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# Outlier-Robust Estimation of GPS Satellite Clock Offsets

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**Abstract**—A new method to predict a GPS satellite’s clock offset is presented. The motivation for this work is to improve the time to first fix and make the clock offset prediction less sensitive to outliers. The proposed method is tested with real data and it is shown to improve prediction accuracy compared to other known methods.

## I. INTRODUCTION

When a standalone GPS receiver is turned on, it may take over 30 seconds until the first position fix. This is a consequence of the streaming rate of the satellite’s broadcast of its orbital parameters. For example, it takes over ten seconds to send the three first subframes and this information is streamed once every 30 seconds [1, p. 127]. However, under nonideal conditions the time to first fix might be even longer, and it may take over one minute before the receiver has enough information to compute the first position fix.

The time to first fix can be reduced by obtaining the satellite’s orbital parameters and its clock offset in some other way than waiting for the data from the broadcast stream. One option is to download this assisting data from a network, for instance the Internet. However, this “assisted GPS” option may not be feasible. For example, an Internet connection might be too expensive from the end-user’s point of view or the wireless network’s signal might be too weak. This motivates the study “self-assisted” methods to predict orbital parameters and clock offsets.

In [2] a method to predict a GNSS satellite’s orbit and clock offset was presented. The satellite’s orbit was predicted by numerically integrating its equation of motion. The clock offset was predicted using the polynomial model provided in the satellite’s broadcast message. In this study it was found that the error in the user position estimate was dominated by the clock offset prediction model, not by the orbit prediction model. The empirical results showed that the inaccuracy of the clock offset prediction approximately doubled the range error in a four day prediction span.

In this paper a statistical approach to predict a satellite’s clock offset is presented. The proposed method applies Bayesian filtering theory to estimate and predict a satellite’s clock offset behaviour. A novel feature is the application of a recently published outlier-robust Kalman filtering algorithm [3] in the clock offset estimation and prediction problem.

The structure of this paper is as follows. The task is formulated as a Bayesian inference problem and a model for the satellite’s clock offset process is presented in section II. Computational methods to perform outlier-robust Kalman filtering and to estimate model parameters are described briefly in section III. The empirical setup and the performance of the proposed algorithm with real data are presented in section IV.

## II. CLOCK MODEL

### A. Dynamical Model

In this work the satellite’s clock dynamics are modelled with the time discretized version of a constant frequency bias model presented in [4] for Cesium atomic clocks. That is, the unknown state vector  $x_k$  consists of clock offset  $\tau_k$  and frequency bias  $b_k$  at time  $t_k$ . The state evolution is written as a stochastic difference equation as follows

$$x_{k+1} = \begin{bmatrix} \tau_{k+1} \\ b_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} x_k + w_k, \quad (1)$$

where  $w_k$  is assumed to be a zero mean Gaussian white noise process with a known covariance matrix  $Q$ .

### B. Measurement Model

A GPS satellite sends coefficients  $a_{f_0}$ ,  $a_{f_1}$  and  $a_{f_2}$  of a second order polynomial in the broadcast message. This polynomial is an approximation of the true path of the clock offset process. With these coefficients a GPS receiver can predict the satellite’s clock offset  $\tau$  at a given time  $t$  as

$$\tau(t) = a_{f_0} + a_{f_1}(t - t_0) + a_{f_2}(t - t_0)^2, \quad (2)$$

where  $t_0$  is the time of ephemeris. In this work the constant term  $a_{f_0}$  is treated as an inaccurate measurement of the clock offset component of the state that evolves according to (1). For simplicity the “inaccurateness” is modelled to be a consequence of an additive measurement noise process  $v_k$ . That is, the measurement model is assumed to have the following form

$$y_k = a_{f_0} = [1 \quad 0] x_k + v_k. \quad (3)$$

Coefficients  $a_{f_1}$  and  $a_{f_2}$  could, at least in principle, be treated as measurements of frequency bias and frequency drift respectively. However, because these coefficients do not change much in the time intervals we are considering, their broadcast values are not used in this work.

With the measurement and dynamical models it is possible to compute the posterior distribution of the state  $x_k$  with Bayesian filtering techniques. That is, one can compute the probability distribution of the current state given the measurements up to the  $k$ th epoch  $x_k|y_{1:k}$ . With the posterior distribution it is possible to compute the predictive distribution  $x_{k+1}|y_{1:k}$  and this constitutes a statistical prediction of the satellite's clock offset behaviour.

In the standard Kalman filtering scheme the measurement error process  $v_k$  is assumed to be a zero mean white noise process. If measurement error and prior distributions are Gaussian, the posterior distribution is Gaussian and the Kalman filter computes the parameters of the posterior distribution.

However, the Kalman filter's estimate is sensitive to occasional large deviations ("outliers") because the Gaussian distribution is thin tailed. Student's  $t$  distribution offers a heavy-tailed alternative to the Gaussian distribution and a possible approach to obtain an estimate that is not so sensitive to outliers in the measurements is to assume an additive Student's  $t$  measurement error process. This estimation problem cannot be solved with a standard Kalman filter and requires more sophisticated computational methods.

### III. COMPUTATIONAL METHODS

#### A. Outlier-Robust Kalman Filtering

Consider a state space model such that

$$x_{k+1} = Ax_k + w_k \quad (4)$$

$$y_k = Hx_k + v_k \quad (5)$$

where  $w_k$  is a zero mean Gaussian process and  $v_k$  is a zero mean Student's  $t$   $\nu$ -degree of freedom distributed random variable. In this work the outlier-robust Kalman filter, presented in [3], is applied to solve the posterior distribution of state  $x_k$  given the measurements  $y_{1:k}$ . The outlier-robust Kalman filter is an iterative algorithm that updates the posterior mean of the state  $x_k$  and the covariance matrix of the measurement error distribution in successions to obtain the posterior distribution of the state  $x_k$ .

#### B. Estimation of Process Noise Covariance

Good filtering performance requires a correct selection of the process noise covariance matrix  $Q$  [5]. In [4] and [6] the process noise covariance matrices are assumed to have a certain functional form and the numerical values of the tuning parameters are chosen with prior knowledge. In this work the Expectation Maximization algorithm is applied to obtain a maximum likelihood estimate of the process noise covariance matrix  $Q$ . The Expectation Maximization algorithm can be written as shown in Algorithm 1 [5].

In estimation theoretical terms the expectation step consists of fixed interval smoothing with a given process noise covariance matrix  $Q_k$ . After smoothing the objective function is maximized with respect to the unknown parameter. Both steps can be carried out analytically for linear dynamical systems with additive Gaussian measurement and process noise. The

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#### Algorithm 1 Expectation Maximization

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Given  $N$  measurements  $y_{1:N}$ .

Set initial value  $Q_0$ .

**while** Not converged **do**

1) Expectation step. Compute expectation

$$L(Q, Q_k) = E_{Q_k}(\log(p(x_{1:N}, y_{1:N}|Q)))$$

2) Maximization step. Solve maximization problem

$$Q_{k+1} = \arg \max_Q L(Q, Q_k)$$

**end while**

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details of the maximum likelihood estimation of the process noise covariance matrix in dynamical systems are given in [5] and [7].

## IV. EMPIRICAL RESULTS

### A. Offline Parameter Estimation

In this work the precise ephemerides (PE) published by the National Geospatial-Intelligence Agency (NGA) were considered as the reference values of the satellite's clock offsets. That is, it is assumed that the residual error of the precise ephemerides is small compared to the prediction and measurement errors. To obtain an estimate of measurement error statistics, the difference between precise and broadcast ephemerides  $a_{f_0}^{PE} - a_{f_0}^{BE}$  was computed for each sample during GPS weeks 1600 – 1604 and Student's  $t$  distribution was fitted to the measurement error distribution in the maximum likelihood sense [8]. The degrees of freedom and the shape parameter of the Student's  $t$  distribution were fitted to the measurement error data individually for each satellite.

A typical clock offset measurement error distribution is given in Figure 1. The cumulative density functions of the maximum likelihood fits are plotted with realized measurement errors in Figure 1. The Gaussian distribution clearly underestimates the frequencies in the tails of the distribution. For example, measurement errors of 5 meters are 5 times more frequent than predicted by the Gaussian model; errors of 10 meters are 1000 times more frequent. In contrast the Student's  $t$  distribution fits the data in the whole range, and large measurement errors are correctly modelled.

Process noise covariance matrices were estimated offline with the Expectation Maximization algorithm individually for each satellite. The training data used in the estimation was selected to be NGA's precise ephemerides from GPS weeks 1600 – 1604. The residual error of precise ephemerides was assumed to be normally distributed.

### B. Prediction Results

The proposed algorithm was tested with real broadcast ephemerides, abbreviated in this paper as BE, from GPS weeks 1605 – 1650. Satellite tracks that had clear step-wise jumps in the clock offset process were excluded, because such steps are not included in the statistical model. The sampling rate was chosen to be 1/day and the prediction was computed after receiving each sample. The prior state  $x_0$  was set to be a

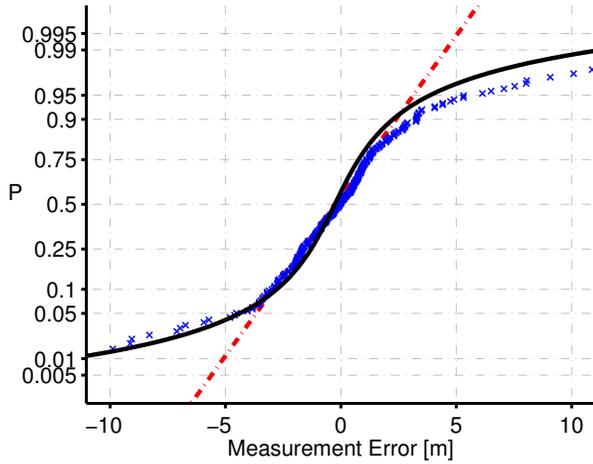


Fig. 1. Normal probability plot of measurement error distribution showing realized measurement errors (blue crosses), a gaussian fit (red dashed line) and a Student's  $t$  fit (solid black line). Data was collected from satellite 30 on GPS weeks 1605 – 1650. Errors are given as error in pseudorange.

Gaussian distribution with mean  $\mu$  consisting of the parameters of NGA's precise ephemerides as follows

$$\mu = \begin{bmatrix} a_{f_0}^{\text{PE}} \\ a_{f_1}^{\text{PE}} \end{bmatrix} \quad (6)$$

The covariance matrix of the prior distribution was set to zero. The posterior predictive mean was reported as the algorithm's prediction and its numerical value was compared with NGA's precise ephemerides. The filter was set to compute only the predictive distribution if no broadcast ephemerides were available for some epoch.

The prediction accuracy of the polynomial models of broadcast ephemerides and NGA's precise ephemerides were also studied. The predictive polynomial was downloaded for each epoch that the outlier-robust Kalman filter received a new measurement. After receiving a predictive polynomial, the prediction was done and the predicted value was compared with the precise clock offset value at the predicted epoch.

GPS positioning is based on pseudorange measurements  $\rho$

$$\rho = \|r - s_i\| + c(\tau - \tau_r) + \epsilon, \quad (7)$$

where  $r$  is the user's position,  $s_i$  is the position of the  $i$ th satellite,  $c$  is the speed of light,  $\tau$  the satellite's clock offset,  $\tau_r$  the receiver's clock offset and  $\epsilon$  is measurement error. If  $\Delta\tau$  denotes the estimation error of the satellite's clock offset, then the error in pseudorange becomes

$$\Delta\rho = c\Delta\tau \quad (8)$$

Applying standard Dilution of Precision (DOP) analysis, the variance of the error in pseudorange is mapped to the variance of actual positioning error  $\Delta P$  as follows [1, p. 207]

$$\sigma_{\Delta P}^2 = \text{DOP} \cdot \sigma_{\Delta\rho}^2 = \text{DOP} \cdot \sigma_{c\Delta\tau}^2, \quad (9)$$

where DOP is a function of the geometry of visible satellites. Therefore the error in pseudorange is considered in this paper

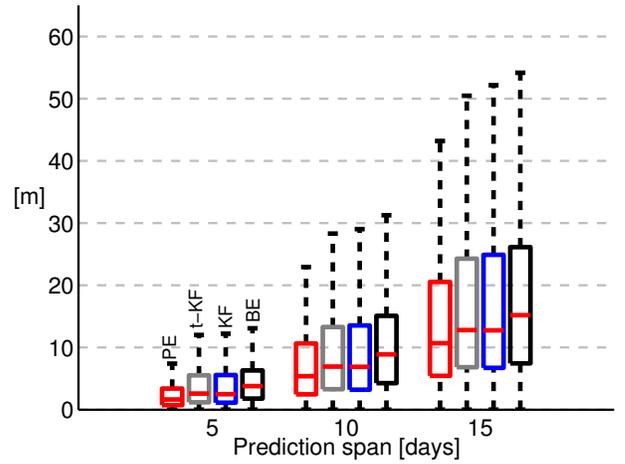


Fig. 2. Prediction error results. The upper and lower edges of the box show the 75% and 25% quantiles of the prediction error respectively. The upper whisker corresponds to 90% quantile. In this figure PE, KF, t-KF and BE corresponds to the prediction error of precise ephemerides, Kalman filter, outlier-robust Kalman filter and broadcast ephemerides respectively.

when the accuracy analysis is carried out because it is a measure that is independent of the current satellite constellation; formula (9) indicates how the prediction accuracy is mapped to the actual positioning accuracy.

The prediction results (Figure 2) show that the proposed method improves the accuracy of the broadcast's clock offset prediction by approximately 5 meters in 15 days prediction (90% quantile). There is also slight improvement in shorter prediction intervals. There are no significant differences between the 25% quantiles of the prediction error distributions of the proposed method and broadcast ephemerides. This can be interpreted as an indication that the predictive polynomials of broadcast ephemerides are good in about 25% cases, and filtering is of benefit in the remaining 75% of the cases.

It was observed that the constant drift assumption of the dynamical model given in (1) was not valid for the whole constellation. For example, the clock offset process of satellite 20 has an evident quadratic trend and thus the linearization increases the error in a long term prediction span. However, the linearization is a trade-off between the model complexity and its generalizability.

### C. Comparison to Other Methods

Empirical tests were repeated using standard Kalman filter as proposed in [4]. As illustrated in Figure 2, the prediction accuracy was overall similar to the results computed with the outlier-robust Kalman filter. However standard Kalman filter was found to be sensitive to large measurement errors. One such case is shown in Figure 3 where Kalman filter takes 14 samples to recover from a single outlier. The measurement error was as large as one millisecond ( $3 \cdot 10^5$  meters) for the outlier observed at the 250th epoch. The time-series plot of the estimation error of the satellite's clock offset process illustrates that there are no significant differences between the estimates of Kalman filter and the outlier-robust Kalman filter

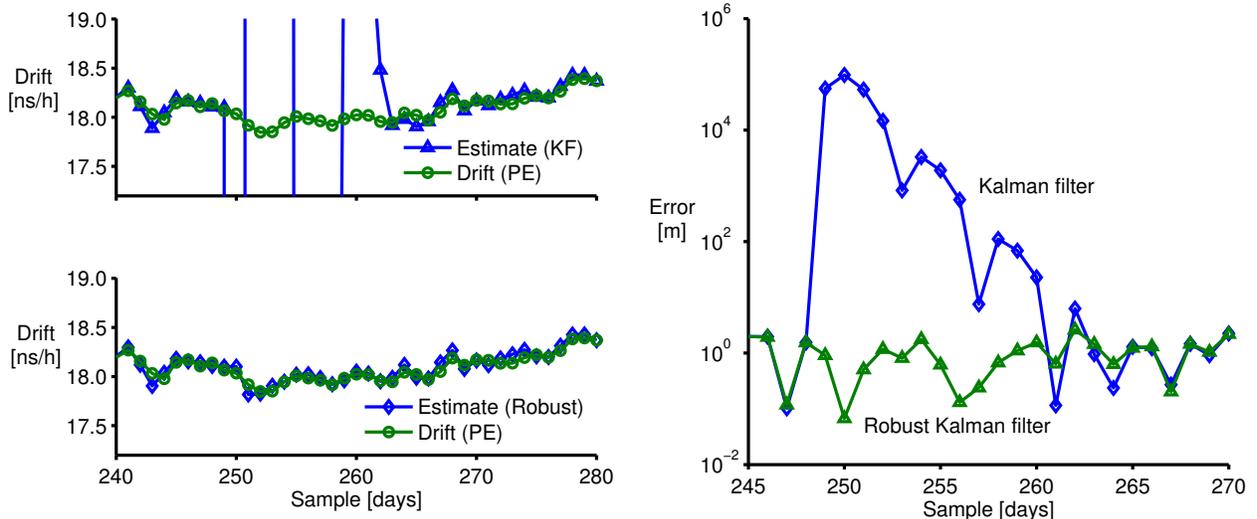


Fig. 3. Effect of outlier on drift process estimation (left) and clock offset process estimation error (right). Data was collected from satellite 3.

before observing the outlier and after the Kalman filter has recovered. The drift and clock offset estimates of Kalman filter become highly oscillating when an outlier is observed and it takes several time steps for the oscillation to damp out. In contrast, the outlier-robust Kalman filter performs correctly, automatically giving less weight to the outlier measurement.

When the measurement was not an obvious outlier, estimates computed with Kalman filter were approximately equal even though some satellites have a heavy tailed measurement error distribution. This might be a consequence of the sparse sampling rate. Since the sampling frequency of one sample per day is considered in this paper, process noise becomes the limiting factor of filtering performance. Therefore outlier-robust Kalman filter could improve the filtering performance if the filter would receive samples more often. However, more frequent sampling rate is not usually possible in the applications of self-assisted GPS but it could be one approach to make offline estimation of satellite's clock offset behaviour when there is lots of data available.

The prediction accuracy of precise ephemerides was found to be most accurate among the four prediction methods considered. For example the prediction accuracy of the precise ephemerides was shown to be seven meters better than the predictions made with outlier-robust Kalman filter (90% quantile). Even though the proposed method gives worse predictions than the predictive polynomials of NGA's precise ephemerides, it is worth mentioning that NGA has more data and that their precise ephemerides are processed offline with also some knowledge about the upcoming data and so use of precise ephemeris is not feasible self-assisted GPS.

Huang and Zhang have recently published a method in [9] to make a GPS satellite's clock offset estimation less sensitive to outliers. They present an adaptively robust Kalman filtering algorithm to control outliers. Their method applies a slightly different dynamical model with an extra acceleration term  $d_k$ . The dynamical model can be expressed in the notation of (1)

as

$$z_{k+1} = \begin{bmatrix} \tau_{k+1} \\ b_{k+1} \\ d_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & \frac{1}{2}(\Delta t)^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} z_k + \tilde{w}_k \quad (10)$$

In their work a connection between the parameters of the process noise covariance matrix and Hadamard total variance and Allan variance is described and the parameters are fitted to match these statistics. The root mean square prediction accuracy for a two day prediction interval was reported to be 3.48 meters for a single satellite (PRN 28). The outlier-robust Kalman filter approach presented in this paper was observed to be accurate up to 5.85 meters (90% quantile) for the whole constellation. Root mean square error for two day prediction interval varied from 0.90 meters to 9.11 meters for different satellites giving root mean square error of 3.63 meters for the whole constellation and 2.96 meters for PRN 28. Therefore our method seems to offer accuracy that is comparable to the recently published method.

## V. CONCLUSIONS

An outlier-robust method to improve satellite's clock offset prediction was presented. The proposed method was shown to improve the prediction accuracy with real data. The method was also shown to be less prone to outliers than the usual Kalman filter approach. The given model is compact, only the process noise covariance matrix and the parameters of Student's  $t$  distribution need be fitted to the data, and no additional threshold values need to be adjusted.

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