# On improvement of transient stage of composite nonlinear feedback control using arbitrary order set point filters

# On Improvement of Transient Stage of Composite Nonlinear Feedback Control Using Arbitrary Order Set Point Filters

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*Abstract*—**This paper studies the generalization of composite nonlinear feedback (CNF) control using arbitrary order set point filters, which focus on the initial stage of the transient response. The set point filters can be used to provide more performance by shortening the rise and settling times of the control system. Furthermore, the filters operate outside the feedback loop, and hence, they do not sacrifice loop robustness. The new method is illustrated by a benchmark problem found in an open literature. The simulation results show that the proposed method improves the set point response more than 10% in terms of settling time.**

*Index Terms*—**Composite nonlinear feedback, robust control, high performance, set point filter, actuator saturation.**

## I. INTRODUCTION

A so-called composite nonlinear feedback (CNF) methodology has been developed to improve the tracking performance of servo systems. The CNF originates from the work of Lin *et al.* [1], and since then it has been extensively studied. For example, Turner *et al.* [2] generalized the CNF control to multivariable systems, Chen *et al.* laid the foundation for measurement feedback in [3], which was further generalized by He *et al.* in [4]. In [5], Lan *et al.* investigated the CNF for a class of nonlinear systems. Discrete time CNF were reported in [6], respectively. Lan *et al.* have also proposed a scaled nonlinear function to achieve robust transient performance to the variation of the step amplitude in [7]. Cheng *et al.* have generalized the CNF for tracking non-step references in [8]. The CNF has been successfully applied to many systems such as hard disk drives (HDD) in [3], [6], [7], [9] helicopter flight systems in [10], servo position systems in [8] and [11], respectively.

Generally, the CNF controller is composed of linear and nonlinear parts, which are designed as follows. First, a lightly damped linear part is designed for fast response without violating the actuator saturation constraints. Then, the nonlinear part is attached to the structure in order to reduce the overshoot caused by the linear part. The nonlinear part provides control error dependent damping when the output is reaching the target set point. By such mechanism, the nonlinear part smoothly relocates the closed-loop poles with large damping ratio, which effectively reduces settling time.

The CNF focuses primarily at the final stage of the transient response by reducing settling time with the nonlinear part. However, the CNF does not pay any special attention to the initial stage: only a lightly damped closed-loop system is designed via linear state feedback. Can we advance the CNF methodology such that an additional design freedom would be available for improving the initial transient? To answer this question, we propose a generalization to the CNF by allowing arbitrary order proper set point filters between the target step reference and feedback control. The overall effect is that the rise and settling times of the control system can be reduced without sacrificing robustness, and without the adverse actuator saturation. The new method is illustrated by a comprehensive HDD benchmark problem.

The material in this paper is organized as follows. In Section 2, the CNF methodology will be generalized with an arbitrary order set point filter for the following cases: A) state feedback, and B) measurement feedback, respectively. Furthermore, in Section 3, the proposed method will be applied to a benchmark problem found in [9]. The results show that the tracking performance of the control system can considerably be improved while maintaining robustness. Finally, the summary of the paper is presented in Section 4.

## II. COMPOSITE NONLINEAR FEEDBACK WITH ARBITRARY ORDER SET POINT FILTERS

In this section, the CNF is generalized with additional arbitrary order set point filters. The filters are placed in the feedforward path between the given reference and control input, as indicated in Fig. 1. We would like to note that the feedforward path will remain invariant w.r.t. feedback control. However, dynamic adjustment of the target step references will affect the feedback control during the transient stage, which requires attention. We will provide conditions when an additional set point filter improves the tracking performance without the actuator saturation. We will also prove the asymptotic stability of the closed-loop system. The following cases will be investigated: A) state feedback and B) measurement feedback.

Consider the following single input single output (SISO) system with an amplitude constrained actuator

$$\begin{cases} \dot{x} = Ax + B\text{sat}(u),\ x(0) = x_0 \\ y = C_y x \\ m = C_m x \end{cases}, \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, $y \in \mathbb{R}$ and $m \in \mathbb{R}^p$ are the state, control input, controlled output and measured output, respectively, whereas $x_0$ is an initial condition. The actuator saturation is represented by

$$\text{sat}(u) = \min\{u_{\max}, |u|\}\,\text{sgn}(u)\ , \tag{2}$$

where $u_{\max}$ is the saturation limit of the input and sgn denotes the sign function. It is required that

- the pair $(A,B)$ is stabilizable,
- the pair $(A,C_m)$ is detectable,
- the triple $(A,B,C_y)$ has no transmission zeros at the origin.

Here, the intention is to design a generalized composite nonlinear feedback control law for the system (1) such that the resulting closed-loop system is asymptotically stable, and is capable to track a step command input of given magnitude swiftly without saturating the actuator and without large overshoot.

*A. State Feedback Case*

Consider the block diagram in Fig. 1, which represents the system to be controlled, a state feedback CNF controller, and a feedforward set point filter from the target reference $r$ to the control signal $u$. It is clear from the Fig. 1 that the feedback control does not affect the feedforward response from the given reference $r$. However, the injection of the target step reference through the filter does affect the feedback loop, which requires attention.

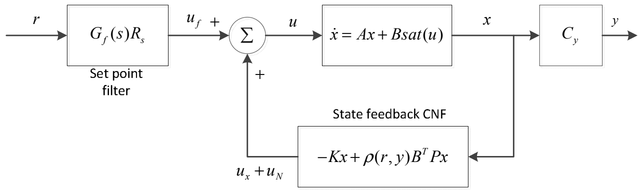


Fig. 1. State feedback CNF controller with set point filter.

The control signal $u$ is the sum of the feedback and feedforward signals given by

$$u = u_L + u_N = u_x + u_f + u_N = -Kx + G_f(s)R_s r + \rho(r,y)B^T Px\ , \tag{3}$$

where $u_L$ represents the linear part consisting of a state feedback gain $K$, a reference tracking gain $R_s$ and a proper set point filter $G_f(s)$. The gain $K$ is chosen such that $(A–BK)$ is asymptotically stable. The nonlinear part $u_N$ is discussed later. The reference tracking gain is a scalar defined by

$$R_s = -[C_y(A - BK)^{-1}B]^{-1}\ , \tag{4}$$

where the inner inverse exists under the given assumptions.

For a step input $r = a_r r_1$ with $a_r$ being the step amplitude and $r_1$ the unit step, the feedforward response $u_f$ through the filter $G_f(s)$ can generally be expressed as

$$u_f(t) = R_s a_r \cdot u_{f,1}(t)\ , \tag{5}$$

where $u_{f,1}$ corresponds to the unit step response. The feedforward response is subject to following conditions, which are clarified later.

- $u_{f,1} \to 1$, $t \to \infty$, i.e. the DC-gain of the filter is 1.
- $u_{f,1}$ is differentiable.
- $\dot{u}_{f,1}$ is uniformly continuous.
- $\dot{u}_{f,1} \to 0$, $t \to \infty$.

The decay speed of $u_{f,1}$ is important in order to ensure that the filter dynamics are appropriate for the closed-loop system for the following reasons. On the one hand, too fast decay speed does not significantly improve the rise or settling times, and it may require high sampling rate. On the other hand, the decay speed cannot be too slow, because it may result in an unacceptable overshoot, which cannot be compensated by the nonlinear part. Nevertheless, it should be noted that the dynamics of $u_f$ is faster than the dominating dynamics of the unfiltered closed-loop control system.

In what follows, we proceed to assign conditions for the traceable magnitudes of the target step references and for the set point filter such that the actuator is not saturated and closed-loop system remains linear. We have the following theorem, which address the properties of $u_L = u_x + u_f$.

***Theorem 1.*** Let $P \in \mathbb{R}^{nxn}$, $P > 0$ be a symmetric and real solution to the following Lyapunov equation

$$(A - BK)^T P + P(A - BK) + Q = 0\ , \tag{6}$$

where $Q \in \mathbb{R}^{nxn}$, $Q^T = Q > 0$. Such $P$ exists since $(A–BK)$ is asymptotically stable. Also, let $c > 0$ be the largest positive scalar satisfying

$$|Kx| \le u_{\max}(1 - \delta),\ \forall x \in \{x : x^T Px \le c\} \triangleq S\ , \tag{7}$$

where $\delta \in (0,1)$. Furthermore, define

$$\begin{cases} H \triangleq -[1 + K(A - BK)^{-1}B]R_s \\ x_d(t) \triangleq R_d a_r \cdot u_{f,1}(t) = -(A - BK)^{-1}BR_s a_r \cdot u_{f,1} \end{cases}, \tag{8}$$

where $x_d(\infty)$ is the desired state as $t \to \infty$. Then, the linear feedback control $u_L$ in (3) tracks a step command $r = a_r r_1$ asymptotically in time without saturating the actuator provided that the initial state $x_0$, $x_d(0)$, $r$ and $u_{f,1}$ satisfy:

$$\tilde{x}(0) \triangleq (x(0) - x_d(0)) \in S \text{ and } |a_r||H| \cdot |u_{f,1}| \le \delta u_{\max},\ \forall t. \quad (9)$$

***Proof.*** Defining the state error through the coordinate translation $\tilde{x} = x - x_d$, we can write the linear control law as

$$\begin{aligned} u_L &= -K\tilde{x} + [1 + K(A - BK)^{-1}B]R_s a_r \cdot u_{f,1} \\ &= -K\tilde{x} - Ha_r \cdot u_{f,1}. \end{aligned} \quad (10)$$

Now, for all $\tilde{x} \in S$ and requiring $|a_r||H| \cdot |u_{f,1}| \le \delta u_{\max},\ \forall t$, the actuator is not saturated because

$$|u_L| = |-K\tilde{x} - Ha_r \cdot u_{f,1}(t)| \le |K\tilde{x}| + |a_r||H| \cdot |u_{f,1}| \le u_{\max},\ \forall t \quad (11)$$

which retains linear closed-loop system. Moreover, the closed-loop system can be expressed as

$$\dot{\tilde{x}} = (A - BK)\tilde{x} + Ax_d - BHa_r \cdot u_{f,1} - \dot{x}_d \quad (12)$$

where

$$Ax_d - BHa_r \cdot u_{f,1} = 0,\ \forall t \quad (13)$$

and $\dot{x}_d \to 0$ as $t \to \infty$. Note that $x_d$ is differentiable as $u_{f,1}$ is differentiable, and $x_d$ has a finite limit as $t \to \infty$, i.e. $x_d(\infty) = R_d a_r$. Also, $\dot{x}_d$ is independent of the system states with faster convergence speed than the closed-loop state error.

Finally, define a quadratic Lyapunov function

$$V(\tilde{x}) = \tilde{x}^T P \tilde{x} \quad (14)$$

along the trajectories of the closed-loop system. It is simple to verify that the time derivative of $V(\tilde{x})$ satisfies

$$\dot{V}(\tilde{x}) = \dot{\tilde{x}}^T P \tilde{x} + \tilde{x}^T P \dot{\tilde{x}} = -\tilde{x}^T Q \tilde{x} - 2\tilde{x}^T P \dot{x}_d \le -\tilde{x}^T Q \tilde{x} \quad (15)$$

which implies that (12) decreases monotonically towards zero along the closed-loop trajectories. Therefore, all trajectories of (12) starting from $S$ remain in $S$ and converge asymptotically to the origin. For $a_r$, $x_0$ and $x_d(0)$ that satisfy (9) leads to

$$\lim_{t\to\infty} x(t) = x_d(\infty) = -(A - BK)^{-1} B R_s a_r,\ u_{f,1} \to 1 \quad (16)$$

$$\lim_{t\to\infty} y(t) = C_y x_d(\infty) = R_s^{-1} R_s a_r = a_r, \quad (17)$$

which implies that the controlled output $y$ converges exactly to the target reference in steady-state. Moreover, if the initial condition $x_0 = 0$, then any step amplitude can be tracked by $u_L$ provided that

$$|a_r| \le \frac{1}{u_{f,1}(t)} \sqrt{\frac{c}{R_d^T P R_d}} \text{ and } |a_r||H| \cdot |u_{f,1}| \le \delta u_{\max},\ \forall t \quad (18)$$

are satisfied. We would like to note that if the set point filter is omitted, then $u_{f,1} = 1$ at all times and the result (18) reduces back to the regular CNF. The result (18) also indicates that we can either increase or decrease the traceable magnitudes of the target step references by utilizing an appropriate filter. For example, a low-pass filter would increase, and a phase-lead filter would decrease the traceable magnitudes of the step references compared to the regular CNF.

Next we focus on the nonlinear feedback part $u_N$. The following theorem provides conditions for asymptotic stability of the closed-loop system together with $u_N$ and the set point filter $G_f(s)$.

***Theorem 2.*** Consider the system (1) and the control law (3). Then for any nonpositive function $\rho(r,y)$ locally Lipschitz in $y$, the CNF controller internally stabilizes the given system, and the output $y$ tracks asymptotically a step input $r$, provided that the conditions (9) are satisfied. The proof follows similar lines of reasoning as presented in [3], but here the additional set point filter requires further attention.

***Proof.*** The closed-loop system under the coordinate translation $\tilde{x} = x - x_d$ and the CNF controller (3) is

$$\dot{\tilde{x}} = (A - BK)\tilde{x} - \dot{x}_d + B\varphi, \quad (19)$$

where

$$\varphi = \mathrm{sat}(-K\tilde{x} - Ha_r \cdot u_{f,1} + u_N) + K\tilde{x} + Ha_r \cdot u_{f,1},\ \forall t. \quad (20)$$

Notice that if the nonlinear term $u_N$ is omitted, then (19)–(20) reduces back to Theorem 1.

Using the quadratic Lyapunov function

$$V(\tilde{x}) = \tilde{x}^T P \tilde{x}, \quad (21)$$

the derivative of $V(\tilde{x})$ along the trajectories of (19) can be written as

$$\dot{V}(\tilde{x}) = \dot{\tilde{x}}^T P \tilde{x} + \tilde{x}^T P \dot{\tilde{x}} = -\tilde{x}^T Q \tilde{x} - 2\tilde{x}^T P \dot{x}_d + 2\tilde{x}^T P B \varphi. \quad (22)$$

It should be noted that

$$\forall \tilde{x} \in S = \{\tilde{x} : \tilde{x}^T P \tilde{x} \le c\} \Rightarrow |K\tilde{x}| \le u_{\max}(1 - \delta). \quad (23)$$

In what follows, the value of (22) along the trajectories of (19) w.r.t. saturation function is calculated. This gives the following three instances; namely, no saturation, saturation at the upper limit, and saturation at the lower limit:

1. If $|-K\tilde{x} - Ha_r \cdot u_{f,1} + u_N| \le u_{\max}$, then $\varphi = u_N = \rho B^T P \tilde{x}$.

2. If $-K\tilde{x} - Ha_r \cdot u_{f,1} + u_N > u_{\max}$, then $0 < \varphi < \rho B^T P \tilde{x}$.

3. If $-K\tilde{x} - Ha_r \cdot u_{f,1} + u_N < -u_{\max}$, then $\rho B^T P\tilde{x} < \varphi < 0$.

Therefore, we can write $\varphi = q\rho B^T P\tilde{x}$, for some $q \in [0, 1]$. To summarize, the derivative of the Lyapunov candidate function satisfies

$$\dot{V}(\tilde{x}) \le -\tilde{x}^T Q\tilde{x},\ \tilde{x} \in S, \tag{24}$$

which implies that every trajectory starting within $S$ remains in $S$ and converge eventually to the origin under the given assumptions. Therefore, the state error converge to zero in time, which ensure the validity of equations (16)–(17) and the proof is complete.

*B. Measurement Feedback Case*

It is impractical to assume that all state variables are measurable. For example, it is uncommon that the velocity is directly measured in position control systems. Therefore, it needs to be estimated if state feedback controllers are used. In what follows, a full-order measurement feedback controller is utilized of the form

$$\begin{cases} \dot{x}_c = (A - FC_m)x_c + Fm + B\text{sat}(u_L) \\ u_L = -K(x_c - x_d) - Ha_r \cdot u_{f,1} + \rho(r,y)B^T P(x_c - x_d) \end{cases}, \tag{25}$$

where $x_c$ is the state of the controller. As usual, the gain matrices $K$ and $F$ are designed such that $(A–BK)$ and $(A–FC_m)$ are asymptotically stable. Again, the set point filter needs some further attention to prove the asymptotic stability of the closed-loop system.

Let $P_s \in \mathbb{R}^{nxn}$, $P_s > 0$ be a symmetric and real solution to the following Lyapunov equation

$$(A - BK)^T P_s + P_s(A - BK) + Q_s = 0, \tag{26}$$

where $Q_s \in \mathbb{R}^{nxn}$, $Q_s^T = Q_s > 0$. Also, let $P_o \in \mathbb{R}^{nxn}$, $P_o > 0$ be a symmetric and real solution to the Lyapunov equation

$$(A - FC_m)^T P_o + P_o(A - FC_m) + Q_o = 0, \tag{27}$$

where $Q_o \in \mathbb{R}^{nxn}$, $Q_o^T = Q_o > 0$. Such $P_s$ and $P_o$ exist since $(A–BK)$ and $(A–FC_m)$ are asymptotically stable. In addition, $Q_o$ must satisfy

$$Q_o > K^T B^T P_s Q_s^{-1} P_s BK. \tag{28}$$

Finally, let $c > 0$ be the largest positive scalar satisfying

$$\left|\begin{bmatrix} K & K \end{bmatrix} x_m\right| \le u_{\max}(1-\delta),\ \forall x_m \in \{x_m : x_m{}^T R x_m \le c\} \triangleq S_m \tag{29}$$

where $\delta \in (0,1)$, $x_m = \begin{bmatrix} x \\ x_c \end{bmatrix}$ and $R = \begin{bmatrix} P_s & 0 \\ 0 & P_o \end{bmatrix} > 0$.

It is also required that the initial states $x(0)$, $x_c(0)$, $x_d(0)$ as well as $a_r$ and $u_{f,1}$ satisfy:

$$\tilde{x}_m(0) \triangleq \begin{pmatrix} x(0) - x_d(0) \\ x_c(0) - x(0) \end{pmatrix} \in S_m \tag{30}$$

and

$$|a_r||H| \cdot |u_{f,1}| \le \delta u_{\max}, \tag{31}$$

where $H$ is defined as in (8). We have the following theorem.

***Theorem 3.*** Consider the system (1) and the control law (25). Then a scalar $\rho^* > 0$ exists such that for any nonpositive function $\rho(r,y)$ locally Lipschitz in $y$ and $|\rho(r,y)| \le \rho^*$, the CNF controller internally stabilizes the given system, and cause the output $y$ to track a step input $r$ asymptotically, provided that the conditions (30) and (31) are satisfied.

***Proof.*** Using the following coordinate translations $\tilde{x} = x - x_d$ and $\tilde{x}_c = x_c - x$, the closed-loop system with the control law (25) can be written as

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{x}}_c \end{bmatrix} = \begin{bmatrix} (A-BK) & -BK \\ 0 & (A-FC_m) \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{x}_c \end{bmatrix} - \begin{bmatrix} \dot{x}_d \\ 0 \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \varphi, \tag{32}$$

where

$$\varphi = \text{sat}\left( -\begin{bmatrix} K & K \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{x}_c \end{bmatrix} - Ha_r \cdot u_{f,1} + \rho \begin{bmatrix} B^T P_s & B^T P_s \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{x}_c \end{bmatrix} \right) + \begin{bmatrix} K & K \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{x}_c \end{bmatrix} + Ha_r \cdot u_{f,1}. \tag{33}$$

Then for $x(0)$, $x_c(0)$ and $x_d(0)$ satisfying (30) and (31) ensure

$$\tilde{x}_m(0) \in S_m. \tag{34}$$

Using the quadratic Lyapunov function

$$V(\tilde{x}_m) = \tilde{x}_m{}^T R \tilde{x}_m \tag{35}$$

and evaluating it's time derivative w.r.t. closed-loop trajectories (32) gives

$$\dot{V}(\tilde{x}_m) = \tilde{x}_m{}^T \left( \begin{bmatrix} -Q_s & -P_s BK \\ -K^T B^T P_s & -Q_o \end{bmatrix} \right) \tilde{x}_m - 2\tilde{x}^T P_s \dot{x}_d + 2\tilde{x}^T P_s \varphi. \tag{36}$$

Therefore as in the state feedback case, the saturation function can be evaluated for the same three instances. We can show that for all possible cases, $\varphi$ can be rewritten as

$$\varphi = q\rho \begin{bmatrix} B^T P_s & B^T P_s \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{x}_c \end{bmatrix} \tag{37}$$

for some $q \in [0, 1]$, $|\rho| \le \rho^*$, and for $Q_o$ satisfying (28).

To summarize, we have

$$\dot{V}(\tilde{x}_m) \le 0,\ \tilde{x}_m \in S_m \tag{38}$$

and all trajectories starting within $S_m$ remain inside $S_m$ and converge eventually to origin. Once again, the equations (16–17) remain valid. Due to the page limit, we omit the reduced-order measurement feedback case.

Next, we select a standard nonlinear function $\rho(r,y)$, which is used to shorten the settling time of the closed-loop system. In this paper, the following nonlinear function is used as in [3], [7] and [9].

$$\rho(e) = -\beta\left|\exp(-\alpha|e|) - \exp(-\alpha|y(0) - r|)\right|,\ |e| = |r - y|, \tag{39}$$

which is a smooth and nonpositive function of $|e|$. The positive parameters $\alpha$ and $\beta$ are chosen such that the closed-loop system is appropriately damped when $|e| \to 0$. In the next section, we illustrate our method by a comprehensive HDD design example.

## III. Design Example: A HDD Benchmark Problem

In this section, the design procedure proposed in Section 2 is applied to a HDD benchmark problem found in the open literature [9]. Please refer to [9] for complete description of the problem. The nominal model of the HDD is captured by

$$\begin{aligned}\dot{x} &= Ax + B(\mathrm{sat}(u) + \omega_{in}) \\ &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 6.4013\cdot 10^7 \end{bmatrix} (\mathrm{sat}(u) + \omega_{in}),\end{aligned} \tag{40}$$

where $\omega_{in} = -3$mV is a constant input bias disturbance and $u$ is the control voltage, which is limited by ±3V. The measurement signal available for feedback is

$$m = \left[\prod_{i=1}^{4} G_{r,i}(s)\right] x_1 + \omega_{out} + \text{ noise} \tag{41}$$

where $G_{r,i}(s)$ are the transfer functions of uncertain resonance modes given in [9]. In order to mitigate the effects of the resonance modes, notch filters are series connected at the process input. Refer to [9] for the description of $G_{\text{notch}}(s)$. The output disturbances in (41) are mainly repeatable runouts (RRO), which can be expressed as $\omega_{out}$ = 0.1sin(110$\pi t$)+ 0.05sin(220$\pi t$)+0.02sin(440$\pi t$)+0.01sin(880$\pi t$). Finally, the measurement noise is assumed to be zero-mean white Gaussian noise having a variance $\sigma_n^2 = 9\cdot 10^{-6}$ (μm)$^2$.

In what follows, we design the generalized CNF controller using the theory from the last section. For fair comparison and in order to demonstrate the performance improvement by utilizing additional set point filters only, the linear and nonlinear feedback gain matrices are kept as in the original problem. Therefore, the generalized CNF controller is given by the following equations

$$\begin{aligned}\begin{bmatrix} \dot{x}_{c,r} \\ \dot{x}_i \end{bmatrix} &= \begin{bmatrix} -4000 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{c,r} \\ x_i \end{bmatrix} + \begin{bmatrix} 6.4013\cdot 10^7 \\ 0 \end{bmatrix} \mathrm{sat}(\tilde{u}) \\ &\quad + \begin{bmatrix} 0 \\ k_i \end{bmatrix} r - \begin{bmatrix} 1.6\cdot 10^7 \\ k_i \end{bmatrix} m\end{aligned} \tag{42}$$

$$\begin{aligned}\tilde{u} &= -\begin{bmatrix} 1.0310\cdot 10^{-5} & 0.12335 & 1.2248 \end{bmatrix} \begin{bmatrix} x_{c,r} + 4000m \\ m \\ x_i \end{bmatrix} \\ &\quad + \rho(e)\begin{bmatrix} 8.4622\cdot 10^{-5} & 0.049449 & 0.61237 \end{bmatrix} \begin{bmatrix} x_{c,r} + 4000m \\ m \\ x_i \end{bmatrix} \\ &\quad + G_f(s)R_s,\ u = G_{\text{notch}}(s)\cdot \tilde{u},\end{aligned} \tag{43}$$

where $\rho(e)$ is as in equation (39), $k_i = 10$ is an integration gain, and $G_f(s)$ is a set point filter, which is designed to improve the transient performance. As an illustration, the following two set point filters are designed

$$G_{f,1}(s) = \frac{5s + 50000}{s + 50000} \tag{44}$$

$$G_{f,2}(s) = \frac{\omega_n^2 (as+1)^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{45}$$

where $a = 3.0\cdot 10^{-5}$, $\zeta = 0.7$ and $\omega_n = 6.0\cdot 10^4$.

The parameters of $\rho(e)$; namely, $\alpha$ and $\beta$ are found by the well-known integral of time-weighted absolute error (ITAE)-criterion using simulation horizon of 4ms. For the filters $G_{f,1}$ and $G_{f,2}$, the ITAE index is minimized with $\alpha_1$ = 0.4, $\beta_1$ = 3.92 and $\alpha_2$ = 0.8, $\beta_2$ = 2.51. The overall effect of such parameterizations are that when $\rho(e) \to \rho(0)$ the closed-loop dominant poles are relocated relatively close to each other at the negative real axis, which almost corresponds to the fastest possible strictly monotonic response.

The set point responses and control signals are depicted in the Fig. 2 along with the original tuning. Referring to the Fig. 2, the tracking performance has clearly been improved by the proposed method. The settling times are $T_{s,1}$ = 0.606ms and $T_{s,2}$ = 0.649ms, whereas the original tuning settles in 0.687ms. Thus, the settling time has been improved over 10% using $G_{f,1}$. The 2nd order filter also offers smaller overshoot compared to the regular CNF.

Next, we evaluate the robustness properties for the case $G_{f,1}$. The results for the case $G_{f,2}$ are relatively similar, which are included into Table 1. The loop robustness is determined by setting the reference input $r$ to 0, and evaluating the nonlinear function at the steady-state when $\rho = \rho(0) = -1.2923$. The gain and phase margins are 8.18dB and 45.8 degrees, respectively. The sensitivity S and the complimentary sensitivity T are depicted in Fig. 3, where the complimentary sensitivity function is evaluated from $r$ to $m$, which does include the set point filter. In the frequency domain, the filters (44) and (45) effectively increase the closed-loop bandwidth from $r$ to $m$, which speed up the transient response.

TABLE I

OVERALL PERFORMANCE MEASURES

| | Original CNF | Generalized CNF with Gf1, α1, β1 | Generalized CNF with Gf2, α2, β2 |
|---|---|---|---|
| overshoot % | 1.6 | 2.2 | 1.3 |
| Ts (ms) | 0.687 | 0.606 | 0.649 |
| GM (dB) | 7.85 | 8.18 | 8.01 |
| PM (deg) | 44.7 | 45.8 | 46.8 |
| Smax (dB) | 4.58 | 4.56 | 4.61 |
| Tmax (dB) | 2.65 | 2.72 | 2.58 |

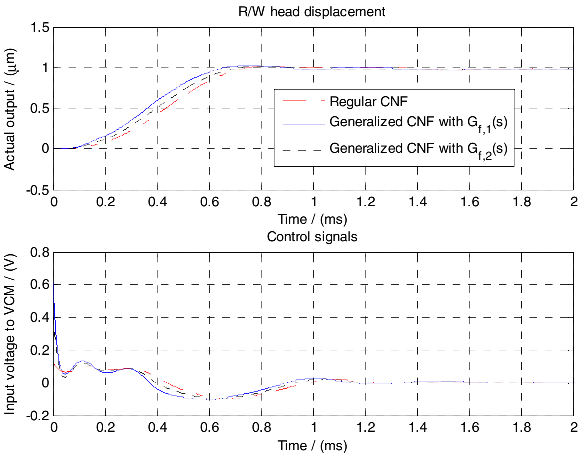


Fig. 2. Set point responses.

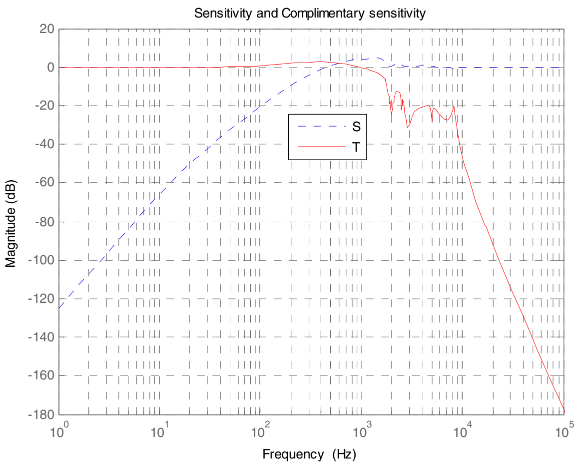


Fig. 3. Sensitivity S and complimentary sensitivity T.

Finally, the response characteristics are summarized into Table 1. It should be noted that in the original benchmark problem, the maximum values of sensitivity and complimentary sensitivity are documented as less than 5dB without accurate measures. We have simulated the sensitivity characteristics and included them into Table 1. Referring to the Table 1, the control loops also introduce slightly better robustness margins compared to the original tuning.

## IV. SUMMARY

A composite nonlinear feedback control with arbitrary order set point filters has been established. The proposed method allows an additional design freedom for improving the tracking performance without sacrificing loop robustness. We believe that the proposed method could be applied to other high-speed servo systems whenever simultaneous fast command following and robustness are required.

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