Converting GNSS satellite orbit segments to GPS-compatible format

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Abstract—An algorithm to convert a GNSS satellite track segment into the generalised Kepler parameter format used in GPS ephemerides messages is presented. A track segment is a sequence of 3D positions in Cartesian coordinates, such as is produced by orbit prediction software for self-assistance, where the format conversion is an essential post-processing step to reduce storage and for compatibility with GPS software and equipment. In this work the conversion task is formulated as a nonlinear curve fitting problem that is solved using the Levenberg-Marquardt algorithm. The algorithm is tested using orbit predictions for GPS and GLONASS satellites and using actual orbits obtained from precise ephemerides. The median error for a two-hour fitting interval is less than 10 cm; for a four-hour fitting interval the median error is less than 40 cm.

I. Introduction

The delay before the first navigation information becomes available to the user when a stand-alone GNSS receiver is turned on is called Time To First Fix (TTFF) and is at least $30\,\mathrm{s}$ but sometimes can be even several minutes. Most of this delay is due to the time it takes to receive the navigation data from the satellite. The message includes ephemeris data, information about the satellite position [1]. Satellite ephemeris data includes more information than a satellite orbit, but this paper focus only on orbit data of ephemeris.

One solution to reduce TTFF is to provide satellite ephemeris data to the navigation device using an alternative source. With this "assistance" data, the device needs to receive only the clock portion of the satellite's message, which takes considerably less time. The ephemeris assistance data also helps to identify the satellites that are in the service area, and this information also helps to reduce TTFF. A widely used method is to provide the assistance data over a data link such as an Internet connection. However, for devices without a data connection, and for users unwilling to pay for the assistance service, some other way of obtaining the assistance data is desired.

Techniques for computing the ephemeris data without a network connection, known as "self-assistance", have been implemented in commercial products and are outlined in the literature [2–4]. In our implementation of self-assistance, described in [5–7], the current satellite position is calculated by integrating the satellite's equation of motion forward starting from the position and velocity at an earlier time instant when the navigation device was operating. The equation of motion includes the gravitational forces of the Earth, the Sun and the Moon and a simple empirical model to account for the effects of the solar radiation pressure.

Because the orbit computation is time-consuming, it is done in advance, and predicted tracks over the next 4 or so days need to be stored for access when the device is next turned on. The orbit prediction software outputs a sequence of Cartesian coordinate points but this format would require a lot of storage space. The ephemerides format used in GPS, based on Kepler parameters and corrections, is much more compact. The GPS format is also convenient because it often the only format that is supported by GNSS hardware and software. For these reasons, conversion to GPS format is a required post-processing stage for the orbit prediction software.

In this paper, the conversion task is formulated as a nonlinear curve fitting problem. GPS ephemerides parameters are sought such that the GPS orbit best fits the predicted orbit track segment at a sequence of points. The paper is organised as follows. In section II the orbit prediction algorithm that produces the track segments is described. Section III reviews the GPS ephemerides format specification. The solution of the optimisation problem using the Levenberg-Marquardt algorithm is described in section IV. Section V presents results of tests using GPS and GLONASS tracks generated by our orbit prediction software, and orbit tracks obtained from precise ephemerides services.

II. Orbit prediction using force model

The orbit prediction algorithm used in this paper is based on the satellite's equation of motion. Because our dynamic model is intended to be used in a consumer grade positioning device that has modest computational resources, we have included only the four forces that have the greatest effect on the satellites orbit.

The acceleration of the satellite is described by the vector differential equation

$$\ddot{\mathbf{r}}_{\text{Sat}} = \mathbf{a}_{\text{Earth}} + \mathbf{a}_{\text{Moon}} + \mathbf{a}_{\text{Sun}} + \mathbf{a}_{\text{SRP}}, \qquad (1)$$

where $\mathbf{a}_{\text{Earth}}$, $\mathbf{a}_{\text{Moon}}$ and $\mathbf{a}_{\text{Sun}}$ are the accelerations caused by the gravitation of the Earth, the Moon and the Sun. The fourth term $\mathbf{a}_{\text{SRP}}$ is the acceleration caused by the solar radiation pressure.

Gravitation acceleration $\mathbf{a}_{\text{Earth}}$, which takes into account the nonuniform distribution of Earth mass, is obtained as the gradient of the geopotential. The geopotential is modelled as a linear combination of spherical harmonic functions. The first term of the spherical harmonic function series corresponds to the ideal potential of a spherical Earth, and the J_2-term takes

into account the flattening at the poles. Higher-order terms account for smaller asymmetries in the geopotential. Our model has terms up to the degree and order 8.

After the Earth's gravitation, the biggest acceleration components are $\mathbf{a}_{\text{Moon}}$ and $\mathbf{a}_{\text{Sun}}$. These can be calculated as functions of the satellite's positions using standard astronomy formulas.

Solar radiation pressure is caused by the satellite's absorption and re-emission of photons coming from the Sun. Our model for solar pressure [7] takes into account the Sun, the Earth and the satellite relative positions, as well as the shadow effects. The model includes empirical satellite-specific correction terms whose parameters have been estimated from historical orbit data.

The orbit is specified by the differential equation (1) and by initial conditions, that is, position and velocity at a given time instant. The position and velocity at the reference instant ('time of ephemeris") that are given in the broadcast ephemeris (BE) are not accurate enough for orbit prediction. Therefore, we compute initial conditions by locally fitting the BE track segment to the dynamic model; see [5] for details. The equations of motion are computed in an inertial coordinate frame and converted to Earth-Centered, Earth-Fixed (ECEF) reference frame. Parameters needed in the reference frame transformation called Earth Orientation Parameters (EOP) are also estimated in the initial condition fitting. The numerical solution of the differential equation initial value problem is then computed using a Runge-Kutta-Nyström method.

III. GPS ORBIT PARAMETERS

There are 16 GPS orbit parameters (Table I). These include the 6 Kepler parameters, which characterise the elliptical track of an idealised satellite subject only to the force of a radially symmetric geopotential. Two of the Kepler parameters, semimajor axis a and eccentricity e, describe the size and the shape of the orbit. Three of the parameters, inclination angle i, argument of perigee ω and longitude of the ascending node Ω, describe the orientation of the orbital plane. The sixth parameter, mean anomaly M, specifies the position of the satellite on the ellipse at the specific time t_{0e}, called the epoch [8].

The remaining GPS orbit parameters are correction terms to account for the other forces. Three rate parameters correct the linear change with time: correction to the mean motion Δn, rate of the inclination angle $\dot{i}$ and rate of the right ascension $\dot{\Omega}$. Three pairs (C_{*s}, C_{*c}) of amplitude terms correct the harmonic change of the satellite.

The GPS orbit parameters specify the satellite position as a function of time t. This position can be computed as follows. First, calculate the corrected mean anomaly using the formula

$$M = M_0 + \left(\sqrt{\frac{\mu}{a^3}} + \Delta n \right) t, \tag{2}$$

where $\mu = 3.986005 \cdot 10^{14}$ m^{3}s^{-2} is the product of Earth's mass and the universal gravitation constant [9, p. 105]. Next, solve the Kepler's equation $M = E - e\sin(E)$ using the Newton-Raphson iteration

$$E_{k+1} = E_k - \frac{E_k - e\sin(E_k) - M}{1 - e\cos(E_k)}, \quad E_0 = M. \tag{3}$$

The true anomaly (Fig. 1) is then given by

$$\nu = \text{atan}_2 \left(\sqrt{1 - e^2} \sin(E), \cos(E) - e \right). \tag{4}$$

Corrections to ascending node, argument of latitude, inclination angle and radius are

$$\Omega = \Omega_0 + (\dot{\Omega} - \Omega_e)t - \Omega_e t_{0e} \tag{5a}$$
$$u = \phi + C_{us}\sin(2\phi) + C_{uc}\cos(2\phi) \tag{5b}$$
$$i = i_0 + \dot{i}\, t + C_{is}\sin(2\phi) + C_{ic}\cos(2\phi) \tag{5c}$$
$$r = a(1 - e\cos(E)) + C_{rs}\sin(2\phi) + C_{rc}\cos(2\phi), \tag{5d}$$
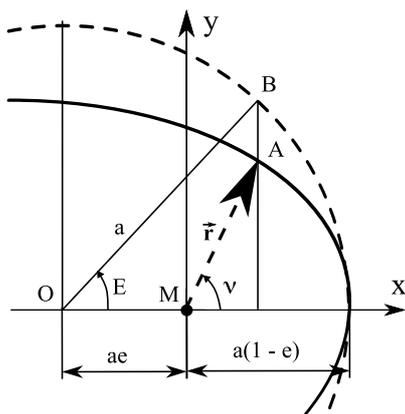

Fig. 1. True anomaly and eccentric anomaly. The center of Earth mass is located at M and the center of the ellipse at O. Point A is the actual position of the satellite in the elliptical orbit (solid curve) and B is the projection of the position to the O-centred circular orbit (dashed curve).
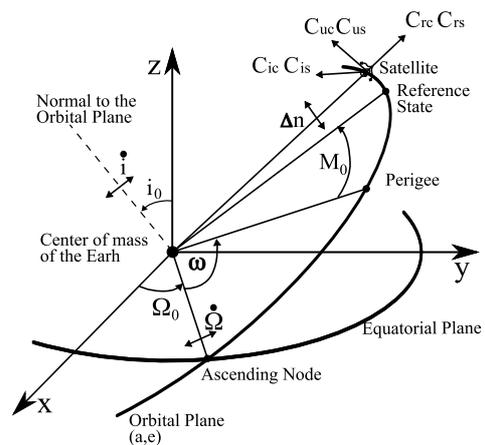

Fig. 2. GPS ephemeris orbit parameters effects to satellite orbit.

TABLE I
GPS ORBIT PARAMETERS

t_{0e}	Reference time of ephemeris
$\sqrt{a}$	Square root of semimajor axis length
e	Eccentricity
i_0	Inclination angle at time t_{0e}
Ω_0	Longitude of the ascending node
ω	Argument of perigee at time t_{0e}
M_0	Mean anomaly at time t_{0e}
Δn	Rate of change of longitude of ascending node
$\dot{i}$	Rate of change of inclination angle
$\dot{\Omega}$	Rate of change of longitude of ascending node
C_{is}, C_{ic}	Amplitudes of sine and cosine correction to inclination angle
C_{rs}, C_{rc}	Amplitudes of sine and cosine correction to orbit radius
C_{us}, C_{uc}	Amplitudes of sine and cosine correction to argument of latitude

where the argument of latitude is calculated from $\phi = v + \omega$ and angular speed of Earth's rotation is $\Omega_e = 7.2921151467 \cdot 10^{-5}$ rad/s [9, p. 105]. Finally, the satellite's position in the ECEF reference frame, using radius and angles from (5), is

$$\mathbf{r}_{\text{ECEF}} = r \begin{pmatrix} \cos(u)\cos(\Omega) + \sin(u)\cos(i)\sin(\Omega) \\ -\cos(u)\sin(\Omega) + \sin(u)\cos(i)\cos(\Omega) \\ -\sin(u)\sin(i) \end{pmatrix}. \quad (6)$$

IV. FITTING GPS ORBIT PARAMETERS TO A TRACK

Let the GPS orbit parameters be collected into the vector

$$\mathbf{x} = \begin{bmatrix} \sqrt{a}/28.86 \\ e/0.0095 \\ i_0/0.0179 \\ \Omega_0/0.0154 \\ \omega/0.0123 \\ M_0/0.0125 \\ \Delta n/5.302 \cdot 10^{-6} \\ \dot{i}/7.702 \cdot 10^{-6} \\ \dot{\Omega}/6.623 \cdot 10^{-6} \\ C_{uc}/0.0212 \\ C_{us}/0.0156 \\ C_{rc}/5.621 \cdot 10^{5} \\ C_{rs}/4.147 \cdot 10^{5} \\ C_{ic}/0.0306 \\ C_{is}/0.0221 \end{bmatrix}. \quad (7)$$

The scaling ensures that numerical values are generally in the range $|x_i| \leq 1$. Let $\mathbf{M}$ denote the function that maps time t, reference time t_{0e} and orbit parameters $\mathbf{x}$ to satellites position in the ECEF reference frame using equations (2-6):

$$\mathbf{M}(t, t_{0e}, \mathbf{x}) = \mathbf{r}(t), \quad t \in \left(t_{0e} - \frac{1}{2}\Delta t, t_{0e} + \frac{1}{2}\Delta t \right). \quad (8)$$

Constant Δt is the length of the fitting window. Let the length-$3m$ vector $\mathbf{y}$ denote satellite positions (e.g. obtained from the orbit prediction software) at the m time points in vector $\mathbf{t}$, and let $\mathbf{M}(\mathbf{t}, t_{0e}, \mathbf{x})$ be the corresponding GPS orbit positions given

by (8). Denoting the residual as $\mathbf{f}(\mathbf{x}) = \mathbf{y} - \mathbf{M}(\mathbf{t}, t_{0e}, \mathbf{x})$, the non-linear minimisation problem can be written as

$$\underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{f}(\mathbf{x})\|^2 = \underset{\mathbf{x}}{\operatorname{argmin}} \frac{1}{2}\mathbf{f}(\mathbf{x})^T\mathbf{f}(\mathbf{x}) = \underset{\mathbf{x}}{\operatorname{argmin}} F(\mathbf{x}), \quad (9)$$

where F is the minimisation problem's objective function.

The nonlinear least squares minimisation problem (9) is solved using the Levenberg-Marquardt (LM) method [10]. In each iteration of this method, one computes the objective function gradient

$$\nabla F(\mathbf{x}) = F'(\mathbf{x})^T = \mathbf{J}(\mathbf{x})^T \mathbf{f}(\mathbf{x}), \quad (10)$$

where $\mathbf{J}(\mathbf{x})$ is the Jacobian matrix of the residual function. The Jacobian matrix can be computed numerically using the central difference formula [10, p. 168].

The Hessian matrix of the objective function is

$$\begin{aligned} \mathbf{H}(\mathbf{x}) = F''(\mathbf{x}) &= \mathbf{J}(\mathbf{x})^T \mathbf{J}(\mathbf{x}) + \sum_{i=1}^{3m} \left(f_i(\mathbf{x}) \mathbf{f}''(\mathbf{x}) \right) \\ &= \mathbf{J}(\mathbf{x})^T \mathbf{J}(\mathbf{x}) + \mathbf{S}(\mathbf{x}), \end{aligned} \quad (11)$$

In the Levenberg-Marquardt method the approximation $\mathbf{S}(\mathbf{x}) \approx \mu \mathbf{I}$ is used, where $\mu \geq 0$ is a damping parameter.

In the minimisation problem, we have to find the stationary point for objective function. At a local minimum, the derivative of the objective function is zero and the Hessian matrix is positive definite. The objective function derivative can be approximated in the neighbourhood of the point $\mathbf{x}$ using the Taylor polynomial

$$F'(\mathbf{x} + \Delta\mathbf{x})^T \approx F'(\mathbf{x})^T + F''(\mathbf{x})\Delta\mathbf{x}. \quad (12)$$

If vector $\mathbf{x}$ is the current approximation to the minimum, we can find better approximation $\mathbf{x} + \Delta\mathbf{x}$ by setting the Taylor approximation (12) to zero and solving for $\Delta\mathbf{x}$. By using the formulas for gradient (10) and Hessian matrix (11) we get

$$\left(\mathbf{J}(\mathbf{x})^T \mathbf{J}(\mathbf{x}) + \mu \mathbf{I} \right) \Delta\mathbf{x} = -\mathbf{J}(\mathbf{x})^T \mathbf{f}(\mathbf{x}^k). \quad (13)$$

If the damping parameter μ is large the Levenberg-Marquardt method resembles the steepest decent method. This is good if the current $\mathbf{x}$ is far from the solution. On the other hand, if μ is small, the method resembles the Gauss-Newton method, whose rate of convergence close to the minimum is faster than steepest descent. During iteration, the damping parameter is continually updated using the gain ratio

$$R = \frac{\mathbf{F}(\mathbf{x} + \Delta\mathbf{x}) - \mathbf{F}(\mathbf{x})}{\mathbf{G}(\mathbf{x} + \Delta\mathbf{x}) - \mathbf{G}(\mathbf{x})}, \quad (14)$$

where $\mathbf{G}(\mathbf{x}) = \frac{1}{2}\|\mathbf{f}(\mathbf{x}) + \mathbf{f}'(\mathbf{x})^T\Delta\mathbf{x}\|^2$ [11]. The gain ratio describes how well the Gauss-Newton algorithm performs at our current point. The damping ratio update formula is

$$\mu_{k+1} = \begin{cases} \frac{1}{2}\mu_k, & \text{if } R_k \geq 0.75, \\ \mu_k, & \text{if } 0.25 < R_k < 0.75, \\ 2\mu_k, & \text{if } R_k \leq 0.25. \end{cases} \quad (15)$$

The LM iteration's initial point $\mathbf{x}_0$ is set as follows. The 6 Kepler parameters at each fitting time point are determined by

Algorithm 1 Levenberg-Marquardt algorithm

Require: $cont = \textbf{true}$; $k = 0$; ε_1; ε_2; ε_3; $k_{\max}$;
$\quad \mathbf{x} = \mathbf{x}_0$; $\mathbf{A} = \mathbf{J}(\mathbf{x})^T \mathbf{J}(\mathbf{x})$; $\mathbf{v} = \mathbf{J}(\mathbf{x})^T \mathbf{f}(\mathbf{x})$; $\mu = 1$;
$\quad$**while** $(cont$ **and** $k < k_{\max})$ **do**
$\quad\quad k = k + 1$;
$\quad\quad$**Solve** $(\mathbf{A} + \mu \mathbf{I}) \Delta \mathbf{x} = \mathbf{v}$;
$\quad\quad \mathbf{x}_{\text{new}} = \mathbf{x} + \Delta \mathbf{x}$;
$\quad\quad$**if** $(\|\Delta \mathbf{x}\| \leq \varepsilon_1 \|\mathbf{x}\|$ **or** $\|F(\mathbf{x}_{\text{new}}) - F(\mathbf{x})\| \leq \varepsilon_2$ **then**
$\quad\quad\quad cont = \textbf{false}$;
$\quad\quad$**else**
$\quad\quad\quad R = (\mathbf{F}(\mathbf{x}_{\text{new}}) - \mathbf{F}(\mathbf{x}))/(\mathbf{G}(\mathbf{x}_{\text{new}}) - \mathbf{G}(\mathbf{x}))$;
$\quad\quad\quad$**if** $R \geq 0.75$ **then**
$\quad\quad\quad\quad \mu = \frac{1}{2}\mu$;
$\quad\quad\quad$**else if** $R \leq 0.25$ **then**
$\quad\quad\quad\quad \mu = 2\mu$;
$\quad\quad\quad$**end if**
$\quad\quad\quad \mathbf{x} = \mathbf{x}_{\text{new}}$;
$\quad\quad\quad \mathbf{A} = \mathbf{J}(\mathbf{x})^T \mathbf{J}(\mathbf{x})$; $\mathbf{v} = \mathbf{J}(\mathbf{x})^T \mathbf{f}(\mathbf{x})$;
$\quad\quad$**end if**
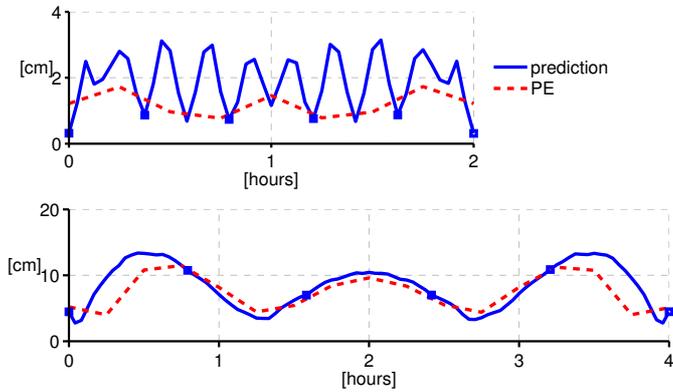$\quad$**end while**
$\quad$**return x**

Fig. 3. Median of fitting error to predicted and precise (PE) orbits for GPS satellites using fitting lengths 2 and 4 h.

the ECEF coordinates using standard formulas, see e.g. [12]. The components (a, e, ω) are set to the average of m values and the components (i_0, Ω_0, M_0) are set to the value at t_{0e}. The rate terms $(\dot{i}, \dot{\Omega}$ and $\Delta n)$ are obtained by fitting quadratic polynomials to the sequence of $(i, \Omega, \Delta n)$. The remaining components of $\mathbf{x}_0$ (sine and cosine corrections) are set to zero.

V. Tests

We tested the method by parametrising all the consecutive Δt-length segments in 10 nonoverlapping 4-day orbit predictions in GPS weeks 1625–1635. Orbits of all usable GPS and GLONASS satellites are predicted. The curve fit is based on $m = 6$ equispaced satellite positions and evaluated using positions at 2.5-minute intervals.

Medians of fitting error (Euclidean distance between positions) over the fitting window for GPS and GLONASS
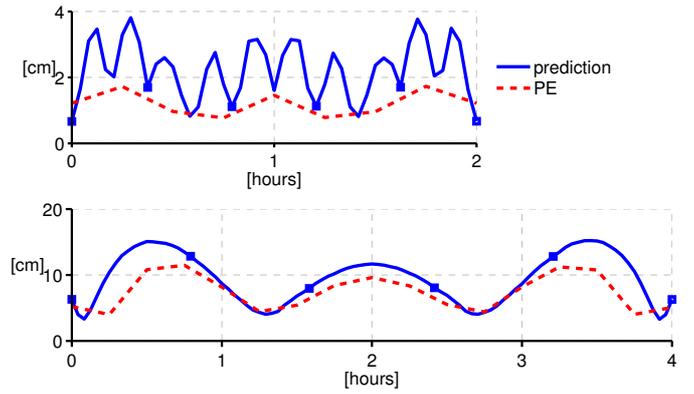

Fig. 4. Median of fitting error to predicted and precise (PE) orbits for GLONASS satellites using fitting lengths 2 and 4 h.
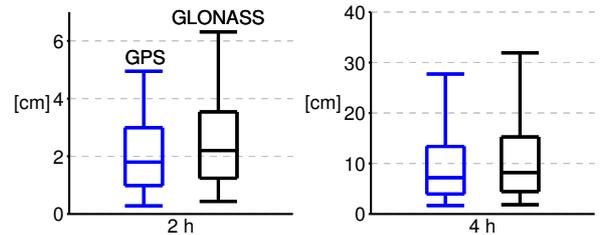

Fig. 5. Fitting error to precise (PE) orbit and predicted orbit for GPS and GLONASS satellites.

satellites are shown in Figures 3–4, and fitting errors are summarised as box plots in Figure 5. Tests show that median error in a 2 hour fitting is about 2 cm and the 95%-quantile error less than 10 cm in both GNSS systems. In the 4 hour fitting the median and 95% errors are 10 cm and 40 cm.

We also tested the method by parametrising the same segments but using precise ephemeris (PE) GPS and GLONASS tracks from IGS. The median fitting errors are shown in Figures 3–4 as dashed lines. The results are similar to those obtained for predicted tracks; the error curves are smoother because the reference points are more widely spaced (15 min).

Additional test results are presented in [13].

VI. Conclusion

In this paper, we presented a method to fit GPS-standard orbit parameters to the GNSS satellite orbits. The parametrisation is intended as a post-processor for orbit prediction algorithms in self-assistance technology, to ease data processing and reduce storage needs. The method achieves a median error of 10 cm over a 4 h fitting window, which is good enough for representing orbit prediction tracks. These tracks currently have median errors on the order of 10 m for 1-day prediction. For applications having more demanding accuracy requirements, e.g. PE tracks, a shorting fitting window can be used: with a 2 h window the median error is under 3 cm.

The conversion algorithm presented here can in principle be applied to fit satellite orbit segments to different ephemeris

formats used in other GNSS systems. For example, the GLONASS satellites' broadcast ephemerides are delivered in a format including the position and the velocity at one epoch. In addition, an acceleration vector is given, which can be used to propagate the trajectory about ± 15 minutes from the reference epoch [14]. The need for conversion between ephemeris formats can be expected to grow as software and hardware for other GNSS systems becomes more prevalent.

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