An Analytical Approach to SINR Estimation in Adjacent Rectangular Cells

An Analytical Approach to SIR Estimation in Adjacent Rectangular Cells*

Vyacheslav Begishev[1], Roman Kovalchukov[1], Andrey Samuylov[1], Aleksandr Ometov[2], Dmitri Moltchanov[2], Yuliya Gaidamaka[1], and Sergey Andreev[2]

[1] Peoples' Friendship University of Russia, Ordzhonikidze str. 3, 115419, Russia
{vobegishev, rnkovalchukov, ygaidamaka}@sci.pfu.edu.ru,
aksamuylov@gmail.com

[2] Tampere University of Technology,
Korkeakoulunkatu 10, FI-33720, Finland
{aleksandr.ometov, dmitri.moltchanov, sergey.andreev}@tut.fi

Abstract. Signal-to-interference-plus-noise ratio (SINR) experienced by a mobile user is the key metric determining the throughput it receives over a certain wireless technology of interest. In this paper, we propose an analytical model for SINR estimation in rectangular-shaped cellscases when noise can be considered negligible. This scenario is common for offices, shopping malls, dormitories, etc. We address uplink and downlink communications showing that in both cases the integral expressions can be derived. Various extensions to the proposed model are discussed. We also elaborate on the numerical results highlighting the important dependencies for the chosen scenario.

1 Introduction

The signal-to-interference-plus-noise ratio (SINR) is one of the most important metrics characterizing the quality of communication and describing the performance of a wireless system. Effectively, via the Shannon's law, it determines the maximum throughput that a user receives over the wireless technology of interest. Introducing additional technology-specific factors pertaining to the type of modulation and coding used at the air interface, one could also derive the instantaneous throughput delivered to the user.

SINR characterizes the wireless channel between a transmitting and a receiving device. Particularly, SINR can be considered as a ratio between the useful energy and the interference-and-noise. Due to the users' mobility, SINR in contemporary wireless systems is often a function of the current location of the user and thus can be considered as a random variable [1]. To this end, SINR depends on the following factors: the distance between the transmitter and the receiver, the set of active transmitters operating over a channel of interest, and the noise power. In other words, SINR at the receiving device indicates the extent of how much the effective signal is superior over various detrimental effects.

* The reported study was partially supported by RFBR, research projects No. 14-07-00090, 15-07-03051, 15-07-03608.

In this work, we analyze the SINR in a characteristic cellular scenario under assumption of zero noise power, that is the signal-to-interference (SIR) ratio. The environment of interest consists of adjacent rectangular cells of certain dimensions, with wireless base stations or access points (APs) deployed in their geometrical centers (one AP per cell). The positions of active mobile users are uniformly distributed over the area of interest. Concentrating on two adjacent cells, and assuming that at most one entity could be transmitting at any time inside a single cell, we apply the methods from stochastic geometry to develop a mathematical framework for SIR estimation in uplink and downlink channels. Our framework allows to obtain integral expressions for SIR values and could be further extended to characterize the throughput received by mobile users or APs in the considered scenario.

The rest of the paper is organized as follows. In Section 2, we describe the system model and introduce our main assumptions. In Section 3, we develop the analytical model and derive expressions for SIR in the uplink channel. The case of the downlink channel is briefly addressed as well. Section 4 illustrates some numerical results and offers a discussion on the key properties of SIR ratio in the considered environment. Conclusions are drawn in the last section.

2 System Model

2.1 Environment of interest

We assume that the mobile users are deployed across the area composed of adjacent rectangular cells (e.g., rooms) with the sides of certain length, as shown in Fig. 1. The APs (access points) are located in the geometrical centers of these cells. In each cell, mobile users are assumed to be uniformly distributed over its area.

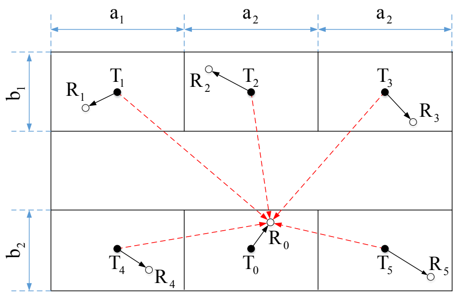


Fig. 1. System operation in scenario with rectangular cells

Even though we do not focus on any particular wireless technology, we adopt a number of assumptions on its characteristic details. First, mobile users operating in adjacent cells are assumed to utilize the common set of frequencies to communicate. This means that all transmitting entities interfere with each other. This is the case, for example, in Wi-Fi systems operating on the same channel or in micro-LTE systems that schedule the same set of resource blocks for users in adjacent cells. In the latter case, the model proposed in this paper corresponds to the worst-case interference scenario. Further, at each time instant t, we assume that at most one entity per cell is allowed to use the channel.

For the described scenario, we focus on both uplink and downlink channels. Whereas there are some differences between these options, the framework developed in this paper may generally address both of them. Due to the space limitations, below we provide full analysis for the uplink operation. In turn, the downlink channel is briefly addressed as an extension to the uplink case.

2.2 SINR details

Formally, SINR can be expressed as:

$$SINR = \frac{S}{\sum_{i=1}^{\mathrm{N}} I_i + \sigma^2}, \tag{1}$$

where S is the received signal power, N is the number of interfering sources, I_i is the interference power from the i^{th} source, and σ^2 is the noise power.

Generally, we can classify the detrimental noises onto (i) the natural noises, including the Johnson–Nyquist noise, the atmospheric noise, the solar noise, etc. and (ii) the artificial noises, e.g. the industrial noises. Occasionally, however, the noise may be assumed to be near-zero and we thus need to investigate SIR:

$$SIR = \frac{S}{\sum_{i=1}^{N} I_i}, \tag{2}$$

where the received signal power S is a function of the distance between the transmitter and the receiver, and the interference power I_i is a function of the distance and the signal between the receivers and the i^{th} interfering user.

The above functions can be specified as:

$$S = S\left(l\right) = gl^{-\alpha},\ I_i = I_i\left(l_i\right) = g{l_i}^{-\alpha_i}. \tag{3}$$

where g is the transmit power assumed to be constant for all the transmitters, l is the distance, and α is the path loss exponent, which ranges from 2, in the case of a Line-of-Sight (LoS) transmission, and up to 6 [2].

Focusing on the interference, we note that the received signal is primarily affected by the signals transmitted at the same and the neighboring frequencies. In today's cellular systems, the interfering users are, on the one hand, mobile devices being in proximity and, on the other hand, the cellular base stations that utilize the common frequency spectrum. The latter is typical for high-density urban deployments.

Here, we analyze the formulation conceptually similar to that in [3], by focusing on a pair of users in adjacent rectangular cells with the side lengths of (a_1, b_1) and (a_2, b_2), respectively (see Fig. 1). The receiver of interest, denoted as Rx_0, is located at the geometrical center of one tagged cell and the coordinates of the corresponding receiver are uniformly distributed over the area of this cell. In Fig. 1, black solid arrows represent logical downlink channels from the transmitter to its respective receiver. The adjacent cells are associated with the transmitters $T_i, i = 1, 2, \ldots, 5$ and the receivers $Rx_i, i = 1, 2, \ldots, 5$ that all use the same wireless technology and the common set of frequencies. Then, red dash lines indicate the interfering signals from the neighboring transmitters.

Further, we *tag* a pair of communicating users. For this pair, we denote $TR_0 =< Tx_0, Rx_0 >$. Similarly, for all the interfering pairs we impose $TR_i =< Tx_i, Rx_i >, i = 1, 2, \ldots, 5$. Let us then denote the distance between the transmitter Tx_0 and the receiver Rx_0 as R_0. Note that the received power is a function of distance between the interfering users, i.e. between Tx_i and Rx_0. We denote this distance as $D_i, i = 1, 2, \ldots, 5$.

Under the above assumptions, (2) can be simplified as:

$$SIR = \frac{S\left(R_0\right)}{\sum\limits_{i=1}^{5} I_i(D_i)}, \tag{4}$$

where

$$S\left(R_0\right) = g R_0^{-\alpha_0}, \qquad \sum_{i=1}^{5} I_i(D_i) = g \sum_{i=1}^{5} {D_i}^{-\alpha_i}. \tag{5}$$

Assuming constant transmit power, (4) reads as:

$$SIR = \frac{g R_0^{-\alpha_0}}{g \sum\limits_{i=1}^{5} {D_i}^{-\alpha_i}} = \frac{R_0^{-\alpha_0}}{\sum\limits_{i=1}^{5} {D_i}^{-\alpha_i}}. \tag{6}$$

3 Proposed Analytical Model

In this section, due to the space limitations, we concentrate primarily on the uplink communication. The downlink channel can be analyzed similarly. In addition, to simplify the resulting expressions, we consider the square cells of the same size.

3.1 Signal-to-Interference Ratio for Uplink

In the uplink case, the interference is effective at the AP side and Fig. 2 reflects the considered scenario, where the tagged cell incorporates the receiver of interest, whereas the interfering user is located in the adjacent cell named the *interfering cell*. Let both cells be of square shape with the side length of c. The receivers denoted by Rx_0 and Rx_1 are positioned in the geometrical centers of their respective cells, while the transmitters (e.g., mobile users) are uniformly distributed in the corresponding cells. In this configuration, Rx_0 is the tagged receiver and Tx_1 is the interfering transmitter. The distance between Tx_1 and Rx_0 is denoted by D_1 and the distance between Tx_0 and Rx_0 is denoted by R_0. A Cartesian coordinate system is chosen, such that the coordinates of the tagged receiver are $(c/2, c/2)$.

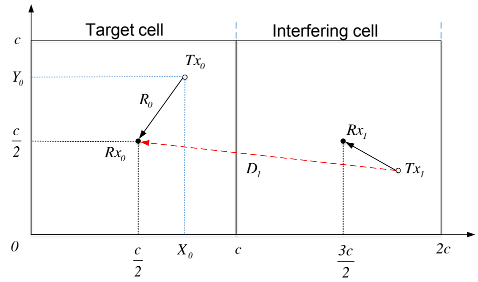


Fig. 2. Uplink SIR estimation details.

For the considered case, SIR takes the following form:

$$SIR = \frac{gR_0^{-\alpha_1}}{gD_1{}^{-\alpha_2}} = \frac{R_0^{-\alpha_1}}{D_1{}^{-\alpha_2}} = \frac{D_1^{\alpha_2}}{R_0^{\alpha_1}}. \tag{7}$$

According to (7), SIR is proportional to the distance D_1 and inversely proportional to R_0. Note that these distances are random variables and they are independent from each other, as the coordinates of the transmitters Tx_0 and Tx_1 are chosen independently. There are two ways of how to obtain the SIR distribution. First, we may determine the distributions of D_1 and R_0, then obtain the distributions of their powers, and finally calculate their ratio. This is completely feasible as the distances D_1 and R_0 are independent from each other and the joint distribution of their powers is simply a product of the individual distributions of their powers. However, this technique is not applicable to the downlink case, as D_1 and R_0 are not independent any more. An alternative approach is to directly employ the coordinates of the transmitter and the interferer. We follow this path for our proposed analysis below.

Let us first introduce the following coordinates:

$$\chi_1 = X_0,\ \chi_2 := Y_0,\ \chi_3 := X_1,\ \chi_4 := Y_1, \tag{8}$$

where (X_0, Y_0) and (X_1, Y_1) are the coordinates of the transmitters Tx_0 and Tx_1, respectively, whereas (χ_1, χ_2) and (χ_3, χ_4) are the particular values that they take. Since the coordinates of each transmitter are uniformly distributed within the square of side length c, the joint probability density function (pdf) of their coordinates is:

$$\begin{aligned} w_{\chi_1,\chi_2}(x_1, x_2) &= f_{X_0,Y_0}(x_1, x_2) = 1/c^2, \\ w_{\chi_3,\chi_4}(x_3, x_4) &= f_{X_1,Y_1}(x_3, x_4) = 1/c^2. \end{aligned} \tag{9}$$

Thereby, our goal is to determine the pdf of the random variables ξ, $W_\xi(y_1)$, denoting the SINR based on the joint densities $w_{\chi_1,\chi_2}(x_1, x_2)$ and $w_{\chi_3,\chi_4}(x_3, x_4)$ of random variables $\chi_1, \chi_2, \chi_3, \chi_4$. Using the Cartesian coordinates, we can write

(7) as:

$$\xi = \frac{D_1^{\alpha_2}}{R_0^{\alpha_1}} = \frac{\left(\sqrt{(\chi_3 - c/2)^2 + (\chi_4 - c/2)^2}\right)^{\alpha_2}}{\left(\sqrt{(\chi_1 - c/2)^2 + (\chi_2 - c/2)^2}\right)^{\alpha_1}}, \tag{10}$$

where c is the square side length.

Following [4, 5], in order to establish the pdf $W_\xi(y_1)$ we need to evaluate the densities of the numerator and the denominator of (10) and then integrate out over auxiliary variables y_2, y_3, y_4. To this end, let us determine the pdf $W_{\eta_1}(y_3)$. Recalling the form of the numerator in (10), we establish:

$$\begin{aligned} y_3 &:= f(x_3, x_4) = \sqrt{(x_3 - c/2)^2 + (x_4 - c/2)^2}, \\ y_4 &= x_4, \end{aligned} \tag{11}$$

where y_4 is an auxiliary variable.

The inverse of (11) takes the following form:

$$x_3 = \varphi(y_3, y_4) = \frac{c \pm \sqrt{-c^2 + 4y_3^2 + 4c\, y_4 - 4y_4^2}}{2}, \tag{12}$$

which is two-valued with the branches $x_{1,i} = \varphi_i\left(y_1, y_2, y_3, y_4\right), i = 1, 2$.

The pdf we are looking for can be determined as in [4]:

$$W_{\eta_1,\eta_2}(y_3, y_4) = \sum_{i=1}^{2} w_{\chi_3,\chi_4}\left(\varphi_i\left(y_3, y_4\right), y_4\right) \left|\frac{\partial \varphi_i\left(y_3, y_4\right)}{\partial y_3}\right|. \tag{13}$$

To deliver $W_\xi(y_1) = W_{\eta_1}(y_1)$, we need to integrate out y_4 in (13) obtaining:

$$W_{\eta_1}(y_3) = \sum_{i=1}^{2} \int_{Y_{2,i}} w_{\chi_3,\chi_4}\left(\varphi_i\left(y_3, y_4\right), y_4\right) \left|\frac{\partial \varphi_i\left(y_3, y_4\right)}{\partial y_3}\right| dy_4, \tag{14}$$

where $Y_{2,i}$ is the region of y_4 for the i^{th} branch of φ_i in (12).

For the branches defined in (12), we have:

$$\frac{\partial \varphi_i\left(y_3, y_4\right)}{\partial y_3} = (-1)^i \frac{2y_3}{\sqrt{-c^2 + 4y_3^2 + 4cy_4 - 4y_4^2}}, \; i = 1, 2. \tag{15}$$

Recalling (9), the integrands in (14) take the following form:

$$\begin{aligned} I_1(y_3, y_4) = w_{\chi_3,\chi_4}\left(\varphi_1(y_3, y_4), y_4\right) \left|\frac{\partial \varphi_1(y_3,y_4)}{\partial y_3}\right| &= \frac{2y_3}{c^2\sqrt{-c^2+4y_3^2+4cy_4-4y_4^2}}, \\ I_2(y_3, y_4) = w_{\chi_3,\chi_4}\left(\varphi_1(y_3, y_4), y_4\right) \left|\frac{\partial \varphi_1(y_3,y_4)}{\partial y_3}\right| &= \frac{2y_3}{c^2\sqrt{-c^2+4y_3^2+4cy_4-4y_4^2}}. \end{aligned} \tag{16}$$

The ranges $Y_{2,i}$, $i = 1, 2$, can be obtained by observing $x_3 \in [c, 2c]$ and $x_4 \in [0, c]$. For example, to calculate $Y_{2,1}$ we need to solve:

$$\begin{cases} c \leq \frac{c + \sqrt{-c^2+4y_3^2+4c\, y_4-4y_4^2}}{2} \leq 2c, \\ 0 \leq y_4 \leq c, \\ y_3 \geq 0. \end{cases} \tag{17}$$

The solution of (17) is in the form $Y_{2,1} = Y^1_{2,1} \cup Y^2_{2,1} \cup Y^3_{2,1}$, where

$$
\begin{aligned}
Y^1_{2,1} &= \begin{cases} \frac{c}{2} < y_3 \le \frac{c}{\sqrt{2}}, \\ \frac{c}{2} - \frac{1}{2}\sqrt{-c^2+4y_3^2} \le y_4 \le \frac{c}{2} + \frac{1}{2}\sqrt{-c^2+4y_3^2}. \end{cases} \\
Y^2_{2,1} &= \begin{cases} \frac{c}{\sqrt{2}} < y_3 \le \frac{3c}{2}, \\ 0 < y_4 \le c. \end{cases} \\
Y^3_{2,1} &= \begin{cases} \frac{3c}{2} < y_3 \le c\sqrt{\frac{5}{2}}, \\ \left[\begin{array}{l} 0 < y_4 \le \frac{c}{2} - \frac{1}{2}\sqrt{-9c^2+4y_3^2} \\ \frac{c}{2} + \frac{1}{2}\sqrt{-9c^2+4y_3^2} < y_4 \le c. \end{array} \right. \end{cases},
\end{aligned}
\tag{18}
$$

Evaluating the integral (14), we establish for $W_{\eta_1}(y_3)$:

$$
\begin{cases}
0, y_3 \le \frac{c}{2} \\
-\frac{y_3}{c^2}\left(\arcsin\left[\frac{-\sqrt{-c^2+4y_3^2}}{2y_3}\right] - \arcsin\left[\frac{\sqrt{-c^2+4y_3^2}}{2y_3}\right]\right), \frac{c}{2} < y_3 \le \frac{c}{\sqrt{2}} \\
-\frac{y_3}{c^2}\left(\arcsin\left[\frac{-c}{2y_3}\right] - \arcsin\left[\frac{c}{2y_3}\right]\right), \frac{c}{\sqrt{2}} < y_3 \le \frac{3c}{2} \\
-\frac{y_3}{c^2}\left(\arcsin\left[\frac{\sqrt{-9c^2+4y_3^2}}{2y_3}\right] - \arcsin\left[\frac{c}{2y_3}\right]\right) - \\
-\frac{y_3}{c^2}\left(\arcsin\left[\frac{-c}{2y_3}\right] - \arcsin\left[\frac{-\sqrt{-9c^2+4y_3^2}}{2y_3}\right]\right), \frac{3c}{2} < y_3 \le c\sqrt{\frac{5}{2}}
\end{cases}
\tag{19}
$$

Further, we estimate the pdf $W_{\eta_3}(y_5)$ of D^{α_2} arriving at:

$$
\begin{cases}
0, y_5 \le \left(\frac{c}{2}\right)^{\alpha_2} \\
\frac{2}{\alpha_2 c^2} y_5^{\frac{2}{\alpha_2}-1}\left(\arcsin\left[\frac{c}{2y_5^{1/\alpha_2}}\right] - \arcsin\left[\frac{\sqrt{-9c^2+4y_5^{2/\alpha_2}}}{2y_5^{1/\alpha_2}}\right]\right), \left(\frac{3c}{2}\right)^{\alpha_2} < y_5 \le \left(c\sqrt{\frac{5}{2}}\right)^{\alpha_2} \\
\frac{2}{\alpha_2 c^2} y_5^{\frac{2}{\alpha_2}-1} \arcsin\left[\frac{\sqrt{-c^2+4y_5^{2/\alpha_2}}}{2y_5^{1/\alpha_2}}\right], \left(\frac{c}{2}\right)^{\alpha_2} < y_5 \le \left(\frac{c}{\sqrt{2}}\right)^{\alpha_2} \\
\frac{2}{\alpha_2 c^2} y_5^{\frac{2}{\alpha_2}-1} \arcsin\left[\frac{c}{2y_5^{1/\alpha_2}}\right], \left(\frac{c}{\sqrt{2}}\right)^{\alpha_2} < y_5 \le \left(\frac{3c}{2}\right)^{\alpha_2}
\end{cases}
\tag{20}
$$

The density of R^{α_1}, $W_{\eta_4}(y_6)$ is obtained by following the same procedure:

$$
\left(\frac{2}{\alpha_1} y_6^{\frac{1}{\alpha_1}-1}\right)
\begin{cases}
\pi \frac{y_6^{1/\alpha_1}}{c^2}, 0 < y_6 \le \left(\frac{c}{2}\right)^{\alpha_1} \\
2\frac{y_6^{1/\alpha_1}}{c^2}\left(\arcsin\left[\frac{c}{2y_6^{1/\alpha_1}}\right] - \arcsin\left[\frac{\sqrt{-c^2+4y_6^{2/\alpha_1}}}{2y_6^{1/\alpha_1}}\right]\right), \left(\frac{c}{2}\right)^{\alpha_1} < y_6 \le \left(\frac{c}{\sqrt{2}}\right)^{\alpha_1}
\end{cases}
$$

$$
\left(\frac{2}{\alpha_1 c^2} y_6^{\frac{2}{\alpha_1}-1}\right)
\begin{cases}
\pi, 0 < y_6 \le \left(\frac{c}{2}\right)^{\alpha_1} \\
2\left(\arcsin\left[\frac{c}{2y_6^{1/\alpha_1}}\right] - \arcsin\left[\frac{\sqrt{-c^2+4y_6^{2/\alpha_1}}}{2y_6^{1/\alpha_1}}\right]\right), \left(\frac{c}{2}\right)^{\alpha_1} < y_6 \le \left(\frac{c}{\sqrt{2}}\right)^{\alpha_1}
\end{cases}
$$

Evaluating the ratio at hand similarly, we produce:

$$W(y_1) := \int_0^{\infty} \sum_{i=1}^{3} I_i(y_1, y_2) dy_2, \tag{21}$$

where the components I_1, I_2, I_3, respectively, are given by:

$$I_1(y_1,y_2) := \begin{cases} y_2 \frac{4}{\alpha_2 c^2} (y_1 y_2)^{\frac{2}{\alpha_2}-1} \arcsin\left[\frac{\sqrt{-c^2+4(y_1y_2)^{\frac{2}{\alpha_2}}}}{2(y_1y_2)^{\frac{1}{\alpha_2}}}\right] \left(\frac{2}{\alpha_1 c^2} y_2^{\frac{2}{\alpha_1}-1}\right) \times \\ \times \left(\arcsin\left[\frac{c}{2y_2^{1/\alpha_1}}\right] - \arcsin\left[\frac{\sqrt{-c^2+4y_2^{2/\alpha_1}}}{2y_2^{1/\alpha_1}}\right]\right), \\ if \left[\frac{c^{\alpha_2-\alpha_1}}{(\sqrt{2})^{2\alpha_2-\alpha_1}} \le y_1 \le \left(\frac{c}{2}\right)^{\alpha_2-\alpha_1} \cap \left(\frac{c}{2}\right)^{\alpha_2} \frac{1}{y_1} \le y_2 \le \left(\frac{c}{\sqrt{2}}\right)^{\alpha_1}\right] \cup \\ \cup \left[\left(\frac{c}{2}\right)^{\alpha_2-\alpha_1} < y_1 \le \frac{c^{\alpha_2-\alpha_1}}{(\sqrt{2})^{\alpha_2-\alpha_1}} \cap \left(\frac{c}{2}\right)^{\alpha_1} \le y_2 \le \left(\frac{c}{\sqrt{2}}\right)^{\alpha_1}\right] \cup \\ \cup \left[\left(\frac{c}{\sqrt{2}}\right)^{\alpha_2-\alpha_1} < y_1 \le \left(\frac{c}{\sqrt{2}}\right)^{\alpha_2} \left(\frac{c}{2}\right)^{-\alpha_1} \cap \left(\frac{c}{2}\right)^{\alpha_1} \le y_2 \le \left(\frac{c}{\sqrt{2}}\right)^{\alpha_2} \frac{1}{y_1}\right]; \\ y_2 \frac{2\pi}{\alpha_2 c^2} (y_1 y_2)^{\frac{2}{\alpha_2}-1} \arcsin\left[\frac{\sqrt{-c^2+4(y_7y_8)^{\frac{2}{\alpha_2}}}}{2(y_1y_2)^{\frac{1}{\alpha_2}}}\right] \left(\frac{2}{\alpha_1 c^2} y_2^{\frac{2}{\alpha_1}-1}\right), \\ if \left[\left(\frac{c}{2}\right)^{\alpha_2-\alpha_1} \le y_1 \le \left(\frac{c}{\sqrt{2}}\right)^{\alpha_2} \left(\frac{c}{2}\right)^{-\alpha_1} \cap \left(\frac{c}{2}\right)^{\alpha_2} \frac{1}{y_1} \le y_8 \le \left(\frac{c}{2}\right)^{\alpha_1}\right] \cup \\ \cup \left[y_1 > \left(\frac{c}{\sqrt{2}}\right)^{\alpha_2} \left(\frac{c}{2}\right)^{-\alpha_1} \cap \left(\frac{c}{2}\right)^{\alpha_2} \frac{1}{y_1} \le y_2 \le \left(\frac{c}{\sqrt{2}}\right)^{\alpha_2} \frac{1}{y_1}\right]; \\ 0, otherwise \end{cases}$$

$$I_2(y_1,y_2) := \begin{cases} y_2 \frac{4}{\alpha_2 c^2} (y_1 y_2)^{\frac{2}{\alpha_2}-1} \arcsin\left[\frac{c}{2(y_1y_2)^{1/\alpha_2}}\right] \left(\frac{2}{\alpha_1 c^2} y_2^{\frac{2}{\alpha_1}-1}\right) \times \\ \times \left(\arcsin\left[\frac{c}{2y_2^{1/\alpha_1}}\right] - \arcsin\left[\frac{\sqrt{-c^2+4y_2^{2/\alpha_1}}}{2y_2^{1/\alpha_1}}\right]\right), \\ if \left[\left(\frac{c}{\sqrt{2}}\right)^{\alpha_2-\alpha_1} \le y_1 \le \left(\frac{c}{\sqrt{2}}\right)^{\alpha_2} \left(\frac{c}{2}\right)^{-\alpha_1} \cap \left(\frac{c}{\sqrt{2}}\right)^{\alpha_2} \frac{1}{y_1} \le y_2 \le \left(\frac{c}{\sqrt{2}}\right)^{\alpha_1}\right] \cup \\ \cup \left[\left(\frac{c}{\sqrt{2}}\right)^{\alpha_2} \left(\frac{c}{2}\right)^{-\alpha_1} < y_1 \le \frac{3^{\alpha_2} c^{\alpha_2-\alpha_1}}{(\sqrt{2})^{2\alpha_2-\alpha_1}} \cap \left(\frac{c}{2}\right)^{\alpha_1} \le y_2 \le \left(\frac{c}{\sqrt{2}}\right)^{\alpha_1}\right] \cup \\ \cup \left[\frac{3^{\alpha_2} c^{\alpha_2-\alpha_1}}{(\sqrt{2})^{2\alpha_2-\alpha_1}} < y_1 \le \left(\frac{3c}{2}\right)^{\alpha_2} \left(\frac{c}{2}\right)^{-\alpha_1} \cap \left(\frac{c}{2}\right)^{\alpha_1} \le y_2 \le \left(\frac{3c}{2}\right)^{\alpha_2} \frac{1}{y_1}\right]; \\ y_2 \frac{2\pi}{\alpha_2 c^2} (y_1 y_2)^{\frac{2}{\alpha_2}-1} \arcsin\left[\frac{c}{2(y_1y_2)^{1/\alpha_2}}\right] \left(\frac{2}{\alpha_1 c^2} y_2^{\frac{2}{\alpha_1}-1}\right), \\ if \left[\left(\frac{c}{\sqrt{2}}\right)^{\alpha_2} \left(\frac{c}{2}\right)^{-\alpha_1} \le y_1 \le \left(\frac{3c}{2}\right)^{\alpha_2} \left(\frac{c}{2}\right)^{-\alpha_1} \cap \left(\frac{c}{\sqrt{2}}\right)^{\alpha_2} \frac{1}{y_1} \le y_2 \le \left(\frac{c}{2}\right)^{\alpha_1}\right] \cup \\ \cup \left[\left(\frac{3c}{2}\right)^{\alpha_2} \left(\frac{c}{2}\right)^{-\alpha_1} < y_1 \cap \left(\frac{c}{\sqrt{2}}\right)^{\alpha_2} \frac{1}{y_1} \le y_2 \le \left(\frac{3c}{2}\right)^{\alpha_2} \frac{1}{y_1}\right]; \\ 0, otherwise \end{cases}$$

$$
I_3(y_1,y_2) := \begin{cases}
y_2 \frac{4}{\alpha_2 c^2} (y_7 y_8)^{\frac{2}{\alpha_2}-1} \left(\arcsin\left[\frac{c}{2(y_1y_2)^{1/\alpha_2}}\right] - \arcsin\left[\frac{\sqrt{-9c^2+4(y_1y_2)^{2/\alpha_2}}}{2(y_1y_2)^{1/\alpha_2}}\right]\right) \times \\
\times \left(\frac{2}{\alpha_1 c^2} y_2^{\frac{2}{\alpha_1}-1}\right) \left(\arcsin\left[\frac{c}{2y_2^{1/\alpha_1}}\right] - \arcsin\left[\frac{\sqrt{-c^2+4y_2^{2/\alpha_1}}}{2y_2^{1/\alpha_1}}\right]\right), \\
if \left[\left(\frac{3c}{2}\right)^{\alpha_2}\left(\frac{c}{\sqrt{2}}\right)^{-\alpha_1} \le y_1 \le c^{\alpha_2-\alpha_1}\sqrt{\frac{5^{\alpha_2}}{2^{\alpha_2-\alpha_1}}} \cap \left(\frac{3c}{2}\right)^{\alpha_2}\frac{1}{y_1} \le y_2 \le \left(\frac{c}{\sqrt{2}}\right)^{\alpha_1}\right] \cup \\
\cup \left[c^{\alpha_2-\alpha_1}\sqrt{\frac{5^{\alpha_2}}{2^{\alpha_2-\alpha_1}}} < y_1 \le \left(\frac{3c}{2}\right)^{\alpha_2}\left(\frac{c}{2}\right)^{-\alpha_1} \cap \left(\frac{3c}{2}\right)^{\alpha_2}\frac{1}{y_1} \le y_2 \le c^{\alpha_2}\left(\sqrt{\frac{5}{2}}\right)^{\alpha_2}\frac{1}{y_1}\right] \cup \\
\cup \left[\left(\frac{3c}{2}\right)^{\alpha_2}\left(\frac{c}{2}\right)^{-\alpha_1} < y_1 \le c^{\alpha_2}\left(\sqrt{\frac{5}{2}}\right)^{\alpha_2}\left(\frac{c}{2}\right)^{-\alpha_1} \cap \left(\frac{c}{2}\right)^{\alpha_1} \le y_2 \le c^{\alpha_2}\left(\sqrt{\frac{5}{2}}\right)^{\alpha_2}\frac{1}{y_1}\right]; \\
y_2 \frac{2\pi}{\alpha_2 c^2} (y_1 y_2)^{\frac{2}{\alpha_2}-1} \left(\arcsin\left[\frac{c}{2(y_1y_2)^{1/\alpha_2}}\right] - \arcsin\left[\frac{\sqrt{-9c^2+4(y_1y_2)^{2/\alpha_2}}}{2(y_1y_2)^{1/\alpha_2}}\right]\right) \times \\
\times \left(\frac{2}{\alpha_1 c^2} y_2^{\frac{2}{\alpha_1}-1}\right), \\
if \left[\left(\frac{3c}{2}\right)^{\alpha_2}\left(\frac{c}{2}\right)^{-\alpha_1} \le y_1 \le c^{\alpha_2}\left(\sqrt{\frac{5}{2}}\right)^{\alpha_2}\left(\frac{c}{2}\right)^{-\alpha_1} \cap \left(\frac{3c}{2}\right)^{\alpha_2}\frac{1}{y_1} \le y_2 \le \left(\frac{c}{2}\right)^{\alpha_1}\right] \cup \\
\cup \left[c^{\alpha_2}\left(\sqrt{\frac{5}{2}}\right)^{\alpha_2}\left(\frac{c}{2}\right)^{-\alpha_1} < y_1 \cap \left(\frac{3c}{2}\right)^{\alpha_2}\frac{1}{y_1} \le y_2 \le c^{\alpha_2}\left(\sqrt{\frac{5}{2}}\right)^{\alpha_2}\frac{1}{y_1}\right]; \\
0, otherwise
\end{cases}
$$

3.2 Discussion and Extensions

While performing our above analysis, we adopted a number of assumptions, including the one on the square cells. Extending these results for the case of rectangular cells is rather straightforward. The most restrictive component is the use of the same propagation exponent α on both paths, D_1 and R_0. For the uplink scenario, the extension is simple as we can obtain the distributions of the numerator and the denominators individually. Instead of working with the coordinates, we may as well obtain the distributions of the distances D_1 and R_0, and then proceed with deriving the distribution of the ratio of their powers.

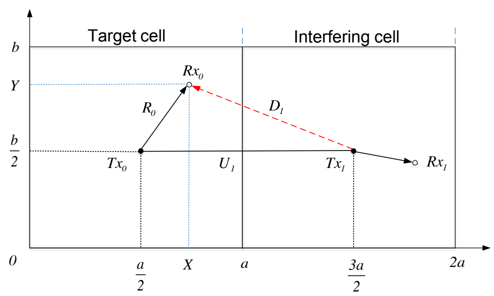


Fig. 3. An illustration of the model for downlink SINR analysis.

The downlink case is illustrated in Fig. 3 and is more involved compared to the uplink. The reason is that the distances R_0 and D_1 are no longer independent, as the coordinates of the receiver Rx_0 determine both of them. Hence, the approach based on the distributions of the distances will not lead to the solution, as we will not be able to determine the joint density of the powers of these distances. However, working with the coordinates similarly to what we did in the case of the uplink scenario is still feasible.

4 Numerical Results

We first verify the proposed analytical framework with our own simulator implemented in R [6]. Then, we demonstrate the impact of different input parameters by highlighting various dependencies. We provide the results for both uplink and downlink cases.

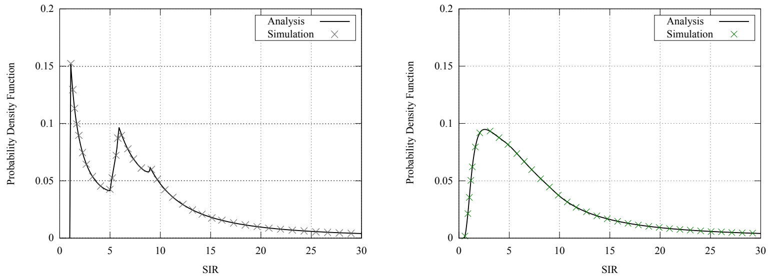
4.1 Verification of the Model

The developed simulator allows to vary all the parameters of interest used in the system model (see Fig. 2 and Fig. 3), including the dimensions of the cells a and b, the path loss exponent α between the transmitter and the receiver, the number of experiments, and the sampling frequency. At the output, our tool provides the empirical density functions, the sample means, the sample standard deviations, and the estimates for the quantiles of random variables R_0, D_1, SIR, and $10 \lg SIR$.

Along these lines, Fig. 4 compares the analytical results against those obtained with simulations for the uplink and downlink scenarios. For both cases, the dimensions of the cells have been set as $a = b = c$, where $c = 1$, while the same path loss exponent $\alpha = 2$ is utilized on both paths. The number of samples used to construct the empirical density has been $10e6$. The simulation data shows a perfect match with the analytical results. In both cases, the Kolmogorov-Smirnov goodness-of-fit statistical test [7] has been performed with the level of significance set to 0.05. It shows that the sample data belongs to the analytical distribution, which allows us to rely on the analytical model in the rest of this section.

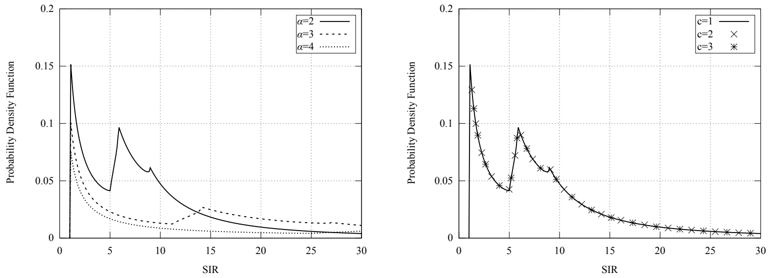
4.2 Performance Evaluation

The considered scenario has a number of interesting properties that could be demonstrated by employing the proposed model. In particular, the results are not very sensitive to the choice of some input parameters. Therefore, Fig. 5(a) illustrates the effect of the different values of α for the downlink scenario with the cell dimensions set as $a = b = 1$. Note that the same α is again used for both directions: the interferer to the receiver and the transmitter to the receiver. As one may observe, the value of α provides the scaling effect on the resulting SIR density. Further, Fig. 5(b) highlights the insensitivity of the model



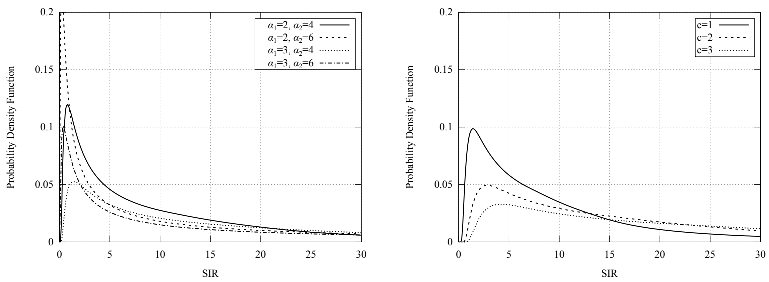
(a) Downlink (b) Uplink

Fig. 4. Validation of the proposed model, $\alpha = 2$, $a = b = 1$.



(a) Effect of α ($a = b = 1$) (b) Effect of dimension, ($\alpha = 2$)

Fig. 5. Insensitivity of the model to the input parameters for downlink scenario



(a) The effect of α ($a = b = 1$) (b) The effect of dimension ($\alpha_1 = 2$)

Fig. 6. Insensitivity of the model to the input parameters for uplink scenario

to the dimensions of interest. The results are demonstrated for two dimensions, $a = b = c$, where $c \in [1, 2, 3]$, while the path loss propagation exponent α is set to 2. It is important to note that these properties hold for the downlink scenario as well, across different a and b, as well as various path loss exponents (describing the path from the transmitter to the receiver and from the interferer to the receiver). However, we do not consider these effects in detail here due to the space limitations.

So far, we provided our results for the same path loss exponent at the propagation paths between the transmitter and the receiver, as well as the interferer and the receiver. These assumptions are only likely to hold in the open-space environments. Whenever there are walls separating the cells, the propagation path between the interferer and the received is characterized by a more severe attenuation. Accordingly, Fig. 6 highlights the effect of different values of α in the uplink scenario with α_1 corresponding to the propagation path between the transmitter and the receiver, and α_2 characterizing the propagation path between the interferer and the receiver. As we observe, this effect is fairly straightforward and self-explanatory: the larger the α_2 is, the better the interference picture at the receiver becomes. Recall that the value of α_2 is primarily determined by the material of walls. Our proposed model thus allows to compute the SIR for different wall materials.

5 Conclusions and Future Work

In this paper, we analyzed the SINR behavior in adjacent rectangular-shaped cells. We demonstrated that for both uplink and downlink channels we can estimate the SINR distribution analytically without the need for time-consuming simulations. Our model allows for a number of useful extensions, including the one taking into account more than one adjacent room. Our numerical investigation reveals that for the square configurations of cells the SINR value is insensitive to the side length of the square for both uplink and downlink scenarios. Further, the impact of the wall material affecting the propagation exponent could be significant. Finally, in our setup the SINR is positive at all times as the transmitting entity is always closer to the receiver compared to the interfering entity. However, this may not be the case if more than one adjacent cell (room) is considered. Developing an analytical model for this case is the subject of our ongoing work.

References

1. J. G. Andrews, R. K. Ganti, M. Haenggi, N. Jindal, and S. Weber, "A primer on spatial modeling and analysis in wireless networks," Communications Magazine, IEEE, vol. 48, no. 11, pp. 156–163, 2010.
2. 3GPP, "3rd Generation Partnership Project; Technical Specification Group Radio Access Networks; Radio Frequency (RF) system scenarios (Release 9)," Technical Report 25.942 v9.0.0, 12-2009.
3. Yu.V. Gaidamaka and A.K. Samouylov, "Method for calculating numerical characteristics of two devices interference for device to device communications in a wireless heterogeneous network," Computer science and its applications, vol. 1, no. 9, pp. 10–15, 2015.
4. S. M. Ross, Introduction to probability models. Academic press, 2014.
5. B. Levin, "Theoretical principles of statistical radiophysics," Sovetskoe Radio, Moscow, 1969.
6. R. C. Team, "R language definition," 2000.
7. F. J. Massey Jr, "The kolmogorov-smirnov test for goodness of fit," Journal of the American statistical Association, vol. 46, no. 253, pp. 68–78, 1951.