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Bayesian Receiver Autonomous Integrity Monitoring Technique

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BIOGRAPHY

Henri Pesonen received his M.Sc. degree from Tampere University of Technology in 2006 and currently is pursuing his PhD studies. His research interests are robust and reliable positioning methods and Bayesian statistical methods.

Robert Piché is professor of mathematics at Tampere University of Technology. He has a Ph.D. in civil engineering from the University of Waterloo (Canada). His scientific interests include mathematical modelling and scientific computing with applications in navigation, finance, and mechatronics.

ABSTRACT

An integrity monitoring/failure detection and identification approach for GNSS positioning that is based on Bayesian model comparison theory is introduced. In the new method the user defines models for no-failure/failure cases and the most plausible model is chosen and used to estimate position. If a channel is contaminated and the corresponding model is chosen then the effect of this channel on the position estimate is attenuated. The posterior probability odds of two models can be used as a measure of how well the models can be distinguished from each other. In the proposed RAIM-technique if none of the model plausibilities stands out from the others, the user is made aware of the situation as the case might be that the effect of a good channel is attenuated and the contaminated one is modeled as a good one. The performances of traditional RAIM/FDE and the new method are compared via simulations. Results of a test with real GPS data are also presented.

INTRODUCTION

Quality monitoring and control techniques are important parts of any position estimation algorithm. As a result, receiver autonomous integrity monitoring (RAIM) has become a basic part of personal positioning receiver architectures [3,8,10]. Integrity of a positioning system refers to the ability of the system to warn the user when a given position estimate cannot be trusted. Autonomous means that the integrity monitoring is carried out using only the signals received by the system. Furthermore RAIM techniques have been enhanced to provide not only valuable information on the quality of the position estimate but also to offer means for detecting satellite failures and enable the exclusion of blunder observations.

Traditional RAIM methods are based on conventional frequentist hypothesis testing, a theory that has been criticised for its convoluted approach and for logical inconsistencies [2]. In frequentist hypothesis testing, one seeks to reject the null hypothesis based on the improbability of the data given that the null hypothesis is true. But often what we are really interested in is whether one hypothesis is better that the other given the data.

Bayesian model comparison allows us to think in this more direct fashion: we compare the probabilities of a model being true given the data and select the model that best describes the data. Bayesian techniques have been used in integrity monitoring by Ober [10] who introduced mixture error models which lead to exact position-domain results in addition to performing data-based integrity monitoring. However, the method introduced relies on improper prior probability densities which should not be used in the particular case of mixture estimation.

We propose to use Bayesian model comparison as an autonomous integrity monitoring/fault detection technique. We refer to it as BRAIM in the rest of this article. The main advantage of the new proposed method is the natural interpretation of the results which appear as odds or probabilities of an assumption being true. Also, the algorithm is computationally light.

We compare the performance of the proposed method to the reliability testing method by [1], which has been often applied to RAIM [5,9]. The technique was designed to be used as a statistical reliability testing procedure in geodetic networks but can be used also in positioning to detect and exclude a failure among the observations. The method performs two tests. First, a global test is carried out to detect a failure by a RAIM method known as least squares RAIM. Second, if the global test detected a failure, a local test is used to identify the faulty observation, after which it can be excluded from the measurement set. Hence the method is sometimes referred to as RAIM/failure detection and exclusion (RAIM/FDE) [8] and we adopt this acronym in this article.

In this article we first introduce briefly the concept of Bayesian model comparison problem, after which we describe Bayesian model comparison-based BRAIM method. We compare the performance of RAIM/FDE and BRAIM using simulations and a test with GPS data and present conclusions.

BAYESIAN MODEL COMPARISON

This section summarizes general Bayesian model comparison theory, see for example [11] for details. Suppose that we have models M_i all of which we consider to be reasonable for the problem we are interested in. Note that we don't necessarily believe that any of the models is the truth. The goal is to choose the most plausible model given the data. We assume that the problem is not new to us so that using our knowledge of the underlying situation, we can assign prior probabilities for the models $P(M_0),\dots,P(M_n)$. The posterior probability of a model M_i being the model that produced data D is

$$P(M_i \mid D) = \frac{P(D \mid M_i)P(M_i)}{P(D)} \tag{1}$$

which we use to compute the posterior ratio of two models

$$O_{ij} = \frac{P(M_i \mid D)}{P(M_j \mid D)} = \underbrace{\frac{P(D \mid M_i)}{P(D \mid M_j)}}_{B_{ij}} \times \underbrace{\frac{P(M_i)}{P(M_j)}}_{P_{ij}} \tag{2}$$

The factor P_{ij} is the prior odds ratio of M_i to M_j. This a priori information represents our personal opinion about the relative plausibility of the models given the background information. Often the prior probabilities for two models are taken to be equal (P_{ij} = 1), representing the case where we don't favor one model over another, but this is not necessary. The second factor B_{ij}, called the *Bayes factor* represents the evidence in favor of M_i as opposed to M_j [7]. The evidence for model M_i is

$$P(D \mid M_i) = \int p(D \mid \theta_i, M_i)\, p(\theta_i \mid M_i)\, \mathrm{d}\theta_i \tag{3}$$

where θ_i is a vector of unknown parameters in the model M_i. The prior probability densities $p(\theta_i \mid M_i)$ are needed to compute the evidence. This sometimes could cause a problem as this information may not be available. On the other hand in many problems some a priori knowledge is available, for example in dynamic problems where models for the evolution of θ_i are readily available. Prior probabilities are a powerful tool for incorporating that information into the model. The posterior odds ratios are used to make decisions. The choice of a meaningful scale depends on the area of application. Jeffreys [6] suggests the following scale for general scientific investigations

O_{ij}	$\log_{10}O_{ij}$	Probability for M_i against M_j
[1,3.2)	[0,0.5)	Not worth more than a mention
[3.2,10)	[0.5,1)	Substantial
[10,31.6)	[1,1.5)	Strong
[31.6,100	[1.5, 2)	Very strong
[100,∞)	[2,∞)	Decisive

Table1. Scales for odds of probability for M_i as suggested by Jeffreys [6].

BAYESIAN INTERGRITY MONITORING TECHNIQUE

In this section we apply the Bayesian model comparison theory described in the previous section to develop an integrity monitoring/failure detection identification technique for GNSS positioning. For the sake of simplicity and possibility of analytical formulations we

$$\begin{aligned} M_0 &: y = H_0 x_0 + v \\ M_i &: y = H_0 x_0 + v + b_i e_i \\ &= \underbrace{\begin{pmatrix} H_0 & e_i \end{pmatrix}}_{H_i} \underbrace{\begin{pmatrix} x_0 \\ b_i \end{pmatrix}}_{x_i} + v, \quad i = 1,\dots,n \end{aligned} \tag{4}$$

where e_i is the i^{th} column of $n\times n$ identity matrix, x_0 is the parameter of m state variables (position, velocity, etc.) and b_i is the bias. Model M_0 corresponds to the situation of no failure component in any of the measurements and in each model M_i the i^{th} measurement has an unknown bias b_i which is taken to be independent of x_0. In general form the measurement equation under the model M_i is

$$y = H_i x_i + v \tag{5}$$

If the prior of the parameter x_i is normal with mean μ_i and covariance P_i and the measurement error has a normal distribution with mean 0 and covariance R, we can write the evidence as

$$\begin{aligned} P(y \mid M_i) &= \int p(y \mid x_i, M_i)\, p(x_i \mid M_i)\, \mathrm{d}x_i \\ &= c_i^* \exp(g_i^*(z_i)) \end{aligned} \tag{6}$$

where

$$z_i = y - H_i \mu_i \tag{7}$$

$$c_i^* = \frac{\sqrt{\det(2\pi (A_i^T \Sigma_i^{-1} A_i)^{-1})}}{\sqrt{\det(2\pi \Sigma_i)}} \tag{8}$$

$$A_i = \begin{pmatrix} H_i \\ I \end{pmatrix} \tag{9}$$

$$\Sigma_i = \begin{pmatrix} R & 0 \\ 0 & P_i \end{pmatrix} = \begin{pmatrix} R & 0 & 0 \\ 0 & P_{x_0} & 0 \\ 0 & 0 & \sigma_b^2 \end{pmatrix} \quad (10)$$

$$g_i^*(z_i) = -\frac{1}{2} z_i^T (H_i P_i H_i^T + R)^{-1} z_i \quad (11)$$

Introducing $S=HP_{x0}H^T+R$, the constant c_i^* can be written as

$$c_i^* = \sqrt{\frac{\det\left(H_0^T R^{-1} H_0 + P_{x_0}^{-1}\right)}{(2\pi)^n \det(R)\det(P_{x_0})}} \times \underbrace{\begin{cases} 1 & ,i=0 \\ (\sigma_b^2 e_i^T S^{-1} e_i + 1)^{-1/2} & ,i \neq 0 \end{cases}}_{c_i}$$

and $g_i^*(\cdot)$ can be expressed as

$$g_i^*(z_i) = -\frac{1}{2} z_i^T S^{-1} z_i + \frac{1}{2} \underbrace{\begin{cases} 0 & ,i=0 \\ \dfrac{(z_i^T S^{-1} e_i)^2}{e_i^T S^{-1} e_i + \sigma_b^{-2}} & ,i \neq 0 \end{cases}}_{g_i(z_i)}$$

Using the above notation, the Bayes factor for the i^{th} model can be computed as

$$B_{ij} = \frac{c_i}{c_j} \times \exp(g_i(z_i) - g_j(z_j)) \quad (12)$$

As an example, compared to the null model, the Bayes factor for i^{th} model is

$$B_{i0} = c_i \exp(g_i(z_i)) \quad (13)$$

when the mean of the bias b_i is zero.

To compute the posterior odds ratio O_{ij} we still need to model the prior odds ratio P_{ij}. We assume that a measurement from a particular channel is contaminated with probability ε and clean with probability 1- ε and the quality of one channel is independent of another. The models that we have constructed in this section correspond to ones with 'no bad channels' and 'exactly one bad channel'. In our model, different channels are contaminated with the same probability so that all the ratios $P_{ij}=1$, $i,j > 0$. Prior odds ratios P_{i0} can be computed as

$$P_{i0} = \frac{P(n-1 \text{ channels are clean and 1 is contaminated })}{P(n \text{ channels are clean})}$$

$$= \frac{P(\text{channel is clean})^{n-1} P(\text{channel is contaminated})}{P(\text{channel is clean})^n}$$

$$= \frac{(1-\varepsilon)^{n-1}\varepsilon}{(1-\varepsilon)^n} = \frac{\varepsilon}{1-\varepsilon}$$

and the posterior odds ratio O_{i0} can be expressed

$$O_{i0} = \frac{\varepsilon c_i g_i(z_i)}{1-\varepsilon}$$
$$O_{ij} = \frac{c_i}{c_j} \times \exp(g_i(z_i) - g_j(z_j)) \quad j \neq 0 \quad (14)$$

The most plausible model can be found by comparing posterior odds, as for most plausible model M: $O_{ij} \geq 1$, for all j.

We can analyze further the properties of the posterior odds O_{ij}. First of all, the maximum odds for $O_{0i} = O_{i0}^{-1}$, for all i is achieved when $y = H_0 y_0$. Thus

$$O_{i0} \leq \frac{1-\varepsilon}{\varepsilon} \sqrt{\sigma_b^2 e_i^T S^{-1} e_i + 1} \quad (15)$$

Let $S_{ii}^{-1} = e_i^T S^{-1} e_i$ and $z = y - H_i\mu_i = y - H_0\mu_0$, and let $T < 1$ be a threshold parameter. Then

$$O_{i0} \leq T$$
$$g_i(z) \leq \ln\left(\frac{(1-\varepsilon)T}{\varepsilon c_i}\right) \quad (16)$$
$$\left|z^T S^{-1} e_i\right| \leq \sqrt{\ln\left(\frac{(1-\varepsilon)T}{\varepsilon}\sqrt{\sigma_b^2 S_{ii}^{-1} + 1}\right)\left(S_{ii}^{-1} + \sigma_b^{-2}\right)}$$

If inequality (16) holds for all i then M_0 is the most plausible model and the odds for it against any other model are at least $1/T$.

For simplicity assume that μ_0 is close to the actual unknown x_0. Then given that M_0 is the most plausible mode, the size of a bias Δ in the k^{th} measurement is bounded as

$$|\Delta| \leq \frac{1}{S_{ki}^{-1}} \sqrt{\ln\left(\frac{(1-\varepsilon)T}{\varepsilon}\sqrt{\sigma_b^2 S_{ii}^{-1} + 1}\right)\left(S_{ii}^{-1} + \sigma_b^{-2}\right)}, \ \forall i.$$

A larger bias in k^{th} observation causes one of the i odds $O_{i0} > 1$. Because of this O_{i0} can be used to detect whether there is a blunder observation among the observation set and the odds O_{i0} are a sensible measure of quality of the null model in a practical sense.

Similar analysis can be carried for all the models. We focus on the posterior odds of a correct model that is the posterior odds for model O_{ij} when there is a bias component in the i^{th} measurement. From the equation (14) we see that the odds depend on bias as

$$g_i(\Delta e_i) - g_j(\Delta e_i) = \Delta^2 \left[\frac{\left(S_{ii}^{-1}\right)^2}{S_{ii}^{-1} + \sigma_b^{-2}} - \frac{\left(S_{ij}^{-1}\right)^2}{S_{jj}^{-1} + \sigma_b^{-2}}\right] \quad (17)$$

We want that a larger bias in i^{th} observation would cause O_{ij} to be larger, however from (17) using the fact that S^{-1}

is a symmetric positive definite matrix it can be shown that this can be guaranteed only if

$$S_{ii}^{-1} > S_{ij}^{-1}, \forall j \qquad (18)$$

If this holds, we can identify a blunder in i^{th} if it is large enough. And because the odds for the correct model increase quadratically with the size of the realized bias element, the odds are a sensible measure of the correctness of the model choice.

We assume that integrity will be attained if a model corresponding to a contaminated channel is selected and that the effect of a contaminated measurement is thereby attenuated. The decision of integrity is therefore based solely on the model space and not on the resulting positioning space. As a result the BRAIM method is based on posterior odds ratios that one model stands out as the best model. The test is declared inconclusive if none of the models stands out. The test is a failure if (18) does not hold and M_0 is not the most plausible model because in this case there is no guarantee that the most plausible model handles the correct observation as a blunder. Otherwise the system is assumed to be working within prescribed standards. The threshold T for posterior odds for different situations can be based on Table 1. The BRAIM algorithm is illustrated in Figure 1.

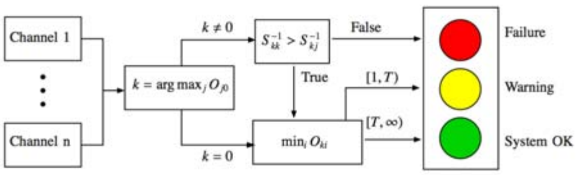

Fig.1. Diagram of the proposed BRAIM method.

Once a model is selected, the information about the state is contained in the normal posterior distribution $p(x_i \mid y, M_i) = N(m_i, C_i)$, where

$$m_i = \mu_i + P_i H_i^T (H_i P_i H_i^T + R)^{-1} (y - H_i \mu_i)$$
$$C_i = P_i - P_i H_i^T (H_i P_i H_i^T + R)^{-1} H_i P_i$$

In the case of models M_i, $i > 0$, the state vector x_i contains the bias element in addition to other state variables. This means that it will be estimated along with other parameters.

Note that in the case of warning or failure the resulting model does not necessarily result in particularly bad position estimate. It is important to note that the only conclusion that can be drawn is that none of the compared models stand out as the best one given the data. Instead of issuing a warning or failure message, one could proceed to further data analysis, expanding the set of models until one model does stand out. This expansion could be for example models with more than one contaminated channel but this is left for future study.

TESTS

The performance of the new proposed method is now compared to that of the classic method of RAIM/FDE as it is discussed for example in [8] in a special case of only one possible outlying observation at a time. Positioning scenarios with various numbers (n) of satellites and sets of measurements with different noise variances are generated. We generate all the observation noises from $N(0,10^2)$ for the duration of 10 epochs and after that one randomly selected satellite generates contaminated observations for the next 10 epochs. The contaminated observation noise has distribution $N(0,\sigma_c^2)$ (Table 2)

Test	n	σ_c^2
A_1,A_2	5	$100^2,200^2$
B_1,B_2	6	$100^2,200^2$
C_1,C_2	7	$100^2,200^2$

Table2. Test parameters.

The track of the target was generated using a constant velocity model [4] using σ_c^2 = 0.01 with an initial state $(0,0,0,1,0,0)^T$. The satellites were generated uniformly on a rectangle $[-10^5,10^5]\times[-10^5,10^5]\times[10^5,10^5+10^2]$.

The prior probability distribution for x_0, which contains position and velocity were propagated using two different motion models from the posterior probability distribution $p(x_0 \mid M_k, y)$ obtained in previous epoch. The model M_k refers to the most plausible model in that epoch. The prior probability for b_i is always taken to be independent of position and velocity and distributed as $N(0,\sigma_b^2)$. The motion models can be written as

$$x_0^{k+1} = \begin{pmatrix} I & I \\ 0 & I \end{pmatrix} x_0^k + Q_j, \; j = 1,2$$

where $Q_1=100^2$I is large in the sense that it results in a prior that influences the results very little and $Q_2=I$ is smaller so that the resulting prior does have an influence.

The parameters of the methods α, β (probabilities of Type I and II errors in RAIM/FDE), ε and σ_b^2 are varied and the performance is reported as the fraction of epochs in which correct faulty channel was identified vs. the fraction of epochs in which no good channels were identified as faulty.

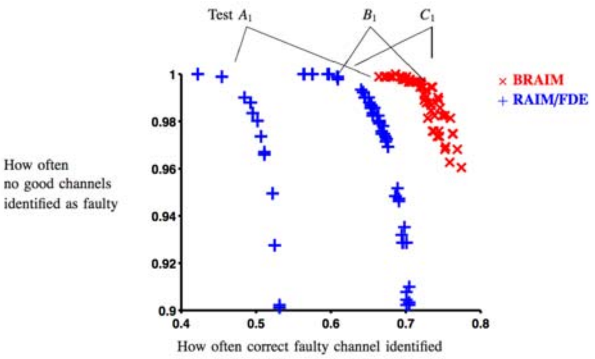

Fig.2. Method performance, more informative prior and smaller observation noise.

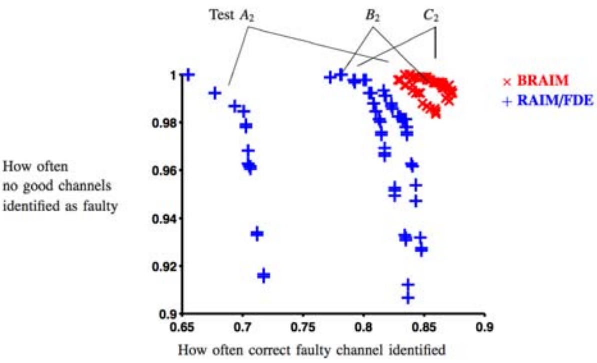

Fig.3. Method performance, more informative prior and larger observation noise.

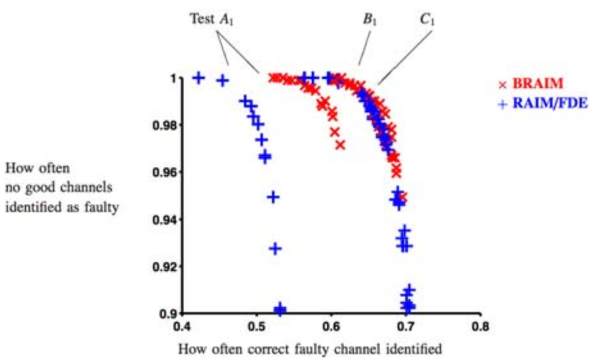

Fig.4. Method performance, less informative prior and smaller observation noise.

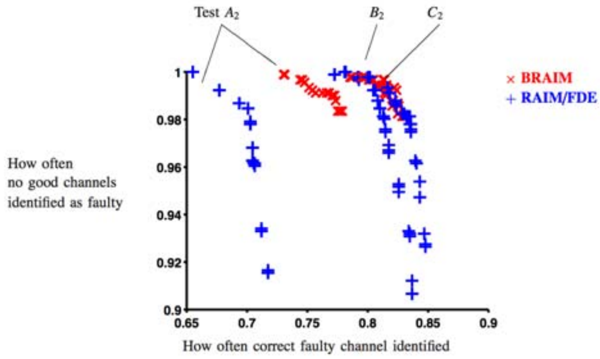

Fig.5. Method performance, less informative prior and larger observation noise.

The results of the simulations are given in Figures 2 - 5 which correspond to different scenarios. Figures 4 and 5 indicate that if prior information for BRAIM is not taken advantage of, the methods have similar performance when there are 6 or more satellites. On the other hand if use of prior information is made, then the BRAIM can perform significantly better than the traditional method, as can be seen from Figures 2 and 3.

	System OK (%)	Warning (%)	Failure (%)
Correct decision	74	26	0
Wrong decision	33	67	0

Table3. Test C_1 results with T=10, $\sigma_b^2=80^2$, ε=0.6 (small-variance prior for parameters).

	System OK (%)	Warning (%)	Failure (%)
Correct decision	69	31	0
Wrong decision	33	67	0

Table4. Test C_1 results with T=10, $\sigma_b^2=80^2$, ε=0.6 (large-variance prior for parameters).

The rates of BRAIM algorithm issuing *system OK* and *warning* flags are given in Tables 3 and 4 in the cases where correct or wrong identification were made. The results show that when correct model is chosen, the system is most often recognized to be working properly and almost no false *warning* flags are given. When wrong model is chosen, system most often issues a *warning* in these particular tests with reported parameters.

The new method was also applied to a real GPS-data test drive in Tampere. The 800 epochs long test route was in an urban area with a relatively clear view of the sky. The test was carried out by including data from at most one satellite with a poor carrier-to-noise ratio (C/N). Although poor C/N of a measurement does not mean that the measurement from that particular satellite is contaminated and high C/N does not mean that a observation is of good quality this situation can be close to the at-most-one bad observation situation that we are considering in this article. The error of the estimated position is illustrated by Figure 6 where the error of the BRAIM estimate and ordinary Kalman filtered position are given. Several significant errors are excluded when the BRAIM method is used.

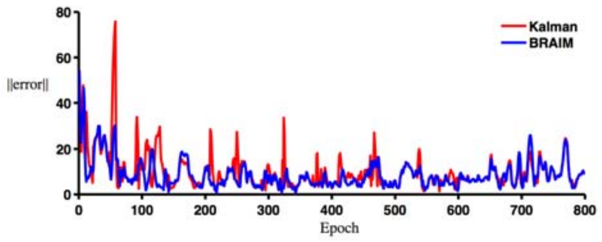

Fig.6. Errors of Kalman filtered position estimate versus the error given by the BRAIM method on a real GPS data vehicular test.

CONCLUSIONS

In the current report we applied Bayesian model comparison theory to GNSS integrity monitoring problem and introduced Bayesian receiver autonomous integrity monitoring technique (BRAIM). It was shown through simulations that the new proposed method obtains similar performance to traditional RAIM/FDE processing method. Better performance can be achieved if good prior information for the unknown parameters is available. The clearest advantage of the new proposed method is its foundations in Bayesian statistics, so that method parameters can be interpreted more easily than the traditional concepts of significance, power of the test etc. Drawback of the method is the requirement to have prior distributions for parameters and prior odd ratios for the models, but on the other hand this can be considered an advantage as this information may well be available (e.g. through filtering) and Bayesian theory enables to use this information.

The method can be developed further by formulating more realistic models than the current ones based on normal distributions and the generalization of the method to handle more than one faulty channel. In this paper we have not discussed position-domain integrity information; such information could be obtained by computing credibility regions, as is standard in Bayesian statistics [10,11].

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