Tekes, under the project “Advanced Techniques for RF Impairment Mitigation in Future Wireless Radio Systems”), the Technology Industries of Finland Centennial Foundation and EUREKA CELTIC E!3187 B21C-Broadcasting for 21st Century.

Authors are with Department of Communications Engineering, Tampere University of Technology, P.O. Box 553, 33101 Tampere, Finland. (email: {ville.syrjala, mikko.e.valkama, yaning.zou, nikolay.n.tchamov, jukka.rinne}@tut.fi).

e.g., in [4], [5] and [6]. However, the oscillator models in the reported studies are typically fixed or restricted to highly-simplified models, such as the free-running oscillator (FRO) [3]. In this paper, the effective signal-to-interference-plus-noise ratio (SINR) is analyzed for OFDM systems under the influence of phase noise caused ICI. The phase noise process in this study is based on arbitrary phase noise spectral mask, thus making the model generally applicable for practical oscillators. Only an assumption of an oscillator with reasonably small phase noise values is made without restricting the spectral shape of the phase noise to any predefined criteria. To authors’ knowledge, analysis without any restrictions to the spectral shape of the oscillator cannot be found in existing literature.

The rest of this article is organized as follows: Section II gives a compact subcarrier-wise OFDM link model including the radio channel and receiver phase noise. This forms the basis for the SINR analysis. Then, Section III formulates and carries out the actual SINR analysis in detail, while Section IV verifies the derived expressions with the help of link computer simulations. Finally, the work is concluded in Section V.

## II. OFDM LINK MODEL

In OFDM systems, the phase noise effects can be divided in two distinct parts [2]. The first effect is called CPE that is the common complex multiplicative effect the phase noise has on all subcarrier symbols within one OFDM symbol. The second effect is called ICI, which is the loss of orthogonality due to frequency spread of the subcarriers on top of each other. The CPE is easily cancelled out in the receiver with help of pilot subcarriers [3]. ICI, however, has more complicated effect on the OFDM systems, thus making its mitigation much more burdensome [3], [7]. This is why the ICI is much more interesting also from the performance analysis point-of-view. Therefore, in this work, we assume that the CPE is known, namely part of the useful signal, while the actual interference is coming from the neighbouring subcarriers. Furthermore, to simplify the analysis, we assume that the interference from adjacent RF bands is negligible, meaning that we focus on the in-band effects only.

Next, to form the basis for SINR analysis, a link model is shortly established taking into account a noisy multipath channel and receiver phase noise. In effect, the observation at subcarrier  $k$  can be written as

$$R_k(m) = X_k(m)H_k(m)J_0(m) + \sum_{l=0, l \neq k}^{N-1} X_l(m)H_l(m)J_{k-l}(m) + Z_k(m). \quad (1)$$

Here,  $R_k(m)$  is the received signal,  $X_k(m)$  is the transmitted subcarrier symbol,  $H_k(m)$  is the channel transfer function,  $J_k(m)$  is the frequency-domain phase-noise complex exponential and  $Z_k(m)$  is additive white Gaussian noise, all at  $m$ th OFDM symbol and  $k$ th subcarrier. Furthermore,  $N$  is the IDFT/DFT length used in the OFDM modulation/demodulation. The frequency domain phase-noise complex exponential is given by

$$J_k(m) = \sum_{n=0}^{N-1} e^{j\varphi_n(m)} e^{-j2\pi nk/N}, \quad (2)$$

where  $\varphi_n(m) = \varphi(m(T_S + T_{CP}) + nT_S/N)$  denotes the actual phase noise value at  $n$ th sample within the studied  $m$ th OFDM symbol. Here,  $T_S$  and  $T_{CP}$  indicate the OFDM symbol length and cyclic prefix length, respectively. In (1), the first term of the right hand side is the CPE corrupted term, and the additive sum term is directly the phase noise incurred ICI. Namely, the DC-bin of the phase noise exponential  $J_0(m)$  is the complex multiplication caused by the CPE, and all the other terms of  $J_k(m)$  summed up denote the effect of the ICI. Since we already concluded that the only relevant effect of the phase noise for this study is caused by the ICI (not by CPE), the SINR due to phase noise can be easily derived from (1). We let the power of the first term of the right hand side to be the useful power of the signal and assume the power of the ICI and noise term to be the interference power. Thus, the SINR is defined as

$$\gamma_k = \frac{E\left[|X_k(m)H_k(m)J_0(m)|^2\right]}{E\left[\left|\sum_{l=0, l \neq k}^{N-1} X_l(m)H_l(m)J_{k-l}(m) + Z_k(m)\right|^2\right]}. \quad (3)$$

Next, we make natural assumptions that  $X_k(m)$ ,  $H_k(m)$ ,  $J_k(m)$  and  $Z_k(m)$  are mutually statistically independent, stationary, and that  $\forall k: X_k(m)$  are independent of each other and zero mean. Therefore, we can write the SINR equation (3) as

$$\gamma_k = \frac{E\left[|X_k(m)|^2\right]E\left[|H_k(m)|^2\right]E\left[|J_0(m)|^2\right]}{\sum_{l=0, l \neq k}^{N-1} E\left[|X_l(m)|^2\right]E\left[|H_l(m)|^2\right]E\left[|J_{k-l}(m)|^2\right] + \sigma_z^2}, \quad (4)$$

where  $\sigma_z^2 = E\left[|Z_k(m)|^2\right]$  denotes the average power of the additive white Gaussian noise term  $Z_k(m)$ . Now, we make two other reasonable assumptions that mean of the average transmit symbol powers and average channel power responses are subcarrier independent. Namely,

$$\forall k: E\left[|X_k(m)|^2\right] = \sigma_x^2, \quad (5)$$

$$\forall k: E\left[|H_k(m)|^2\right] = \sigma_h^2. \quad (6)$$

Now, stemming from assumptions (5) and (6), we can write

$$\gamma_k = \frac{\sigma_x^2 \sigma_h^2 E\left[|J_0(m)|^2\right]}{\sigma_x^2 \sigma_h^2 \sum_{k=1}^{N-1} E\left[|J_k(m)|^2\right] + \sigma_z^2}. \quad (7)$$

Equation (7) can be further simplified to form

$$\gamma_k = \frac{E\left[|J_0(m)|^2\right]}{\sum_{k=1}^{N-1} E\left[|J_k(m)|^2\right] + \frac{\sigma_z^2}{\sigma_x^2 \sigma_h^2}}. \quad (8)$$

Now by denoting the received signal-to-noise ratio (SNR) without phase noise by  $\rho = \sigma_x^2 \sigma_h^2 / \sigma_z^2$ , we can write

$$\gamma_k = \frac{E\left[|J_0(m)|^2\right]}{\sum_{k=1}^{N-1} E\left[|J_k(m)|^2\right] + \frac{1}{\rho}}. \quad (9)$$

Here,  $J_k(m)$  is simply DFT of complex exponential by definition. Since sum of squared absolute values of  $N$  complex exponentials is naturally  $N$ , we know according to Parseval's theorem, that

$$\sum_{k=0}^{N-1} |J_k(m)|^2 = N^2. \quad (10)$$

Now, from the linearity of the expectation operator, it directly stems that

$$\sum_{k=0}^{N-1} E\left[|J_k(m)|^2\right] = N^2. \quad (11)$$

Thus by combining (9) and (11), we can finally write the SINR as

$$\gamma_k = \gamma = \frac{E\left[|J_0(m)|^2\right]}{N^2 - E\left[|J_0(m)|^2\right] + \frac{1}{\rho}}. \quad (12)$$

As (12) shows, the achievable average SINR is independent of the subcarrier index  $k$ , even though ICI is also present, and depends only on the received SNR and the second-order statistics of the CPE. Thus to complete the analysis, the average power of the CPE,  $E\left[|J_0(m)|^2\right]$ , needs to be

addressed. This will be done in closed-form as a function of the used oscillator spectral shape in the following sections.

### III. OSCILLATOR MODELLING

Usually in studies like this, free-running or otherwise constrained oscillator model is used. In this paper, however, we use a so-called frequency-masking approach to model the oscillator phase noise. In this model, white noise is first transformed to frequency domain using discrete Fourier transform (DFT), filtered with arbitrary phase-noise frequency-mask and then transformed back to time-domain with inverse DFT [8]. This allows easy modelling of arbitrary oscillators used in OFDM systems, where the used DFT length can be conveniently selected, e.g., as the number of used subcarriers  $N$ . Practical shapes for the masks can be derived, e.g., with help of point single-sideband (SSB) phase-noise laboratory measurements of the oscillators [9].

Now based on (12), we must derive the average power of  $J_0(m)$  for arbitrary oscillator defined by phase-noise frequency-mask. The mask is here defined with subcarrier-specific scaling variables  $\lambda_k$ ,  $k = 0 \dots N-1$ . Then, if we assume that the values of the phase noise  $\varphi_n(m)$  are relatively small,  $\varphi_n(m) \ll 1$  rad, as they are in any practical oscillator, we can first make an approximation  $e^{j\varphi_n(m)} \approx 1 + j\varphi_n(m)$ . The scaled version of this, keeping unit variance, can be written as

$$e^{j\varphi_n(m)} \approx \frac{1 + j\varphi_n(m)}{\sqrt{1 + \sigma_\varphi^2}}. \quad (13)$$

Here,  $\sigma_\varphi^2$  is the average power of the phase noise  $\varphi_n(m)$ , derived in the Appendix as a function of the spectral mask.

The first frequency bin of the process  $e^{j\varphi_n(m)}$  is the multiplicative effect of the CPE, the average power of which we are interested in. Now, stemming directly from (2),  $J_0(m)$  can be written as

$$J_0(m) = \sum_{n=0}^{N-1} e^{j\varphi_n(m)}, \quad (14)$$

Uniting the approximation (13) and the equation (14), we can write

$$J_0(m) \approx \frac{N + j \sum_{n=0}^{N-1} \varphi_n(m)}{\sqrt{1 + \sigma_\varphi^2}} = \frac{N + j\Phi_0(m)}{\sqrt{1 + \sigma_\varphi^2}}, \quad (15)$$

where  $\Phi_0(m)$  is the DC-bin of DFT of the  $\varphi_n(m)$ . Then, as shown in the Appendix, the average power of  $J_0(m)$  can be written as

$$E[|J_0(m)|^2] \approx \frac{N^2 + \sigma_w^2 \lambda_0^2}{1 + \sigma_\varphi^2} = \frac{N^2 + \sigma_w^2 \lambda_0^2}{1 + \frac{\sigma_w^2}{N^2} \sum_{k=0}^{N-1} \lambda_k^2}. \quad (16)$$

The expression in (16) describes the average power of the CPE as a function of some elementary variables. By combining (12) and (16), and by applying some simple manipulations, we end up in the following form for the SINR given by

$$\gamma \approx \frac{N^2 + \sigma_w^2 \lambda_0^2}{\sigma_w^2 \sum_{k=1}^{N-1} \lambda_k^2 + \frac{\sigma_w^2}{\rho N^2} \sum_{k=0}^{N-1} \lambda_k^2 + \frac{1}{\rho}}. \quad (17)$$

The variable  $\gamma$  can then be used to evaluate the SINR for arbitrary phase noise mask defined by  $\lambda_k$ ,  $k = 0 \dots N-1$ . This form is already usable. However, quantities  $\psi_k^2 = \sigma_w^2 \lambda_k^2$  are easier to connect to actual oscillator spectral measurements, as they are the energies of the phase noise around the  $k$ th subcarrier, as depicted in Fig. 1. These energies can be approximated by the common dBc/Hz measurement values of the oscillator phase noise spectrum at the centre of the  $k$ th subcarrier multiplied by the subcarrier spacing  $1/T_s$ , i.e.,

$$\psi_k^2 = \sigma_w^2 \lambda_k^2 \approx \text{PSD}_\varphi \left( k \frac{1}{T_s} \right) \frac{1}{T_s}. \quad (18)$$

Here,  $\text{PSD}_\varphi$  is the power spectral density function of the phase noise, that is also depicted in Fig. 1. Now, by using  $\psi_k^2$  in (17), the SINR can be rewritten as a direct function of the spectral measurements as

$$\gamma \approx \frac{N^2 + \psi_0^2}{\sum_{k=1}^{N-1} \psi_k^2 + \frac{1}{\rho N^2} \sum_{k=0}^{N-1} \psi_k^2 + \frac{1}{\rho}}. \quad (19)$$

Finally, by nullifying the additive noise contribution ( $1/\rho = 0$ ), we are able to achieve alternative very simple form

$$\gamma \approx \frac{N^2 + \psi_0^2}{\sum_{k=1}^{N-1} \psi_k^2}. \quad (20)$$

This form can be used in calculation of SIR due to oscillator effects alone in noise-free OFDM-system, forming and upper-

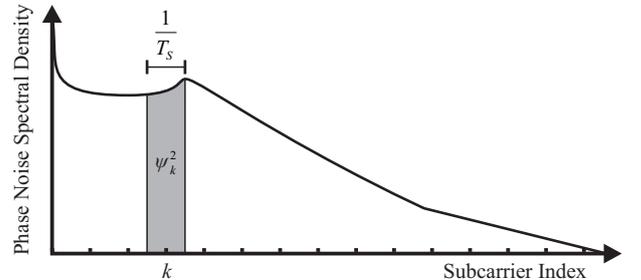


Fig. 1. An example power spectral density function of phase noise.

bound for SIR, which might also be interesting for some oscillator designers.

#### IV. VERIFICATION OF THE MODEL AND SIMULATION ANALYSIS

In this Section, the tenability of the above model is verified by comparing the SINR given by the analytical formula (17) to the SINR given by simulations. In addition, the simulation results are shortly analyzed. In this study, we use phase-locked-loop (PLL) based oscillator as an example. We use PLL oscillator because the idea is well-known and the phase-noise frequency-mask can be easily connected to real world oscillator measurements, e.g., by using formulae given in [9]. As mentioned, however, the SINR expression in (17) is applicable to any phase noise spectral shape, so the usage is by no means restricted to the special case of PLL-oscillator.

In simulations, 1000 realizations of OFDM symbols, with 1024 subcarriers with 15 kHz subcarrier spacing and 63 sample cyclic prefix, are generated with 16QAM subcarrier modulation. They are then transmitted through independent realizations of extended ITU-R Vehicular A multipath-channel. Additive white Gaussian noise and receiver phase noise are then modelled to the system as well as OFDM demodulation. From these modelled signal components, then, average powers of useful signal and ICI are numerically calculated, and SINR is then evaluated using these powers, and compared against the analytical results.

In these experiments, SIR and SINR are studied as functions of phase-noise spot measurements  $L_F$  and  $L_W$ . A principal illustration is given in Fig. 2. These parameters characterize phase-noise power-spectral-density of the free-running voltage-controlled oscillator that is integrated in the PLL.  $L_F$  corresponds to the 1/f-noise-dominated region measurement at 10 kHz offset from the oscillation frequency, whereas  $L_W$  corresponds to the thermal-noise-dominated region measurement at 1 MHz offset. Here we assume that the spot thermal-noise region measurement for the reference oscillator used in PLL is  $-160$  dBc/Hz at 1 MHz offset, and assume that no significant flicker noise levels are present. Refer to [7] and [9] for more details on the oscillator modelling. Notice that the studies of this paper are applicable for any realistic oscillator model. This oscillator design parameterization above is used here only to reflect a practical example scenario for simulations.

For easier interpretation and visualization, we first consider the case where channel noise is set to zero, i.e.,  $1/\rho = 0$ . Fig. 3 gives the contour plot of the SIR-performance in the simulations. Fig. 4 then shows the difference in simulated SIR-performance and the analytical SIR-performance as formulated in (17). As Fig. 4 depicts, over the studied values of parameters  $L_W$  and  $L_F$ , the simulated and theoretical performances are very near to each other. Maximum performance difference in the studied parameter region is about 1.6 dB. This is excellent accuracy especially because we are having nearly 0 dB SIR in the worst case region as shown in Fig. 3. In regions, where SIR is over 10 dB, the theoretical

results match the simulated ones almost perfectly. As one can remember, the theoretical analysis developments were stemming from the small phase noise (practical oscillator) assumption, so the small performance difference in 0-10 dB SIR range is understandable as the oscillator gets very noisy and the small phase noise assumption,  $\phi_n(m) \ll 1$  rad, is violated.

According to SINR results in Fig. 5, when additive channel noise is also considered present in the system, the formula (17) still gives accurate results. Fig. 5 shows the simulated performance versus theoretical performance in fixed PLL-oscillator with example parameters  $L_F = -82$  dBc/Hz and  $L_W = -120$  dBc/Hz, the spectrum of which can be seen in Fig. 2. According to simulations, with these parameters the PLL-oscillator causes SIR of 40 dB in case without additive noise as shown in Fig. 3. In Fig. 5, we can see the expected 40 dB

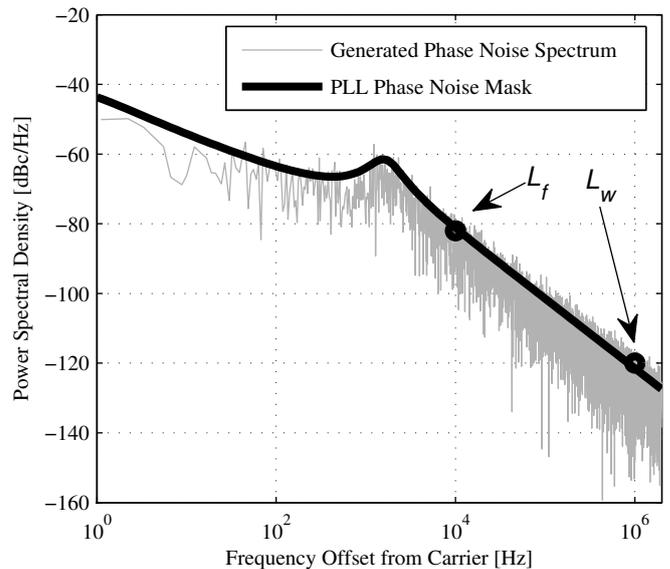


Fig. 2. Example spectrum of the oscillator phase noise generated with parameters  $L_F = -82$  dBc/Hz and  $L_W = -120$  dBc/Hz.

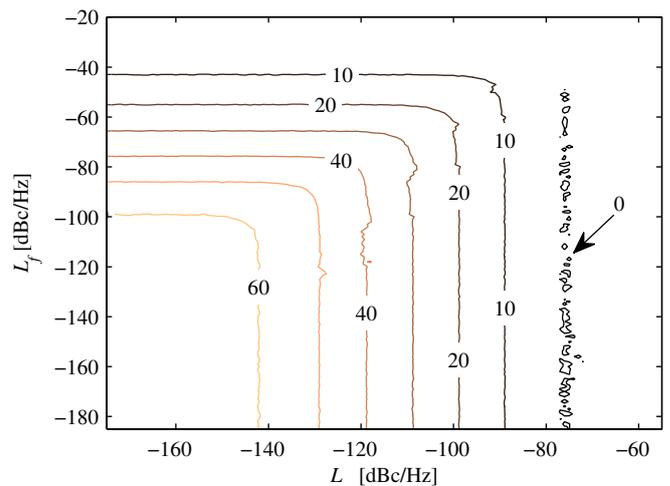


Fig. 3. Contour plot for the value of SIR (in dB) as a function of  $L_F$  and  $L_W$ , given by the simulations. No additive channel noise.

upper performance boundary and the 37 dB SINR when the received SNR is 40 dB. In that operating point, ICI and channel noise are equally strong implying thus 3 dB penalty compared to no phase noise (ideal receiver) case.

Already from Fig. 3, one can see that there is indeed margin to play when designing the down-converting oscillator for OFDM receiver under the assumed PLL topology. One can see that with fixed  $L_f$ , one can play with  $L_w$  quite much without practically affecting the SIR, and vice versa. The derived formula is thus quite interesting from receiver design point-of-view, meaning that one can trade higher phase noise spectral densities at certain frequencies without essentially affecting the receiver detection performance. This was actually observed empirically already in [9] and is now analytically justified.

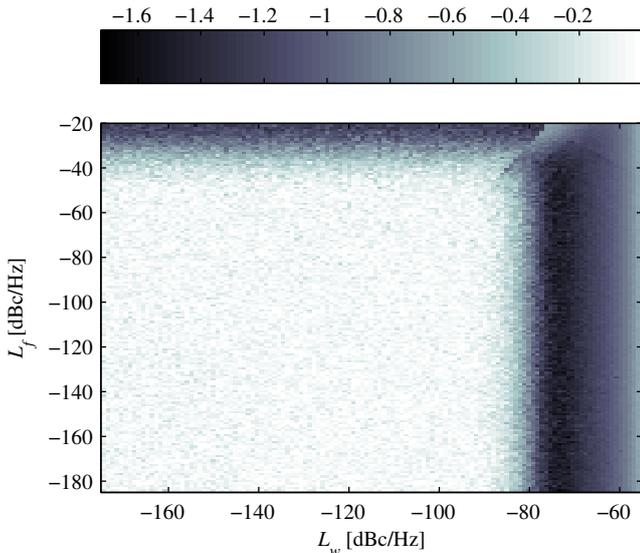


Fig. 4. SIR error given for PLL-type example oscillator as a function of  $L_f$  and  $L_w$ . Absolute value of the difference between the theoretical SIR given by (20) and simulated SIR is depicted. Error is given in dB scale. No additive channel noise.

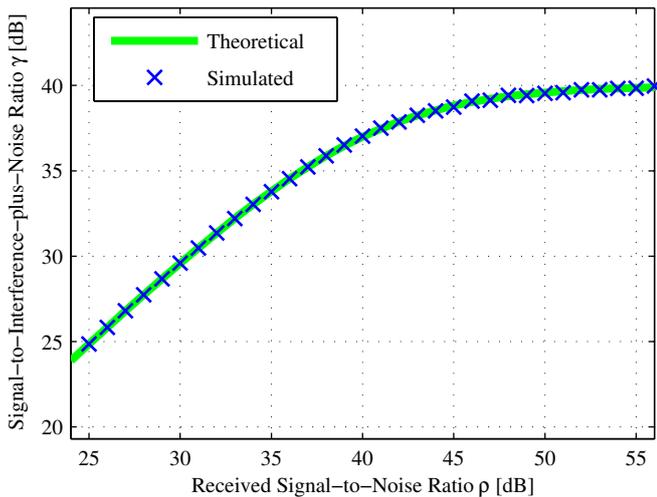


Fig. 5. Theoretical and simulated SINR as a function of SNR compared in case with additive noise present. Fixed oscillator with parameters  $L_f = -82$  dBc/Hz and  $L_w = -120$  dBc/Hz.

## V. CONCLUSION

Phase noise is one of the main impairments in OFDM radio receivers. In this paper, a simple and efficient analytical tool was derived for oscillators with arbitrary spectral shape to calculate average subcarrier-level SINR due to ICI stemming from down-converting oscillator phase-noise. The tool derivation is based on using a sampled phase-noise frequency-mask to characterize the phase noise spectral density. Also, OFDM system was simulated in order to verify the accuracy of the tool. Simulation results verified that the performance given by the tool corresponds accurately to the performance given by the simulations. In practice, the analysis results can be used to derive RF design requirements for the receiver oscillator module in terms of the allowable phase noise spectral density at given frequencies.

### APPENDIX: DERIVATION OF PHASE NOISE VARIANCE

Here we derive the variance of the phase noise generated by an arbitrary frequency-domain phase-noise mask vector  $\boldsymbol{\lambda} = [\lambda_0, \lambda_1, \dots, \lambda_{N-1}]^T$ . Initially, in phase noise vector generation, we only have white noise vector  $\mathbf{u}$  with covariance  $\sigma_u^2 \mathbf{I}$ . After transforming this to frequency-domain with DFT matrix  $\mathbf{A}$ , we have still white noise, since the covariance of  $\mathbf{w} = \mathbf{A}\mathbf{u}$  is of the form

$$\mathbb{E}[\mathbf{w}\mathbf{w}^H] = \mathbf{A}\mathbb{E}[\mathbf{u}\mathbf{u}^H]\mathbf{A}^H = \mathbf{A}\sigma_u^2\mathbf{I}\mathbf{A}^H = N\sigma_u^2\mathbf{I} = \sigma_w^2\mathbf{I}. \quad (21)$$

Then, we apply the frequency-domain phase-noise mask to this frequency-domain white noise. This results into signal  $\mathbf{q} = \boldsymbol{\Lambda}_M \mathbf{w}$ , where  $\boldsymbol{\Lambda}_M$  is a diagonal matrix corresponding to elements of the phase noise mask  $\lambda_k, k = 0 \dots N-1$ . The corresponding covariance matrix is then given by  $\sigma_w^2 \boldsymbol{\Lambda}_M^2$  as is easy to show. Finally, after inverse DFT we end up having the phase noise vector  $\boldsymbol{\phi} = \mathbf{B}\mathbf{q}$ , where  $\mathbf{B}$  is the inverse-DFT matrix. The covariance matrix for phase noise vector  $\boldsymbol{\phi}$  is then given by

$$\begin{aligned} \mathbb{E}[\boldsymbol{\phi}\boldsymbol{\phi}^H] &= \mathbf{B}\mathbb{E}[\mathbf{q}\mathbf{q}^H]\mathbf{B}^H = \sigma_w^2 \mathbf{B}\boldsymbol{\Lambda}_M^2 \mathbf{B}^H \\ &= \sigma_w^2 \sum_{k=0}^{N-1} \lambda_k \mathbf{b}_k \mathbf{b}_k^H, \end{aligned} \quad (22)$$

where  $\mathbf{b}_k$  is the  $k$ th column of the inverse DFT matrix  $\mathbf{B}$ . From (22), we can see that the variance of the  $l$ th sample of the phase noise vector  $\boldsymbol{\phi}$  can be written as

$$\begin{aligned} \sigma_\phi^2(l) &= \mathbf{e}_l^T \mathbb{E}[\boldsymbol{\phi}\boldsymbol{\phi}^H] \mathbf{e}_l = \sigma_w^2 \sum_{k=0}^{N-1} \lambda_k^2 |\mathbf{b}_k(l)|^2 \\ &= \frac{\sigma_w^2}{N^2} \sum_{k=0}^{N-1} \lambda_k^2 = \sigma_\phi^2, \end{aligned} \quad (23)$$

where  $\mathbf{e}_l$  is the  $l$ th unit vector (column vector of all zeros except for  $l$ th element which equals one) and  $N$  is the length of vectors  $\boldsymbol{\varphi}$ ,  $\mathbf{b}_k$  and  $\mathbf{e}_l$ . We can see that variance given by (23) is actually not dependent on the sample index  $l$ , and depends essentially only on the mask values.

#### REFERENCES

- [1] G. Fettweis, "Dirty-RF: A new paradigm", in *Proc. 16th International Symposium on Personal, Indoor and Mobile Radio Communications, 2005 (PIMRC'05)*, Berlin, Germany, September 2005, pp. 2347-2355, Vol. 4.
- [2] P. Robertson, and S. Kaiser, "Analysis of the effects of phase-noise in orthogonal frequency division multiplex (OFDM) systems," in *Proc. IEEE International Conference on Communications (ICC'95)*, Seattle, WA, June 1995, pp. 1652-1657, Vol. 3.
- [3] D. Petrovic, W. Rave, and G. Fettweis, "Effects of phase noise on OFDM systems with and without PLL: Characterization and compensation," *IEEE Transactions on Communications*, Vol. 55, No. 8, pp. 1607-1616, August 2007.
- [4] C.-H. Yih, "BER analysis of OFDM systems impaired by phase noise in frequency-selective Rayleigh fading channels," in *Proc. Global Telecommunications Conference 2008 (GLOBECOM'08)*, New Orleans, LO, December 2008.
- [5] M. Krondorf, S. Bittner, and G. Fettweis, "Numerical performance evaluation for OFDM systems affected by phase noise and channel estimation errors," in *Proc. Vehicular Technology Conference 2008 (VTC'08-Fall)*, Calgary, Canada, September 2008.
- [6] T. Schenk, *RF Impairments in Multiple Antenna OFDM: Influence and Mitigation*, PhD dissertation, Technische Universiteit Eindhoven, 2006. ISBN 90-386-1913-8. 291 p.
- [7] V. Syrjälä, M. Valkama, N. N. Tchamov, and J. Rinne, "Phase noise modelling and mitigation techniques in OFDM communications systems," in *Proc. Wireless Telecommunications Symposium 2009 (WTS'09)*, Prague, Czech Republic, April 2009.
- [8] Y. Ferdi, A. Taleb-Ahmed, and M. R. Lakehal, "Efficient generation of  $1/f^\beta$  noise using signal modelling techniques," *IEEE Transactions on Circuits and Systems—I: Regular Papers*, Vol. 55, No. 6, pp. 1704-1710, July 2008.
- [9] N. N. Tchamov, J. Rinne, V. Syrjälä, M. Valkama, Y. Zou, and M. Renfors, "VCO phase noise trade-offs in PLL design for DVB-T/H receivers," in *Proc. IEEE International Conference on Electronics, Circuits and Systems (ICECS'09)*, Yasmine Hammamet, Tunisia, December 2009.