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Robust Kalman Filter for Positioning with Wireless BS Coverage Areas

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***Abstract*—A robust Kalman filter method for positioning using a database of wireless base station coverage areas is presented. In tests with simulated and real data, the proposed filter is found to be more accurate than static positioning or conventional Kalman filtering.**

I. Introduction

This paper addresses the problem of estimating the state variables of a dynamic system using noisy measurements in positioning systems. The state estimation problem has attracted the interests of many researchers in the past three decades. One of the popular methods is the celebrated Kalman filter approach, which is based on minimizing the variance of the estimation error. The filtering algorithm assumes perfect knowledge of the dynamic model for the signal generating system, and noise sources to be white with known statistics. Thus, the standard Kalman filter (KF) may not be robust against unmodeled dynamics and disturbances. This has motivated many studies on robust Kalman filter (RKF) design [1, 2]. In this paper a new variant of RKF is introduced.

The application considered in this paper is coverage-area based positioning [3, 4]. Its basic idea is to determine the location of the user terminal (UT) using a radio map. A radio map is a database of coverage area information of wireless base stations (BS). Here, coverage area means the region in the plane where signals from the BS can be received by the network user. Coverage areas are represented by ellipses. This has the advantage that only five parameters have to be stored in the radio map for each BS. In the online positioning phase the coverage areas of the heard BSs are used to infer the position of the UT. In [3], ellipse-shaped coverage areas are fitted by modeling fingerprints as having a Normal (Gaussian) distribution. However, the Normal regression model is well known to be overly sensitive to outliers, i.e. outliers produce coverage areas that are too large. The Student-t distribution, is an alternative to the Normal distribution that, due to its heavy tails, is better suited as a model of data that may contain outliers. Hence, in this paper ellipse-shaped coverage areas are modeled using a Student-t model distribution as explained in [5].

The basic RKF used in this paper is similar to the Kalman filter which was presented in our earlier papers [3, 4]. In this paper, practical consideration in designing the filter leads us to the proposed RKF filter which shows a good performance in real scenario. Moreover, different from our earlier works, in this paper ellipse shaped coverage area models that are fitted to data using Student-t regression are used. The positioning algorithm presented in this paper is tested using both simulated and real positioning data. The positioning performance using the RKF is compared to the static positioning solution and the standard KF solution. The results indicates the superior positioning performance using RKF.

The remainder of this paper is organized as follow: The RK filter is derived in section II. Section III describes the application of the filter for positioning. The positioning results using static solution, KF and RKF are compared in section IV-A and IV-B. Concluding remarks are given in section V.

Notations: The superscript T denotes matrix transposition. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $||\cdot||$ refers to Euclidean vector norm. Capital letters stand for matrices.

II. Filter Design

In this section, formulation of Kalman filter as the solution of a least squares problem is reviewed. This problem is reformulated using M-estimators. The problem's numerical solution algorithm, which leads to the proposed re-weighted Kalman filter (Robust KF), is then introduced.

A. System Model

The state of the dynamic process is modeled as a stochastic process $x_k, k \in \mathbb{N}\backslash\{0\}$. The process dynamics is described by a discrete-time equation

$$x_k = \mathrm{G}_{k-1}x_{k-1} + w_{k-1}, \tag{1}$$

where x_k is the state at time step t_k and the linear state transition matrix $\mathrm{G}_k \in \mathbb{R}^{n_x \times n_x}$ describes the evolution of the state, e.g. position, velocity, etc., in time. Here, w_k is a zero mean white noise process with covariance matrix Q_k. Let the initial state be $x_0 \sim \mathrm{Normal}(\hat{x}_0, \mathrm{P}_0)$. The noise w_k is assumed to be independent of x_0.
The measurement equation describes the connection between the state and the measurements y_k

$$y_k = \mathrm{H}_k x_k + v_k, \tag{2}$$

where the measurement function is denoted by $\mathrm{H}_k \in \mathbb{R}^{n_{y_k} \times n_x}$, and n_{y_k} is the dimension of the measurement vector at time step t_k. The measurement noise v_k is assumed to be white with covariance matrix R_k and independent of x_0 and the state noise w_k.

B. The Kalman Filter

In Bayesian filtering the posterior distribution of the state given all available data can be solved in two steps, prediction and update. It follows from the whiteness of the noise processes and Bayes' rule that the prior and posterior probability density functions (pdf) may be formulated as

$$f_{x_k^-}(x_k|y_{1:k-1}) = \int f(x_k|x_{k-1})f(x_{k-1}|y_{1:k-1})\,\mathrm{d}\,x_{k-1} \quad (3)$$

$$f_{x_k^+}(x_k|y_{1:k}) = \frac{f(y_k|x_k)f(x_k|y_{1:k-1})}{\int f(y_k|x_k)f(x_k|y_{1:k-1})\,\mathrm{d}\,x_k} \quad (4)$$

where x_k^- and x_k^+ are prior and posterior states respectively. Notation $y_{1:n}$ stands for $\{y_1, ..., y_n\}$ and denotes the measurement history. The subscripts of some pdf's are left out for readability.

The prior density function may be written as Normal distribution $f_{x_k^-}(x_k|y_{1:k-1}) = \mathrm{Normal}(x_k; \hat{x}_k^-, \hat{\mathrm{P}}_k^-)$. Mean and covariance of the prior density are obtained according to the state model (1)

$$\hat{x}_k^- = \mathbb{E}(x_k|y_{1:k-1}) = \mathrm{G}_{k-1}\hat{x}_{k-1} \quad (5)$$

$$\hat{\mathrm{P}}_k^- = \mathbb{V}(x_k|y_{1:k-1}) = \mathrm{G}_{k-1}\hat{\mathrm{P}}_{k-1}\mathrm{G}_{k-1}^T + \mathrm{Q}_{k-1} \quad (6)$$

The distribution of the predicted measurement is also Normal distribution

$$f_{y_k^-}(y_k|y_{1:k-1}) = \mathrm{Normal}\left(y_k; \mathrm{H}_k\hat{x}_k^-, \mathrm{H}_k\hat{\mathrm{P}}_k^-\mathrm{H}_k^T + \mathrm{R}_k\right)$$

The measurement likelihood function may be written as

$$\begin{aligned} f_{y_k|x_k}(y_k|x_k) &= f_{v_k}(y_k - \mathrm{H}_k x_k) \\ &= \mathrm{Normal}(y_k - \mathrm{H}_k x_k; 0, \mathrm{R}_k) \end{aligned} \quad (7)$$

Inserting prior density and predicted measurement density functions into (4) yields the posterior density

$$f_{x_k^+}(x_k|y_{1:k}) = \mathrm{Normal}(x_k; \hat{x}_k, \hat{\mathrm{P}}_k) \quad (8)$$

where

Posterior mean	$\hat{x}_k = \hat{x}_k^- + \mathrm{K}_k(y_k - \mathrm{H}_k\hat{x}_k^-)$	(9a)
Posterior cov.	$\hat{\mathrm{P}}_k = (\mathrm{I} - \mathrm{K}_k\mathrm{H}_k)\hat{\mathrm{P}}_k^-$	(9b)
Filter gain	$\mathrm{K}_k = \hat{\mathrm{P}}_k^-\mathrm{H}_k^T(\mathrm{H}_k\hat{\mathrm{P}}_k^-\mathrm{H}_k^T + \mathrm{R}_k)^{-1}$	(9c)

C. Robust Kalman Filter

In the linear-Gaussian case KF solves the posterior pdf state analytically. Since the posterior pdf is Gaussian, the posterior mean estimate is the value that maximizes the posterior pdf

$$\begin{aligned} \hat{x}_k &= \underset{x_k}{\mathrm{argmax}}\, f_{x_k^+}(x_k|y_{1:k}) \\ &\overset{\text{Eq. (4)}}{=} \underset{x_k}{\mathrm{argmax}} \Big(\mathrm{Normal}(x_k; \hat{x}_k^-, \hat{\mathrm{P}}_k^-)\cdot \\ &\qquad \mathrm{Normal}(y_k - \mathrm{H}_k x_k; 0, \mathrm{R}_k)\Big) \\ &= \underset{x_k}{\mathrm{argmin}} \left(\frac{1}{2}\left(||x_k - \hat{x}_k^-||^2_{(\hat{\mathrm{P}}_k^-)^{-1}} + ||y_k - \mathrm{H}_k x_k||^2_{\mathrm{R}^{-1}}\right)\right) \\ &\overset{\star}{=} \underset{x_k}{\mathrm{argmin}} \left(\frac{1}{2}\left(||n||^2 + ||l||^2\right)\right) \\ &= \underset{x_k}{\mathrm{argmin}} \left(\sum_{j=1}^{n_x} \frac{1}{2}n_j^2 + \sum_{i=1}^{n_y}\frac{1}{2}l_i^2\right) \end{aligned} \quad (10)$$

where in $\star$ the notations $n = \mathrm{N}(x_k - \hat{x}_k^-)$ and $l = \mathrm{L}(y_k - \mathrm{H}_k x_k)$ are used, where N and L are matrices such that $(\hat{\mathrm{P}}_k^-)^{-1} = \mathrm{N}^T\mathrm{N}$ and $\mathrm{R}_k^{-1} = \mathrm{L}^T\mathrm{L}$. It will be shown that it is not necessary to compute these matrices.

It is seen from (10) that the posterior mean estimate of the KF is a recursive solution to an ordinary least squares problem. To make the filter robust, the second quadratic cost function in (10) is replaced by ρ-function of an M-estimator introduced in [1, 6–8]. The aim is to make the measurement model more robust, hence only the second sum is modified. Since the score function is assumed to be convex, the minimum is found by setting the gradient of the modified sum to zero (Here and in the following the time subscript k is omitted for simplicity.)

$$\sum_{j=1}^{n_x} \nabla_x \frac{1}{2}n_j^2 + \sum_{i=1}^{n_y} \nabla_x \rho_i(l_i) = 0 \quad (11)$$

Here it should be noted that various different M-estimator have been developed and it is possible to invent more. The best way to choose an M-estimator for a specific problem would be to empirically find the best weight-function using a suitable optimization algorithm. This, however, is out of the scope of this paper. The DHA Hampel estimator, presented in [1, 6–9], is considered in the robust filter design in this paper. The parameters of the influence function ψ and weight function w are then optimized for the purpose of this paper.

$$\psi_{DHA}(\theta) = \begin{cases} \theta, & |\theta| < k_1 \\ k_1\mathrm{sign}(\theta), & k_1 \le |\theta| < k_2 \\ \frac{k_1 k_2}{|\theta|}\mathrm{sign}(\theta), & |\theta| \ge k_2 \end{cases} \quad (12)$$

Now, denoting the derivative of the score function with ψ-function of an M-estimator, we have

$$\nabla_x\left(\frac{1}{2}n_j^2\right) = n_j\mathrm{N}^T e_j \quad (13a)$$

$$\nabla_x \rho_i(l_i) = -\psi_i(l_i)\mathrm{H}^T\mathrm{L}^T e_i \quad (13b)$$

where e_i is the ith column of the identity matrix.

Since ψ is in general a non-linear function, (11) has to be solved numerically. Simultaneously, the computationally convenient properties of the KF should be preserved. In [10, 11] the equation is replaced by a linear approximation. Thus, the minimization problem in (11) is equal to the matrix equation

$$
\begin{aligned}
&\mathrm{N}^T \begin{bmatrix} n_1 \\ \vdots \\ n_x \end{bmatrix} - \mathrm{H}^T\mathrm{L}^T \begin{bmatrix} w_1(\hat{l}_1^-)l_1 \\ \vdots \\ w_{n_y}(\hat{l}_{n_y}^-)l_{n_y} \end{bmatrix} = 0 \\
\Leftrightarrow &\mathrm{N}^T\mathrm{N}(x-\hat{x}^-) + \mathrm{H}^T\mathrm{L}^T\mathrm{W}_\psi\mathrm{L}(\mathrm{H}x - y) = 0
\end{aligned} \tag{14}
$$

where $\hat{l}^- = \mathrm{L}(y - \mathrm{H}\hat{x}^-)$ and the weights are given by

$$
w_i(\hat{l}_i) = \begin{cases} \frac{\psi_i(\hat{l}_i^-)}{\hat{l}_i^-}, & \hat{l}_i^- \neq 0 \\ 1, & \hat{l}_i^- = 0 \end{cases}. \tag{15}
$$

The diagonal matrix is

$$
\mathrm{W}_\psi = \begin{bmatrix} w_1(\hat{l}_1^-) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & w_{n_y}(\hat{l}_{n_y}^-) \end{bmatrix}.
$$

Defining $\mathrm{R}_\mathrm{W} = (\mathrm{L}^T\mathrm{W}_\psi\mathrm{L})^{-1}$, (14) may be written as

$$
(\hat{\mathrm{P}}^-)^{-1}(x - \hat{x}^-) + \mathrm{H}^T\mathrm{R}_W^{-1}(\mathrm{H}x - y) = 0 \tag{16}
$$

Eq. (16) is a solution for the minimization problem

$$
\hat{x}_k = \underset{x}{\operatorname{argmin}} \left(||(x - \hat{x}^-)||^2_{(\hat{\mathrm{P}}^-)^{-1}} + ||y - \mathrm{H}x||^2_{\mathrm{R}_\mathrm{W}^{-1}} \right) \tag{17}
$$

Eq. (17) is similar to (10) which was derived from the posterior mean estimate of the KF. The only difference is that the measurement covariance R is replaced by the weighted measurement covariance R_W. Thus, the solution of (17) is obtained using the posterior mean relation of the KF as

"Posterior" mean	$\hat{x} = \hat{x}^- + \mathrm{K}_\mathrm{W}(y - \mathrm{H}\hat{x}^-)$	(18a)
"Posterior" cov.	$\hat{\mathrm{P}} = (\mathrm{I} - \mathrm{K}_\mathrm{W}\mathrm{H})\hat{\mathrm{P}}^-$	(18b)
Filter gain	$\mathrm{K}_\mathrm{W} = \hat{\mathrm{P}}^-\mathrm{H}_k^T(\mathrm{H}\hat{\mathrm{P}}^-\mathrm{H}^T + \mathrm{R}_\mathrm{W})^{-1}$	(18c)

The derived robust Kalman filter (RKF) consists of computing the transformed innovation $\hat{l}^- = \mathrm{L}(\mathrm{H}\hat{x}^- - y)$ and weighting the measurement covariance matrix accordingly. The prior covariance remains unchanged since only the measurement model is modified. Thus, the filter derived here may be considered as a robust KF which modifies the given measurement covariances according to the innovation variable so that the larger transformed innovations result in larger variances.

Based on Eq. (18) it can be seen that it is not necessary to compute the matrix N. Now it is shown how to implement the filter without computing the matrix L. Matrix L is not unique because ($\hat{\mathrm{L}} = \mathrm{QL}$) also satisfies equation $\mathrm{R}_k^{-1} = \mathrm{L}^T\mathrm{L} = \hat{\mathrm{L}}^T\hat{\mathrm{L}}$, where Q is any orthogonal matrix. It is possible to select matrix L so that

$$
\begin{aligned}
\hat{l}^- &= \mathrm{L}(y - \mathrm{H}\hat{x}^-) = ||\mathrm{L}(y - \mathrm{H}\hat{x}^-)||e_1 \\
&= \sqrt{||y_k - \mathrm{H}_k x_k||^2_{\mathrm{R}^{-1}}}\, e_1.
\end{aligned} \tag{19}
$$

Using Eq. (19), one gets

$$
\begin{aligned}
\mathrm{R}_\mathrm{W} &= (\mathrm{L}^T\mathrm{W}_\psi\mathrm{L})^{-1} = \mathrm{L}^{-1}\mathrm{W}_\psi^{-1}\mathrm{L}^{-T} \\
&= \mathrm{R} + \mathrm{L}^{-1} \begin{bmatrix} \frac{1}{w_1(\hat{l}_1^-)} - 1 & 0 & \cdots \\ 0 & 0 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \mathrm{L}^{-T} \\
&= \mathrm{R} + \left(\frac{1}{w_1(\hat{l}_1^-)} - 1 \right) \mathrm{L}^{-1}e_1e_1^T\mathrm{L}^{-T} \\
&\overset{\text{Eq. (19)}}{=} \mathrm{R} + \alpha\mathrm{L}^{-1}\mathrm{L}(y - \mathrm{H}\hat{x}^-)(y - \mathrm{H}\hat{x}^-)^T\mathrm{L}^T\mathrm{L}^{-T} \\
&= \mathrm{R} + \alpha(y - \mathrm{H}\hat{x}^-)(y - \mathrm{H}\hat{x}^-)^T
\end{aligned} \tag{20}
$$

where

$$
\alpha = \left(\frac{1}{||y_k - \mathrm{H}_k x_k||^2_{\mathrm{R}^{-1}}} \left(\frac{1}{w_1(\sqrt{||y_k - \mathrm{H}_k x_k||^2_{\mathrm{R}^{-1}}})} - 1 \right) \right).
$$

Hence, it is not necessary to compute the matrix L.

III. Robust KF Applications in Positioning

A. The State Model of the User Terminal

In this paper, the state of the user terminal (UT) consists of 2-dimensional position vector p and velocity $\dot{p}$ of the UT. The UT may be a laptop, a mobile phone, or any other device connected to a wireless network. The state model follows (1) where the state vector is denoted by

$$
x = \begin{bmatrix} p \\ \dot{p} \end{bmatrix} \tag{21}
$$

The velocity is modeled as a random walk process [12] and thus the state transition matrix become

$$
\mathrm{G}_k = \begin{bmatrix} \mathrm{I}_{2\times2} & \Delta t_k \mathrm{I}_{2\times2} \\ 0_{2\times2} & \mathrm{I}_{2\times2} \end{bmatrix} \tag{22}
$$

where $\Delta t_{k+1} = t_{k+1} - t_k$. The state noise is $w_k \sim \mathrm{Normal}(0, \mathrm{Q}_k)$ and the covariance matrix

$$
\mathrm{Q}_k = \begin{bmatrix} \frac{1}{3}\Delta t^3\mathrm{I}_2 & \frac{1}{2}\Delta t^2\mathrm{I}_2 \\ \frac{1}{2}\Delta t^2\mathrm{I}_2 & \Delta t\mathrm{I}_2 \end{bmatrix} \sigma^2 \tag{23}
$$

where σ represent the velocity error and its value tells how much the variance of the velocity error increases during one second. Similar to [13], here it is chosen to set $\sigma^2 = (1.41\frac{\mathrm{m}}{\mathrm{s}})^2\mathrm{s}^{-1}$.

B. The Measurement Model

In this work, measurements from terrestrial base stations are used. A BS may be a radio station, a TV station, a cellular network BS, a WLAN access point or some other sensor node in a wireless network. The UT continuously measures which BS:s it can hear. If a certain BS is heard then it is assumed that the UT is within that BS's coverage area. A database of coverage areas, each of which is indexed with the identification (ID) number of a heard BS, is used to infer the 2-dimensional position and velocity state of the UT. Estimation of coverage area parameters is described in section VI. The mean and the covariance matrix of the i-th coverage area are denoted with $\mu_i \in \mathbb{R}^2$ and $\Sigma_i \in \mathbb{R}^{2\times 2}$ respectively. Coverage area parameters of all BSs are saved in the radio map. Assuming that coverage area IDs $1, \ldots, n$ are heard at time-stamp k, measurement vector, measurement function matrix and measurement error covariance become

$$y_k = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_n \end{bmatrix}_{2n\times 1} \quad (24)$$

$$\mathrm{H}_k = \begin{bmatrix} \mathrm{I}_{2\times 2} & 0_{2\times 2} \\ \vdots & \vdots \\ \mathrm{I}_{2\times 2} & 0_{2\times 2} \end{bmatrix}_{2n\times 4} \quad (25)$$

$$\mathrm{R}_k = \begin{bmatrix} \Sigma_1 & 0 & \cdots & 0 \\ 0 & \Sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma_n \end{bmatrix}_{2n\times 2n} \quad (26)$$

Note that it is also possible that no coverage area is heard.

IV. RESULTS

A. Simulation

For evaluating the performance of the proposed positioning algorithm, simulations using Matlab are done. Brief explanation of the simulation is given in this section, for more details please see [14]. One hundred different test tracks are simulated. For each simulation (i.e. each test track), 10,000 base station positions (for good geometry cases) or 3,500 (for poor geometry cases) are simulated as 2-dimensional vectors, and uniformly distributed in a 15 km by 15 km area around the simulated track. Simulations are repeated for two different geometries: poor geometry, which simulates only one BS measurement per time step, and usually this measurement is exactly the same as in the previous time step and good geometry, which simulates up to six BS measurements per time step.

In the simulations, the size of the cell coverage area is chosen based on earlier studies and our experimental knowledge. In some earlier works, the dimensions of different cell types are reported [15, 16]. Mobile phone cell size may range from 10 m for nanocell to 3 km for macrocell. In [15], the experimental results of cell-ID location technique are reported. The average distance between cell-ID location estimate and GPS location estimate is reported as 800 meters in US and 500 meters in Italy. For the purpose of this paper, the semi-major axis of coverage-area ellipse is set to the average of these values, i.e. 650 meters.

Table I compares the positioning performance of static positioning and positioning using the KF for good and poor geometry. The static solution is given in [3]. In Table I , "95%" is 95% error quantile, "Mean" and "Med" are mean and median of error respectively. The column "Cons. %" is calculated using Gaussian consistency test [17, p. 54].

Since no outlier observation is simulated, the performance of the KF (non-robust) and RKF are quite similar, and hence only the results of KF solution are given in the table. Simulation results shows that filtering provides more accurate positioning estimate than static solution.

Table I: Results of the simulated data

Geometry	Solver	Mean	Med	95%	Cons. %
Good	Static	423	368	933	61
	KF	246	224	508	37
Poor	Static	702	643	1445	86
	KF	625	572	1304	37

B. Real Data

Fig. 1 shows an example of measured coverage areas of 6 observed WLAN APs in a specific time and specific position. Test track is shown in Fig. 2. A static coverage area based positioning (with no time-series) method, presented in [3], is applied to estimate the position. The results of the test track are shown in Tables II. KF solution uses the solutions of the static coverage area based positioning technique as linear measurements. KF is compared to Robust KF. RKF enhances the positioning performance compared to KF and static solutions.

Table II: Results of the measured track

Solver	Mean	Med	68%	95%	Cons.%
Static	108	69	100	377	77
KF	89	63	88	307	79
RKF	59	50	67	130	92

V. CONCLUDING REMARKS

In this paper, the measurements are the coverage area of wireless BS that the user-terminal observes at each time-stamp. In the real scenario, the UT might measure BS-IDs that are far away from its location (outlier measurements). Static positioning results in discontinuous and jumpy position estimate. Filtering the static position solutions, e.g. with a Kalman filter, improves the position estimations. This paper proposes a robust KF which is less sensitive to those outliers. The RKF is applied to the real data. RKF outperforms the KF solution and static solutions.

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APPENDIX I: COVERAGE AREA ESTIMATION

Ellipse-shaped coverage areas by modeling fingerprints using a Student-t distribution is explained in [5]. Since fitting the ellipse using a Student-t model for fingerprints is less sensitive to the outliers than Gaussian model, this approach

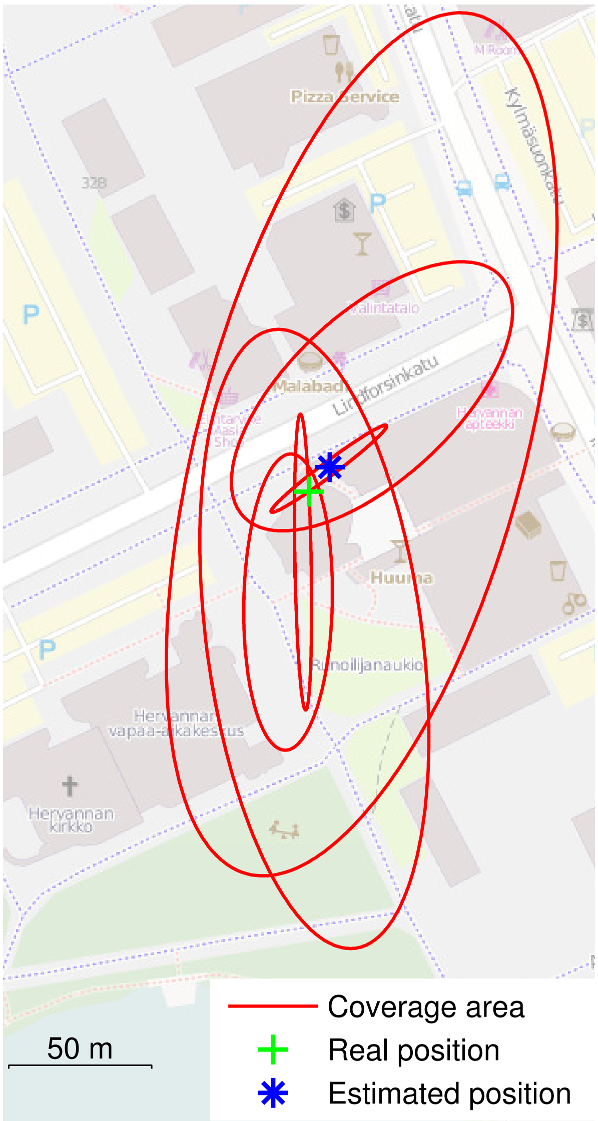

Figure 1: Coverage areas fitted as ellipses for a set of fingerprints. Each of the six ellipses is the coverage area of a heard WLAN AP. The real position is the GPS position.

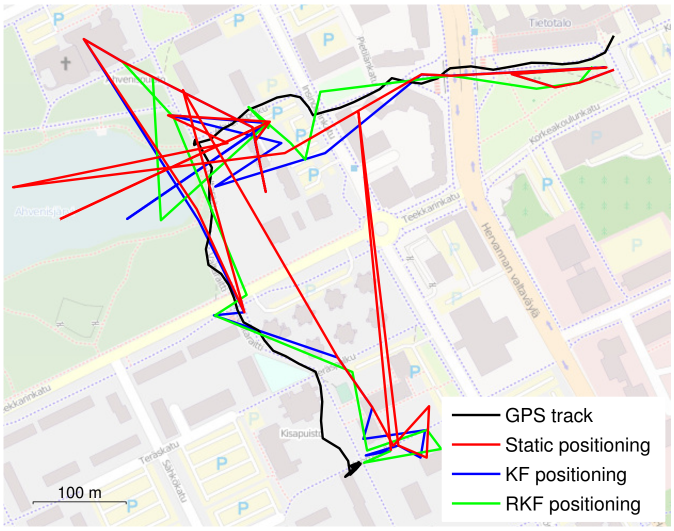

Figure 2: Positioning using different algorithms. The solid black line shows the GPS track. The red track is obtained using static solution method. The blue and green tracks are obtained using coverage-area method respectively with KF and RKF algorithms. Filtering provides a smoother estimate of the user's track.

is applied for estimating the coverage areas in this paper. For $d = 2$ dimensions, the parameters μ and Σ can be interpreted as center and covariance matrix of an ellipse. Here, each observation is modeled as a d-variate Student-t random vector z_n with location μ, shape Σ^{-1}, and ν degree of freedom:

$$z_n|\mu,\Sigma^{-1} \sim \text{Student}(\mu,\Sigma^{-1},\nu)$$
$$f(z_n|\mu,\Sigma^{-1}) \propto |\Sigma^{-1}|^{\frac{1}{2}} \left(1+\frac{1}{\nu}(z_n-\mu)^T\Sigma^{-1}(z_n-\mu)\right)^{-\frac{\nu+d}{2}} \quad (27)$$

This distribution can be obtained as mixture of N Normal distributions by marginalizing an auxiliary weight parameter u_n having the prior distribution [5]

$$u_n \sim \text{Gamma}\left(\frac{\nu}{2},\frac{\nu}{2}\right), \quad f(u_n) \propto u_n^{\frac{\nu}{2}-1} e^{-\frac{\nu}{2}u_n} \quad (28)$$

out of $\quad z_n|\mu,\Sigma^{-1},u_n \sim \text{Normal}\left(\mu,(u_n\Sigma^{-1})^{-1}\right)$

Assuming that N observations are conditionally independent and given the model parameters, the likelihood density is

$$f(z_{1:N}|\mu,\Sigma^{-1},u_{1:N}) = \prod_{n=1}^{N} f(z_n|\mu,\Sigma^{-1},u_n)$$
$$\propto |\Sigma^{-1}|^{\frac{N}{2}} \cdot \prod_{n=1}^{N} u_n^{\frac{d}{2}} \cdot e^{-\frac{N\bar{u}}{2}(\mu-c)^T\Sigma^{-1}(\mu-c)-\frac{1}{2}\text{tr}\Sigma^{-1}\text{S}} \quad (29)$$

where $\bar{u} = \frac{1}{N}\sum_{n=1}^{N} u_n$ and the weighted empirical mean and residual sum of squares are

$$c = \frac{\sum_{n=1}^{N} u_n z_n}{\sum_{n=1}^{N} u_n}, \quad \text{S} = \sum_{n=1}^{N} u_n(z_n-c)(z_n-c)^T \quad (30)$$

respectively. Assuming that $u_1,...,u_n,\mu,\Sigma^{-1}$ are a-priori jointly independent, with uninformative prior

$$f(\mu,\Sigma^{-1}) \propto |\Sigma^{-1}|^{-(d+1)/2} \quad (31)$$

leads to the posterior density

$$\begin{aligned}&f(\mu, \Sigma^{-1}, u_{1:N}|z_{1:N})\\&\propto f(z_{1:N}|\mu, \Sigma^{-1}, u_{1:N})f(\mu, \Sigma^{-1})f(u_{1:N})\\&\propto |\Sigma^{-1}|^{\frac{N-d-1}{2}} \prod_{n=1}^{N} \left(u_n^{\frac{d+\nu}{2}-1} e^{-\frac{1}{2}u_n(\nu+(z_n-\mu)^T\Sigma^{-1}(z_n-\mu))} \right)\end{aligned} \tag{32}$$

The posterior conditional weights are conditionally independent Gamma-distributed

$$\begin{aligned}u_n|z_{1:N}, \mu, \Sigma^{-1} &\sim \mathrm{Gamma}(\tfrac{d+\nu}{2}, \tfrac{\nu+(z_n-\mu)^T\Sigma^{-1}(z_n-\mu)}{2})\\ f(u_{1:N}|z_{1:N}, \mu, \Sigma^{-1}) &= \textstyle\prod_{n=1}^{N} f(u_n|z_{1:N}, \mu, \Sigma^{-1})\end{aligned} \tag{33}$$

with posterior conditional means

$$\mathrm{E}(u_n|z_{1:N}, \mu, \Sigma^{-1}) = \frac{d+\nu}{\nu + (z_n-\mu)^T\Sigma^{-1}(z_n-\mu)} \tag{34}$$

The posterior conditional location and shape parameters are Normal-Wishart distributed

$$\mu|z_{1:N}, \Sigma^{-1}, u_{1:N} \sim \mathrm{Normal}(c, (N\bar{u}\Sigma^{-1})^{-1}) \tag{35}$$

$$\Sigma^{-1}|z_{1:N}, u_{1:N} \sim \mathrm{Wishart}(\mathrm{S}^{-1}, N-1) \tag{36}$$

The joint posterior distribution of $\mu, \Sigma^{-1}|z_{1:N}$ is $(\hat{\mu}, \hat{\Sigma}^{-1})$ contains all relevant inference about the parameters. For our model, the coverage area estimate is completely specified by the posterior modes

$$\hat{\mu} = c, \quad \hat{\Sigma}^{-1} = (N-d-1)\mathrm{S}^{-1} \tag{37}$$

The posterior modes can be computed from observation data in a recursive fashion, see [5] for details.

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