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Nonlinear Robust Control for DC-DC Converters

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***Abstract*—This paper introduces a robust nonlinear controller for a switch mode DC-DC boost converter. This innovative controller is employed to achieve the desired dynamic performance during steady state and transient operations. Step changes are considered in the load, the input and the desired output voltages, while unknown, but bounded disturbances corrupt the output load and the supply voltage. Practical stability of the converter is assured in the presence of those disturbances provided their maximum possible variation is known. Simulation results are provided to substantiate the applicability of the proposed control scheme.**

Keywords- robust control, nonlinear control, dc-dc converter

I. INTRODUCTION

DC-DC boost converters are widespread power electronics circuits with significant advantages, such as small size, light weight, and high efficiency, used as interfaces between DC systems of different voltage levels. Examples of their applications are power supplies in computers, and other electronic equipment, as well as PV's and DC motor drive systems. Despite their advantages, they have inherent drawbacks. The main disadvantage established in the literature [1] is the difficulty in controlling the converter. This difficulty derives from its hybrid nature as its switched circuit topology implies different modes of operation, each with its own related affine continuous-time dynamics.

The main control objective is to regulate the output voltage at a reference value despite changes in the load and the voltage source. In recent years many control techniques have been proposed focusing on this target. Many of them rely on PID controllers based on linearized average models [2],[3]. Different nonlinear control strategies are given in [4],[5] but they lack in presenting the hybrid dynamics of the DC-DC boost converter. Other proposed nonlinear techniques [6],[7] guarantee stability and robustness, but their inherently high and variable switching frequency causes excessive power losses, electromagnetic-interference (EMI) generation, and filter design complication.

This paper examines DC-DC boost converters whose mathematical representation involves uncertain inputs. These are the fluctuations of the supply voltage and the variations of the load which occur in an unpredictable manner. In particular, we consider an unknown, but bounded variation superimposed on the constant supply voltage as a disturbing input. This may consist of step changes due to say aging or faulty operation accompanied by random, but norm bounded, parasitics. Similarly, it is assumed that there is an unknown variation at the load level. Unknown parameters and other uncertainties may be easily incorporated at the expense of more control requirements. The control technique described in this paper yields a feedback controller whose characteristics depend on knowledge of the bounds of the uncertain quantities in the system description. The proposed deterministic nonlinear robust controller assures arbitrarily small deviations of the voltage delivered in the presence of the uncertainties.

II. MODEL DESCRIPTION

A model of the boost converter considered is shown in Fig.1.

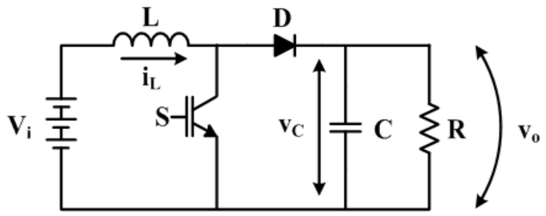

Figure 1. DC-DC boost conveter topology

The circuit has the following mathematical representation:

$$\dot{i}_L = -(1-q(t))\frac{1}{L}v_o(t) + \frac{1}{L}v_s(t) \qquad (1)$$

$$\dot{v}_o = (1-q(t))\frac{1}{C}i_L(t) - \frac{1}{CR(t)}v_o(t) \qquad (2)$$

where $R(t)$ is the load resistance, C the filter capacitance, L the circuit inductance, $i_L(t)$ the inductor current, $v_o(t)$ the output voltage, and $v_i(t)$ the supply voltage. The switch position function $q(t)$ acts as the control input and takes values from the set $\{0,1\}$. Given the supply voltage $v_i(t)$, the output $v_o(t)$ should be regulated at some desired value with suitable choice of control.

In the following, the time argument t will be omitted when no confusion is likely to arise. Assuming the state to be $x = [i_L \ v_o]^T$, and considering (1) and (2), the state space of the system in average, is given in matrix form as in [8]

$$\dot{x} = \begin{pmatrix} 0 & -\frac{1-d_R(t)}{L} \\ \frac{1-d_R(t)}{C} & -\frac{1}{CR(t)} \end{pmatrix} x + \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} v_i \tag{3}$$

where $d_R(t)$ is the duty cycle represented by a continuous function which takes values in $[0\ 1)$. The equilibrium point which represents steady state operating conditions can be determined from the following

$$v_o = \frac{v_i}{1-d_R} \quad \text{and} \quad i_L = \frac{v_o}{R(1-d_R)} = \frac{v_i}{R(1-d_R)^2} \tag{4}$$

for some given value of d_R. Equations (1) and (2) may be linearized about some chosen operating point where the values of i_L, v_o, d_R are designated by $i_{L,e}, v_{o,e}, d_{R,e}$ respectively. The linearized equations are of the form

$$\dot{x} = Ax + Bu + Cv_d \tag{5}$$

where

$$A = \begin{pmatrix} 0 & -\frac{1-d_{\mathrm{Re}}}{L} \\ \frac{1-d_{\mathrm{Re}}}{C} & -\frac{1}{CR} \end{pmatrix}, \quad B = \begin{pmatrix} \frac{v_{o,e}}{L} \\ -\frac{i_{L,e}}{C} \end{pmatrix},$$

$$C = \begin{pmatrix} \frac{1}{L} & 0 \\ 0 & \frac{v_{o,e}}{CR^2} \end{pmatrix}$$

and $u = d_R$ is the control input, and $v_d = [v_i\ R]^T$ are the unknown disturbances. Based on this formulation a suitable control, d_R, is designed in order for the state x of (5) to approach sufficiently close to the desired reference level in the presence of the unknown disturbances v_d. In conformity with the controller theory in [9] we assume that $v_d \in V$, a known compact set. This assumption is reasonable, since maximum values of the supply voltage fluctuation v_i^{max} and of the load variations R^{max} are known for the worse cases. Thus, $V = \left\{ v_d \in R^2 \mid |v_i| \le v_i^{\max}, |R| \le R^{\max} \right\}$.

III. Controller Design

It is readily verified that the system described by (5) does not satisfy the matching assumption $C = BF$ as in [9]. According to [10], one may proceed by defining constant matrices C_m and $\tilde{C}$ such that $C = C_m + \tilde{C}$. This decomposition divides C into two parts. One matched, $C_m = BF$ for some F which yields uncertainty that can be controlled by the existing technique, and the mismatched portion $\tilde{C}$ which represents residual uncertainty. The above decomposition is not unique and one such choice results from $F = [1/v_{o,e}\ \ 0]$. It is shown in [10] that practical stability (see Appendix) can be assured by considering the matched portion of the uncertainty alone. Explicit expressions for the uniform boundedness and uniform ultimate boundedness constants are contained in [10].

The matrix pair (A, B) is controllable and $\bar{A} = A + BK$ is an appropriate choice for some designed K which yields satisfactory convergence characteristics. Thus, the control that assures practical stability for the system of (5), for all disturbances v_d whose values range in V, given $\varepsilon > 0$, is

$$d_R = Kx + p(x) \tag{6}$$

where

$$p(x) = \begin{cases} \frac{-B^T Px}{\|B^T Px\|}\rho & \text{if } \|B^T Px\| \ge \varepsilon \\ \frac{-B^T Px}{\varepsilon}\rho & \text{if } \|B^T Px\| < \varepsilon \end{cases} \tag{7}$$

and P is the solution of

$$P\bar{A} + \bar{A}^T P + Q = 0 \tag{8}$$

for given $Q > 0$ and $\rho = \max_{v_d \in V} |Fv_d| = v_i^{\max} / v_{o,e}$.

The control given by (6) refers to the linear system (5) which approximates the nonlinear system (3) near some desired operating point. In order to drive the nonlinear system at this point where the linearization takes effect we need to supplement control (6) with the corresponding additional term d_{Re}. The resulting control action is:

$$d_{Rtotal} = d_{\mathrm{Re}} + d_R \tag{9}$$

The control given by (9) is applied to the nonlinear system described by (3) using a PWM modulator and can be readily implemented since all pertinent parameters are known a priori. Information about the state can be obtained in real time during operation. The control strategy followed for the simulations is to apply the control given by (9), which is adjusted on line. This involves a new linearization for each of the desired output voltages $v_{o,e}$, and the calculation of a corresponding P in (8).

IV. Simulation Results

In order to demonstrate the feasibility of the proposed nonlinear controller, the following performance conditions are investigated by means of computer simulations using the Matlab/Simulink software: (i) output reference voltage step changes, (ii) supply voltage variations and (iii) load changes. During all operating conditions the switching frequency, f_s, is constant at $50\,kHz$ and the parameters $L = 600\,\mu H$ and $C = 220\,\mu F$. It is assumed that there are unknown disturbances superimposed on the nominal values of the load and the input voltage. These undesired inputs are bounded by prespecified constants and their variation is shown in Figs. 2 and 3.

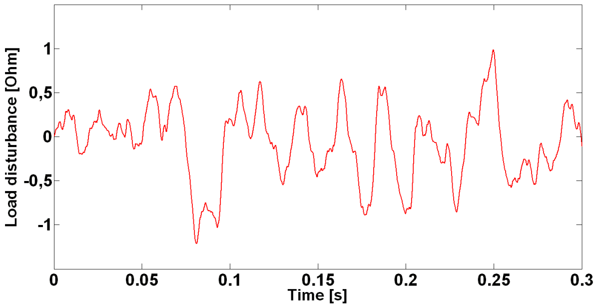

Fig. 2. Unknown load disturbance

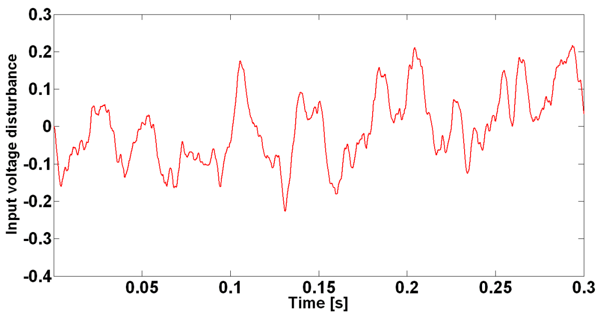

Fig. 3. Unknown input voltage disturbance

A. Output reference voltage step changes

In order to investigate the dynamic response of the proposed controller due to changes of the desired output level, the nominal values of the load and the input voltage are taken to be $55\,Ohm$ and $20\,V$ respectively. Initially, the output reference is set at $40\,V$ with subsequent step changes to $60\,V$ at $0.1\,s$ and $50\,V$ at $0.2\,s$. During those periods, the DC output voltage is regulated accurately at the desired values. It exhibits satisfactory transient and steady state characteristics. The results are shown in Fig. 4.

B. Supply voltage variations

For this part, the desired output level is set at $50\,V$. The nominal value of the load is $55\,Ohm$ and the supply voltage is allowed to take a number of step and ramp changes as shown in Fig. 5. In particular, the input voltage is held initially around $20\,V$ until $0.1\,s$ when it is dropped to about $15\,V$. At $t = 0.15\,s$ it experiences a continuous increase until $t = 0.25\,s$ when it reaches $20\,V$ near which it remains thereafter. The corresponding output voltage time history indicates small digressions from the desired value.

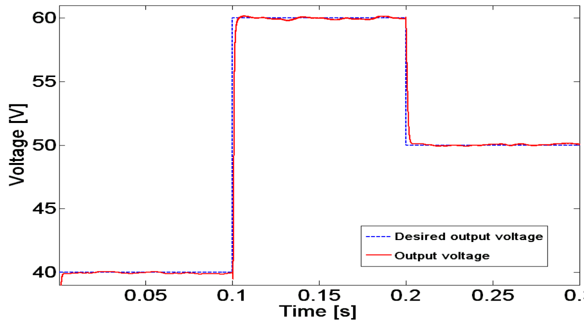

Fig. 4. Reference and actual output voltage step changes

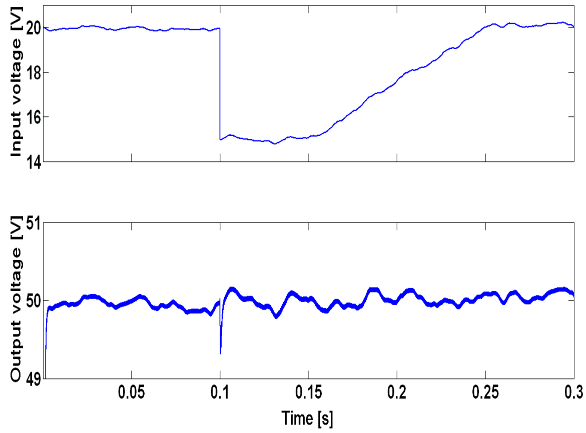

Fig. 5. Input voltage variations and corresponding output voltage

C. Load changes

The robustness of the proposed controller is further tested under a load step change occurring at $t = 0.15\,s$. The step change lowers the value of the resistance from about $55\,Ohm$

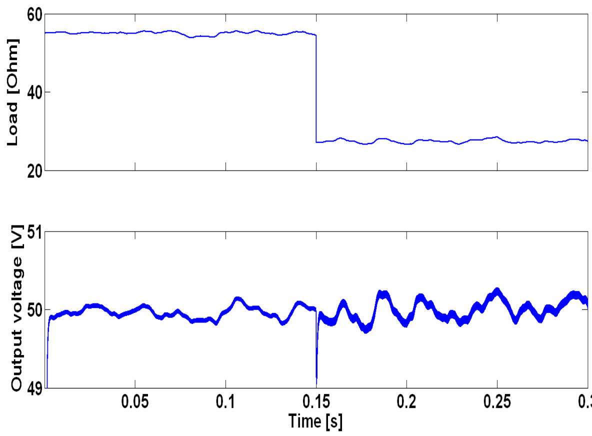

Fig. 6. Load resistance variations and corresponding output voltage

to approximately half its value. Again, the desired output level is $50\ V$ and the simulation results demonstrate that the output voltage remains close to that value irrespective of the load step and the disturbing inputs. This is illustrated in Fig. 6.

V. CONCLUSIONS

The proposed nonlinear controller differs from other customary controllers in a number of respects. It is designed to operate in an uncertain environment where, in the case examined, the voltage and load resistance fluctuations are considered as unknown. The only information needed about the uncertainties is the maximum of their possible range of variation, a figure readily available and no knowledge of the exact characteristics is assumed. In particular, an a priori statistical description is not required. This is advantageous since there is no reliable statistical description for such uncertainties.

The output voltage shows reasonable set point tracking characteristics with no overshoot and minimal deviations from the desired value. The transition times while moving between the desired output values are reasonably short and can be influenced by the controller gains. The required control input is readily implemented in real time.

APPENDIX

Property P1: *Uniform Boundedness*. Given $x_o \in R^n$, there is a positive $d(x_o) < \infty$ such that, for all solutions $x(\cdot): [t_{0,}t_1) \to R^n, x(t_0) = x_o$, $\|x(t)\| < d(x_o), \forall\, t \in [t_0, t_1)$.

Property P2: *Uniform Ultimate Boundedness*. Given $x_o \in R^n$ and $J = \{x \in R^n \mid \|x\| \leq \delta > 0\}$, there is a nonnegative $T(x_o,J) < \infty$such that, for all solutions $x(t) \in J$, $\forall\, t \geq t_0 + T(x_o,J)$.

Loosely speaking, uniform boundedness implies that every solution emanating from initial state x_o remains within a bounded neighborhood whose radius may depend on x_o. Uniform ultimate boundedness implies that every solution starting at x_o will enter and remain within a neighborhood of prescribed radius δ, after a finite time which may depend on x_o and δ. These two properties, sometimes stated in a slightly different but equivalent form, are the main ingredients of practical stability [9], [10].

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