Kalman-Type Filters and Smoothers for Pedestrian Dead Reckoning

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***Abstract*—In this paper, we present a method for device localization based on the fusion of location data from Global Navigation Satellite System and data from inertial sensors. We use a Kalman filter as well as its non-linear variants for real-time position estimation, and corresponding smoothers for off-line position estimation.**

In all filters we use information about changes of user's heading, which are computed from the acceleration and gyroscope data. Models used with Extended and Unscented Kalman filters also take into account information about step length, whereas Kalman Filter does not, because the measurement is non-linear. In order to overcome this shortcoming, we introduce a modified Kalman Filter which adjusts the state vector according to the step length measurements.

Our experiments show that use of step length information does not significantly improve performance when location measurements are constantly available. However, in real situations, when location data is partially unavailable, information about step length and its appropriate integration into the filter design is important, and improve localization accuracy considerably.

I. Introduction

Personal positioning is an actively researched area covering technologies from well-known Global Navigation Satellite Systems (GNSS) to radio based localization techniques based on cellular networks/WiFi, Bluetooth Low Energy (BLE) and Ultra Wide Band radio interfaces.

Whereas GNSS systems work globally and provide very accurate locations outdoors, they suffer from poor signal quality in urban canyons and indoors. WiFi and Cellular signals have good availability indoors and can be used for accurate positioning. However, most network-based localization methods require a precise radio map of the indoor spaces. In most cases, such radio maps are created manually, with the device measuring radio signals, and a person entering geographic references for the radio data. This is what makes indoor radio mapping laborious in the local scale, and infeasible in the global scale. Therefore, to enable radio map generation in the global scale, geo-referencing has to be done automatically, without manual input of any kind.

Being essentially a problem of pedestrian localization, it was considered in a numerous papers. In general pedestrian localization is based on a combination of information derived from inertial sensors and location data provided by GNSS. While some of the approaches use speed computed by double integration of acceleration and gyroscope measurements, others rely on step counting and tacking of user heading to predict user position in the absence of location data.

In this paper we focus on the latter ones. In such approaches the Kalman Filter is used to estimate the position and step vector of the user, where step vector is collinear with the heading of the user, and its norm is equal to the step length of the user [1]. Alternatively, no-linear filters, such as Extended and Unscented Kalman Filters or Particle Filters, can be used to estimate position, heading and step length of the user, where heading and step lengths are represented by separate components of the state vector [1] [3].

In [1] authors incorporate heading changes into the linear state transition model in order to better track user heading and hence provide more accurate position estimations. This is reasonable as humans can make abrupt turns, in which case the assumption of a constant velocity model used in e.g. [2] does not hold. In addition to linear Kalman Filter authors consider Extended and Unscented Kalman filters which also take step length changes into account.

In [3], position, heading and step length changes are modeled by non-linear functions, and a particle filter is applied for state estimation. Particle filters require more computational resources than Kalman filter and suffer from particle degeneracy problem, which may cause the filter to get stuck in a wrong state. This problem is solved by running a Kalman filter as a backup solution in parallel with the particle filter.

In our research we extend the linear model proposed in [1] by incorporating information about step length in addition to heading changes, and propose a robust method to detect unreliable heading change measurements. In our modified Kalman Filter we directly apply step length measurements to adjust length of the estimated step vector after the prediction step. Since for radio mapping it is not required to calculate geo-references instantly, we focus on smoothers instead of filters. We consider Rauch-Tung-Striebel smoothers based on the Kalman Filter and its non-linear approximations for loca-

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tion estimation. We also emphasize the cases when location data is only partially available, in the beginning or in the end of the estimated track. Because mostly such tracks are the source for location probes indoors, where GNSS signals are not available. We then compare performance of the proposed smoothing algorithms based on experiments with simulated and real data.

The remainder of the paper is organized as follows: Section II describes the Kalman filter algorithm, its non-linear approximations, and corresponding Rauch-Tung-Striebel smoother algorithms. In section III we provide details about models used for pedestrian dead reckoning. In section IV we present and discuss experimental results based on simulated and real data. Section V concludes the paper.

II. Kalman filter/smoother algorithms

Kalman Filter estimates a state based on the measurement model, which describes functional relation between the measurement and state components, and the state transition model, which describes how the state evolves from one time epoch to another. In general, measurement and state transition models have the following form:

$$x_k = f_k(x_{k-1}) + w_k \tag{1}$$
$$z_k = h_k(x_k) + v_k \tag{2}$$
$$w_k \sim N(0, Q_k) \tag{3}$$
$$v_k \sim N(0, R_k) \tag{4}$$

where x_k is a state vector, z_k is a measurement vector at time moment k, f_k is a function defining the transition from the previous state x_{k-1} at time $k-1$ to the current state x_k at time k, h_k defines the measurement function which maps measurement z_k to the state x_k, w_k, v_k model the system noises and are assumed to be zero-mean and Gaussian, with covariances Q_k and R_k respectively. All the variables, and numerical values are indexed according to the time index k. The goal is to estimate the distribution of state x_k based on the given sequence of measurements, state transition and measurement models, and assumed process and measurement noises of the system.

A. Kalman filter and its approximations

If the state transition and measurement models are linear functions of the state, and noise components are Gaussian and independent, then system model (1-2):

$$x_k = F_k x_{k-1} + w_k \tag{5}$$
$$z_k = H_k x_k + v_k \tag{6}$$

and the Kalman filter algorithm [4] provides the optimal solution in closed form.

The Kalman Filter recursively estimates the state at time k conditional on prior state at time 0 and measurements up to time k. The algorithm consists of the prediction step, during which the current state is estimated from the previous state according to the state transition model, and the update step, during which the measurement model is used to refine the predicted state estimate. Denoting by $\bar{x}_{k|k-1}$, $\bar{P}_{k|k-1}$ the estimated state mean and covariance at time k based on information up to time $k-1$, and using the above notation for the model parameters, the Kalman Filter algorithm is defined by the following recursive formulas:

$$\bar{x}_{0|0} = \bar{x}_0, \bar{P}_{0|0} = \bar{P}_0 \tag{7}$$
$$\bar{x}_{k|k-1} = F_k \bar{x}_{k-1|k-1} \tag{8}$$
$$\bar{P}_{k|k-1} = F_k \bar{P}_{k-1|k-1} F_k^T + Q_{k-1} \tag{9}$$
$$\eta_k = y_k - H_k \bar{x}_{k|k-1} \tag{10}$$
$$S_k = R_k + H_k \bar{P}_{k|k-1} H_k^T \tag{11}$$
$$K_k = \bar{P}_{k|k-1} H_k^T S_k^{-1} \tag{12}$$
$$\bar{x}_{k|k} = \bar{x}_{k|k-1} + K_k \eta_k \tag{13}$$
$$\bar{P}_{k|k} = (\mathrm{I} - K_k H_k) \bar{P}_{k|k-1} \tag{14}$$

If the state transition or measurement model is non-linear, Extended Kalman Filter can be used instead of Kalman Filter. In Extended Kalman Filter, non-linear models are approximated by a Taylor expansion of function f_k in the predicted state, and function h_k in predicted measurement. The algorithm is defined by the following recursive formulas:

$$\bar{x}_{0|0} = \bar{x}_0, \bar{P}_{0|0} = \bar{P}_0 \tag{15}$$
$$\bar{x}_{k|k-1} = f_k\left(\bar{x}_{k-1|k-1}\right) \tag{16}$$
$$[F_k]_{i,j} = \left.\frac{\delta f_i}{\delta x_j}\right|_{\bar{x}_{k-1|k-1}} \tag{17}$$
$$\bar{P}_{k|k-1} = F_k \bar{P}_{k-1|k-1} F_k^T + Q_{k-1} \tag{18}$$
$$\eta_k = y_k - h_k \bar{x}_{k|k-1} \tag{19}$$
$$[H_k]_{i,j} = \left.\frac{\delta h_i}{\delta x_j}\right|_{\bar{x}_{k|k-1}} \tag{20}$$
$$S_k = R_k + H_k \bar{P}_{k|k-1} H_k^T \tag{21}$$
$$K_k = \bar{P}_{k|k-1} {H_k}^T S_k^{-1} \tag{22}$$
$$\bar{x}_{k|k} = \bar{x}_{k|k-1} + K_k \eta_k \tag{23}$$
$$\bar{P}_{k|k} = (\mathrm{I} - K_k H_k) \bar{P}_{k|k-1} \tag{24}$$

Another non-linear approximation of the Kalman Filter is the Unscented Kalman filter, which uses the Unscented transform for state and measurement prediction. The Unscented transform approximates the distribution of function $g(x) + e$, $e \sim N(0, E)$ of a Gaussian random vector X with mean m and covariance matrix P as a Gaussian random vector with mean μ and covariance S. The Unscented transform is based on choosing set of sigma points $\{\chi^i\}$, $i = 0 \ldots 2N$ that represents the distribution of X, and compute the mean and covariance matrix of transformed variable based on the values of the function $g(\chi^i)$, $i = 0 \ldots 2N$ at the sigma points. One step of unscented transform is done according to the formulas below, and result in the approximated mean μ and covariance matrix S of transformed random variable $Y = g(x) + e$, as well as covariance matrix S of the joint distribution of X and

$Y = g(x) + e$:

$$\lambda = \alpha^2(n+\kappa) - n \tag{25}$$
$$\chi^0 = m \tag{26}$$
$$\chi^i = m + \sqrt{n+\lambda}\left[\sqrt{P}\right]_i, i = 1, ..., n \tag{27}$$
$$\chi^{i+n} = m - \sqrt{n+\lambda}\left[\sqrt{P}\right]_i, i = 1, ..., n \tag{28}$$
$$W_0^{(m)} = \frac{\lambda}{n+\lambda} \tag{29}$$
$$W_0^{(P)} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta) \tag{30}$$
$$W_i^{(m)} = \frac{\lambda}{2(n+\lambda)}, i = 1, ..., 2n \tag{31}$$
$$W_i^{(P)} = \frac{\lambda}{2(n+\lambda)}, i = 1, ..., 2n \tag{32}$$
$$Y^i = g(\chi^i), i = 0, ..., 2n \tag{33}$$
$$\mu = \sum_{i=0}^{2n} W_i^{(m)} Y^i \tag{34}$$
$$S = \sum_{i=0}^{2n} W_i^{(P)} (Y^i - \mu)(Y^i - \mu)^T + E \tag{35}$$
$$C = \sum_{i=0}^{2n} W_i^{(P)} (\chi^i - m)(Y^i - \mu)^T. \tag{36}$$

Parameters α and κ determine the spread of the sigma points, and β is used to incorporate prior information about X [5], [6].

By enclosing 26-36 into a single function:

$$[\mu, S, C] = UT(m, P, g, E), \tag{37}$$

the Unscented Kalman Filter algorithm is defined by the following recursive formulas:

$$\bar{x}_{0|0} = \bar{x}_0, \bar{P}_{0|0} = \bar{P}_0 \tag{38}$$
$$\left[\bar{x}_{k|k-1}, \bar{P}_{k|k-1}, D_k\right] = UT(\bar{x}_{k-1|k-1}, \bar{P}_{k-1|k-1}, f_k, Q_{k-1}) \tag{39}$$
$$[\mu_k, S_k, C_k] = UT(\bar{x}_{k|k-1}, \bar{P}_{k|k-1}, h_{k-1}, R_{k-1}) \tag{40}$$
$$K_k = C_k S_k^{-1} \tag{41}$$
$$\bar{x}_{k|k} = \bar{x}_{k|k-1} + K_k\left[y_k - \mu_k\right] \tag{42}$$
$$\bar{P}_{k|k} = \bar{P}_{k|k-1} - K_k S_k K_k^T \tag{43}$$

Note that in some cases only one of the models (state transition or measurement model) may be non-linear, while the other is linear, as well as models may be different at different times. Therefore, appropriate propagation of the state estimate, must be made at each predict/update step of the filter.

B. Raugh-Tung-Striebel smoother

The Rauch-Tung-Striebel smoother [5] is based on the Kalman filter algorithm and consists of the forward and backward passes. During the forward pass the state of the device is estimated based on the Kalman filter (or EKF, UKF) according to formulas above, and prior and posterior covariances as well as state transition matrices are recorded and used later in the backward pass. During the backward pass the smoother recursively refines the estimates starting from the last one. The backward pass of Rauch-Tung-Striebel smoother is defined by the following recursive formulas:

$$\tilde{x}_{n|n} = \bar{x}_{n|n}, \tilde{P}_{n|n} = \bar{P}_{n|n} \tag{44}$$
$$G_k = P_{k|k} F_{k+1}^T P_{k+1|k}^{-1} \tag{45}$$
$$\tilde{x}_{k|n} = \bar{x}_k + G_k[\tilde{x}_{k+1|n} - \bar{x}_{k+1|k}] \tag{46}$$
$$\tilde{P}_{k|n} = \bar{P}_{k|k} + G_k[\tilde{P}_{k+1|n} - \bar{P}_{k+1|k}]G_k^T, \tag{47}$$

where $\bar{x}_{n|n}$, $\bar{P}_{n|n}$ are the mean and covariance matrix of the last estimate with index n, and $k = n-1, n-2, ..., 1$. In a similar way, smoothing can be done for the non-linear Extended and Unscented Kalman Filters.

Formulas for Extended Kalman Smoother are:

$$\tilde{x}_{n|n} = \bar{x}_{n|n}, \tilde{P}_{n|n} = \bar{P}_{n|n} \tag{48}$$
$$G_k = P_{k|k} F_{k+1}^T P_{k+1|k}^{-1} \tag{49}$$
$$\tilde{x}_{k|n} = \bar{x}_{k|k} + G_k[\tilde{x}_{k+1|n} - \bar{x}_{k+1|k}] \tag{50}$$
$$\tilde{P}_{k|n} = \bar{P}_{k|k} + G_k[\tilde{P}_{k+1|n} - \bar{P}_{k+1|k}]G_k^T \tag{51}$$

Formulas for Unscented Kalman Smoother are:

$$\tilde{x}_{n|n} = \bar{x}_{n|n}, \tilde{P}_{n|n} = \bar{P}_{n|n} \tag{52}$$
$$G_k = D_{k+1} P_{k+1|k}^{-1} \tag{53}$$
$$\tilde{x}_{k|n} = \bar{x}_{k|k} + G_k[\tilde{x}_{k+1|n} - \bar{x}_{k+1|k}] \tag{54}$$
$$\tilde{P}_{k|n} = \bar{P}_{k|k} + G_k[\tilde{P}_{k+1|n} - \bar{P}_{k+1|k}]G_k^T \tag{55}$$

III. Filters for pedestrian dead reckoning

In this section we present different system models specific for pedestrian dead reckoning. For Kalman Filter we define the user state as a four-component vector with the first two components representing location and third and fourth components representing the step vector, where step vector is collinear with the heading of the user, and its norm is equal to the step length of the user. For Extended and Unscented Kalman filters user state is defined as a four-component vector, with the first two components representing location, and third and fourth components representing user heading and step length respectively. The information available for estimation process include location measurements from GNSS module, and step measurements and heading changes from the inertial measurements of the device.

In the standard Kalman Filter models are linear, and step length cannot be taken into account neither in state transition nor in measurement model [1]. Only position propagation and heading change can be reflected in the state transition model. In this case state transition model is defined by the matrix

$$F = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & \cos(\theta) & -\sin(\theta) \\ 0 & 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}, \tag{56}$$

where θ is the angle by which heading of the device has changed between times k and $k-1$. And position measurement model is defined by

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{57}$$

In EKF and UKF models may be non-linear and we define the state vector so that the first two components of the state vector represent position, and the third and fourth components represent heading and step length respectively. In this case the state transition function is of the form:

$$f_k(x) = \begin{bmatrix} x_1 + x_4 \cdot \cos(x_3) \\ x_2 + x_4 \cdot \sin(x_3) \\ x_3 + \theta \\ x_4 \end{bmatrix}, \tag{58}$$

and the measurement function is of the form:

$$h_k(x) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \tag{59}$$

Due to the convenient state vector definition in EKF and UKF, namely because step length is a separate component rather than a function of the state components, it can be directly measured by a linear measurement function. However, in this case state transition model become non-linear, and hence requires the use of non-linear Kalman filter approximation. Additional advantage of modeling step length and heading as separate state components is that their corresponding process and measurement noises may be modeled separately, unlike the case when step length and heading are described by a 2D vector.

We propose variant of the filter which is the same as the standard Kalman Filter, but add additional heuristic step. After each predict/update phase of the filter, length of the step vector is adjusted to match with the measured step length:

$$\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = l \cdot \frac{\begin{bmatrix} x_3 \\ x_4 \end{bmatrix}}{\left\| \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \right\|}, \tag{60}$$

where l is a measured step length. This is done in order to maintain correct length of the step vector even when it cannot be estimated by location measurements (during GPS outage). An alternative approach would be to use EKF-type measurement model to correct the length of the step vector when a step measurement is available.

Note that all the state transition and measurement models assume that state components are changing and measured at the same time. In practice this is not the case, since sensor measurements become available at different times. Therefore, the order of the prediction and update phases, as well as state transition and measurement models must be modified according to the information available. For example, if there is a location or step length measurement, an update step is done, if inertial measurements are available (heading change or step event), a prediction step is done. Also, measurement function (59) will have only first and second component if only location measurement is available, and only third component if step length measurement is available. Similarly, if heading change measurement is available, then only velocity components are modified by the state transition function, and if a step measurement is available, only position components are modified by the state transition function. This also means that described algorithms can be applied to a stream of ordered location, inertial and step measurements without a need to synchronize them.

For testing we used consumer-grade mobile device: LG Nexus 5. We use GPS fixes as a measurement of device location, and embeded inertial sensors to derive information about user motion, such as heading change, step occurence and step length. Heading change is estimated based on gyroscope and accelerometer measurements. Namely, we estimate the gravity vector as a moving average of acceleration measurements, the axis and angle of rotation from gyroscope measurements, and compute heading change as a rotation angle multiplied by projection (scalar product) of rotation axis onto gravity vector, where both vectors are normalized. Steps are detected by the operating system of the device, and step lengths are computed based on the maximum standard deviation of the components of the acceleration signal. Namely, we compute step length as a linear function of the maximum standard deviation of the acceleration signal,where standard deviation is computed over a sliding window. We use sliding window of 1 second, which should be enough to capture acceleration signal within one step, which is representative of the user pace. Another algorithm for step detection and step length estimation can be found in [7].

In real life, reliable estimation of the heading change is quite challenging, as the user may change orientation of the mobile device without changing his/her own heading. In this paper we propose a heuristic method to detect such cases and handle them appropriately. The method is based on the assumption that in majority of the cases when person changes the orientation of the device with respect to the body frame, the gravity vector changes considerably as well. The only exceptions are the cases when user turns the device strictly around the vertical axis, which is not a typical user move (even when changing between portrait and landscape modes). Therefore, we can use angular change of the gravity vector as a measure of reliability of the heading change. In other words, the greater the gravity change is, the less reliable is heading change measurement. It is possible to define the mapping between gravity change to the variance of the heading change so that uncertainty of the estimate grows faster every time orientation of the device is changed. In our tests we used the following function for heading change and corresponding uncertainty:

$$\sigma_\theta = \triangle \bar{g}, \tag{61}$$

where $\triangle \bar{g}$ is an angular difference between gravity vectors in the beginning and end of the sliding window. In our experiments we used 1 s sliding window. In addition, heading change may be completely ignored (set to zero), if corresponding gravity change exceeds a certain threshold. This will result in losing the track estimation, however, estimation errors will remain consistent with the estimate uncertainty. On the other hand, solution will provide more informative estimates for the cases when the user keeps the device relatively steady. This allows not to make assumptions about fixed location of the mobile device with respect to the user body, as in practice user may change it at any time. Instead we assume that device is kept in the same pose with respect to the user body for some time during which we can estimate user locations effectively, and re-initialize the smoother when change of the device's orientation is detected. Yet in the experiments, mobile device is kept in hand in the landscape orientation, for the sake of simplicity.

IV. Experiments

In this section, we present estimation results for different filters based on simulated and real data. We also outline the performance and robustness of filters under different conditions.

A. Simulated data

For simulations we generated the required measurements including approximate device positions, heading changes and step lengths, based on a real user track from outdoors to indoors. Exact values of device position, heading change and step length were derived from the true user locations, and corresponding measurements were simulated by adding zero-mean Gaussian noises with specified variances to the true user locations, heading changes and step lengths. As a result, filter inputs include a set of regular location measurements, and measurements of heading change and step length.

We simulated filter inputs with noises of increasing magnitudes for location, heading change and step length in order to investigate the robustness of the algorithm under different noise conditions. Noise standard deviations for location estimates, heading, and step length changes are denoted by $\sigma_z, \sigma_\theta, \sigma_l$ respectively. Standard deviations are expressed in meters, degrees and meters respectively.

We then estimate device positions within each generated track with different filters and express location error as a distance between true and estimated position, and evaluate 0.95-consistency as in [8]. Average location error and 0.95-consistency for simulated data are summarized in Tables I and II.

As seen from the results in Table I, all filters have similar average error, with Unscented and Extended Kalman filters being slightly better than Kalman Filters. On the other hand Kalman Filter is noticeably better then other filters in terms of 0.95-consistency. In this case location estimates are available every second throughout the whole track, which means that step length was constantly observed and estimated by the filter.

In real scenarios, location estimates are not always available or reliable, e.g. when user enters the building and GNSS fixes become unavailable or unreliable. In this case, step length estimates as well as heading may not be observed from GNSS locations and are only changed by the state transition model. Therefore it is important that the state transition model account not only for heading changes but for changes of step length as well, especially as it may change considerably when the user goes from outdoors to indoors. To model such scenario, we use location measurements only when the user is outdoors. As a result position is estimated purely based on the motion model when user is indoors. Average accuracy and consistency for the scenario with partial GNSS availability are presented in the Tables III, IV.

As seen from Table III, Kalman filter performs considerably worse than other filters in case position measurements are only partially available. This is because step length cannot be estimated with position measurements and remains constant if not changed by the state transition model. As a result, Kalman Filter overestimates or underestimates the traveled distance if user changes the pace. Therefore it is important to use state

TABLE I
Average localization error for simulated data with regular GNSS availability

$\sigma_z, \sigma_\theta, \sigma_l$	KS	KS SL	EKS	UKS
$10, 1^\circ, 0.1$	2.45	3.07	2.16	2.24
$15, 2^\circ, 0.2$	3.37	5.66	4.73	3.21
$20, 3^\circ, 0.3$	4.08	3.75	2.87	3.64
$25, 4^\circ, 0.4$	5.56	3.85	4.4	3.82
$30, 5^\circ, 0.5$	6.02	5.43	6.56	5.59
average error	4.30	4.35	4.14	3.70

TABLE II
Localization error 0.95-consistency for simulated data with regular GNSS availability

$\sigma_z, \sigma_\theta, \sigma_l$	KS	KS SL	EKS	UKS
$10, 1^\circ, 0.1$	0.93	0.47	0.8	0.37
$15, 2^\circ, 0.2$	0.93	0.59	0.83	0.52
$20, 3^\circ, 0.3$	1	0.93	0.94	0.56
$25, 4^\circ, 0.4$	0.96	0.87	0.86	0.62
$30, 5^\circ, 0.5$	1	0.90	0.82	0.52
average error	0.96	0.75	0.85	0.51

TABLE III
Average localization error for simulated data with partial GNSS availability

$\sigma_z, \sigma_\theta, \sigma_l$	KS	KS SL	EKS	UKS
$10, 1^\circ, 0.1$	11.4	5.94	3.21	3.65
$15, 2^\circ, 0.2$	11.13	6.39	3.12	3.53
$20, 3^\circ, 0.3$	16.32	8.14	4.45	5.83
$25, 4^\circ, 0.4$	32.65	18.94	19.95	20.68
$30, 5^\circ, 0.5$	39.09	11.18	12.54	16.31
average error	22.11	10.11	8.65	10

TABLE IV
LOCALIZATION ERROR 0.95-CONSISTENCY FOR SIMULATED DATA WITH PARTIAL GNSS AVAILABILITY

$\sigma_z, \sigma_\theta, \sigma_l$	KS	KS SL	EKS	UKS
$10, 1°, 0.1$	0.94	0.42	0.72	0.48
$15, 2°, 0.2$	1	0.76	0.94	0.55
$20, 3°, 0.3$	1	0.95	0.99	0.66
$25, 4°, 0.4$	0.98	0.55	0.91	0.51
$30, 5°, 0.5$	0.99	0.94	0.99	0.60
average error	0.98	0.72	0.91	0.56

transition models which account for step length changes if position measurements are partially unavailable or unreliable.

On the other hand in [1] we noted the importance of the initial state for non-linear filters EKF and UKF. This is the case in these experiments as well. Without accurate initial state EKF and UKF do not provide acceptable results, whereas Kalman Filter and its modification with step length adjustment perform robustly even without initial state information. This is because in non-linear filters state transition models depend on the estimated state of the filter, and if that is not accurate then model may be incorrect, which makes further estimation even worse. In order to resolve this problem for UKF and EKF, firstly filters run backwards from the last location measurement in order to estimate the initial state, and then start over again from the estimated initial state. This, however, causes additional algorithm complexity, namely all the measurements must be buffered, and additional backward filtering is required.

As previously, Kalman filter outperforms other filters in terms of 0.95-consistency even though it has the largest estimation error.

B. Real data

In order to test filter performance in real case scenarios, four similar user tracks were recorded and estimated based on recorded sensor and location measurements. Each track is approximately 500 meters long, and covers indoor and outdoor parts in equal proportions. Test tracks were started inside the building, continued outdoors, and finished again inside the building. This means that in the beginning and in the end of the track user location was not estimated by GPS and position estimation was based purely on the user trajectory. In practice, this is the most typical scenario when indoor location probes can be inferred. One of the test tracks is presented on the Fig. 1, along with corresponding location references, location estimates, and locations provided by GPS.

True user positions were obtained by means of interpolation from the set of positions indicated by user throughout the track traversal. We then estimated user positions throughout the tracks with different filters, and computed estimation error and 0.95-consistency. Estimation results for real data are summarized in the Tables V and VI.

As seen from Table V, Kalman Filter with step length adjustment noticeably outperforms the other methods. Kalman

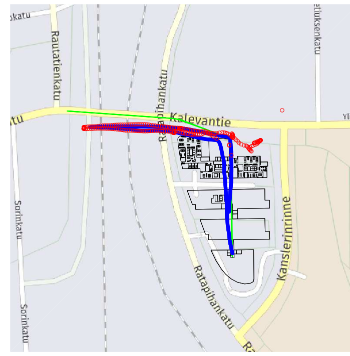

Fig. 1. Example test track with corresponding location references (green), location estimates (blue), and locations provided by GPS (red).

TABLE V
AVERAGE LOCALIZATION ERROR FOR REAL DATA

	KS	KS SL	EKS	UKS
track 1	36.9608	14.1578	21.2462	28.3694
track 2	27.8456	12.6445	25.2197	32.1426
track 3	35.1727	11.4269	22.2905	31.3898
track 4	21.0482	17.6744	12.2995	51.0284
average error	30.2568	13.9759	20.2640	35.7325

Filter performance corresponds to the results from the simulated cases, where filter cannot track the step length changes in the absence of location measurements. Moreover, filter performance may be affected by poor GPS performance during indoor-outdoor transition phase, when GPS may remain at one location or scatter around large area, which underestimates or overestimates step length considerably. EKF and UKF show decrease in performance compared to performance based on simulated data. The problem may also be in inaccurate and sometimes inconsistent GPS fixes during transition phases, which may cause wrong state estimates and hence wrong state transition models, which makes further filter behavior unstable.

All filters, except for UKF, perform worse in terms of estimation consistency. Inconsistent estimates may be caused by inconsistency in GPS fixes or the worse consistency of EKF in case of large nonlinearities [9].

Based on the experiments with simulated and real data we can conclude that Kalman filter with step length adjustment behaves better than other considered filters. This is because

TABLE VI
LOCALIZATION ERROR 0.95-CONSISTENCY FOR REAL DATA

	KS	KS SL	EKS	UKS
track 1	0.80	0.39	0.69	0.76
track 2	0.71	0.40	0.40	0.80
track 3	0.71	0.54	0.51	0.78
track 4	0.97	0.68	0.94	0.87
average error	0.80	0.50	0.63	0.80

it is robust to unreliable position fixes, and also takes into account information about step lengths, which is important in the absence of the absolute location fixes.

V. Conclusion

In this paper we considered Kalman Filter, its non-linear approximations and corresponding Rauch-Tung-Striebel smoothers for inference of indoor location probe data.

Based on the experiments with simulated and real data we can conclude that step length must be modeled and measured properly. This becomes even more evident in the cases when location observations are partially unavailable or unreliable. This is an important conclusion especially for tracks that are partially inside the buildings where GNSS fixes are either completely unavailable or unreliable, i.e. tracks from which indoor location probes can be inferred. Step length measurements maintain the correct propagation of the position estimates even when location measurements are uncertain and unreliable, preventing the filter from overestimating or underestimating the distance traveled.

In order to take step length into account non-linear state transition models or measurement models must be used. From previous discussion, non-linear filters are less robust since state mean propagation heavily depend on previous state mean or covariance. Additionally, according to experimental results, Kalman Filter has better estimation consistency, which is an advantage that must be considered in order to make further inference based on the indoor location probes reliable. Moreover, non-linear filters are more complex from implementation point of view, and are not robust against uninformative initial state estimate.

In summary, it is advisable to use filters with as few non-linear transformations as possible in order to improve filter accuracy, robustness, consistency, and complexity. We suggest to use Kalman Filter with explicit step length correction or EKF-type step length measurement as such filter. It has minimal number of non-linear approximations, it has almost the same complexity as Kalman Filter, and yet provides fairly good estimation accuracy compared to EKF and UKF.

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