Danai Skournetou

Mitigation of Dominant Channel Propagation Effects in GNSS-based Positioning

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Danai Skournetou

**Mitigation of Dominant Channel Propagation Effects in GNSS-based Positioning**

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Abstract

Global Navigation Satellite Systems (GNSSs) have significantly changed the landscape of positioning. The development of the first GNSS, Global Positioning System (GPS), has been a major milestone in the history of navigation; not only because of its technological superiority over the existing positioning technologies of that time but also because it has turned the access to positioning signals from a privilege of a few into a public utility. In addition, the modern advancements in mobile devices have revolutionized the way positioning information is used; besides navigation, location information is used in a wide spectrum of applications such as to track people or goods, authenticate individuals, facilitate fleet management, optimize farming activities, find the nearest restaurant, assist elderly or disabled people, etc. While GNSS-based positioning has several advantages over other positioning technologies, such as global coverage and high availability, it also has its challenges. Most of these are related to the physical channel which consists of various error sources that affect the quality of the received satellite signals, and degrade the receiver’s positioning performance.

In this dissertation, the focus is placed on the dominant propagation effects that take place in GNSS-based positioning. Precisely, these are the (1) ionospheric and the (2) multipath propagation effects. The former depends on the electron content along signal’s route and the latter, on the arrangement of the objects surrounding the GNSS receiver. For example, both effects introduce delays which make the satellite-receiver distance to appear longer than in reality and consequently, decrease the positioning accuracy. To mitigate such unwanted effects, an algorithmic approach is followed. Specifically, this dissertation studies the impact of multipath errors in the estimation of the ionosphere-corrected range and suggests a new algorithm for estimating the ionosphere-corrected range in dual-frequency receivers. The proposed algorithm achieves higher range estimation accuracy than alternative dual-frequency methods in the presence of multipath errors without raising the im-
plementational complexity. Furthermore, this dissertation proposes new code and carrier tracking algorithms for improving directly the receiver’s tracking accuracy under multipath propagation effects. These methods yield better code and carrier tracking performance in exchange for higher complexity. Moreover, this thesis suggests novel algorithms for distinguishing two Carrier to Noise Ratio (CNR) ranges, such as those characterizing indoor and outdoor environments, and for estimating the CNR. Both methods are based on the level crossing rate information of the averaged cross-correlation function and can be used to improve the receiver’s code or carrier tracking performance, for example by adjusting certain parameters or switching between different algorithms.

This dissertation is a collection of nine publications which include detailed descriptions of the proposed algorithms, comparison with state of the art methods and performance analysis. In addition, this thesis contains an introductory part which provides readers with a beginning knowledge on the principles of GNSS-based positioning, the error sources that degrade receiver’s positioning performance and a literature overview on the methods developed for mitigating the dominant environmental effects caused by ionosphere and multipath-propagation.
Preface

Most of the research work that led to this dissertation has been carried out in the Department of Communications Engineering, Tampere University of Technology (TUT), Finland, during the years 2007-2011. Part of my research work has been also performed at the University of California, Los Angeles (UCLA), U.S., where I had been a visiting researcher for a six-month period in year 2008.

The work presented in this dissertation would not have this form without the support of many people. I would like to take this opportunity and thank all those who contributed to the successful completion of this dissertation. First of all, I would like to express my deepest gratitude to my supervisor, Adjunct Professor, Dr. Elena Simona Lohan, for her dedication, continuous guidance and encouragement. I would also like to thank her for being a wonderful role model because besides her being a great supervisor and scientist, she has been always a warm and caring person. I am also very grateful to Prof. Markku Renfors who gave me the opportunity to work in this Department and to get involved in various research projects. In addition, I would like to thank him for acting as my supervisor in the period when Simona was on maternity leave and for his continuous and invaluable support through all of these years. I would also like to express my appreciation to Prof. Ali H. Sayed from the Faculty of Electrical Engineering in UCLA for his hospitality and insightful guidance during my research visit.

I am especially indebted to the reviewers of this dissertation, Dr. Armin Dammann from the German Aerospace Center and Dr. Lauri Wirola from Nokia for agreeing to review my dissertation and for their constructive comments. I am also very grateful to Assistant Professor, Dr. Francesco Benedetto from "Roma TRE" University for agreeing to act as the opponent in the public defense of my dissertation.

One main reason why the Department of Communications Engineering has been such a pleasant place to work is because of its people. I would like
to thank my office mates, friends and colleagues Toni Levanen, Yaning Zou, Stanislav Nonchev, Tero Isotalo and Ville Syrjälä for our numerous discussions and for sharing both the good and the difficult moments. I am also very thankful to the present and the former members of our team, Zahidul Bhuiyan, Alexandru Rusu Casandra, Nazmul Islam, Elina Laitinen, Farzan Samad, Shweta Shrestha, Bashir Siddiqui, Jukka Talvitie, Hu Xuan and Jie Zhang for the encouraging working atmosphere as well as for continuous helpfulness. Moreover, I would like to devote special thanks to the Head of the Department, Prof. Mikko Valkama, as well as to Tarja Erälaukko, Daria Ilina, Mariana Jokila, Saara Kallio, Sari Kinnari, Pertti Koivisto, Antti Niemistö, Elina Orava, Ulla Siltaloppi, Jani Tuomisto and Kirsi Viitanen for their kindness and help with practical matters.

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Finally, I would like to thank my family especially, my mother Vicky, my father George, my sister Julia and my aunt Despina for their unconditional support and love. I am also very thankful to my friends in Greece who didn’t let the distance separate us and my friends in Finland especially, Petros, Sofoklis, Noora and Olga for making Finland feel like home. Last but not least, I would like to thank my loved, Artem, for truly supporting me, helping me, advising me, believing in me, caring and loving me.

Tampere, November 2011

Danai Skournetou
# Contents

Abstract

Preface

Table of Contents

List of Publications

List of Abbreviations

1 Introduction

1.1 Background and Motivation

1.2 Research Objectives and Outcomes

1.3 Thesis Outline

2 History of Navigation and GNSS

2.1 Before GNSS

2.2 GPS: The Beginning of GNSS Era

2.3 Current and Future Global Navigation Satellite Systems (GNSS)

2.3.1 GLONASS

2.3.2 Galileo

2.3.3 CNSS and QZSS

3 Fundamentals of GNSS-based Positioning and GNSS Receiver Operation

3.1 Trilateration Principle

3.2 Receiver front-end and ADC

3.3 Signal Acquisition

3.4 Signal Tracking

3.4.1 Code Tracking
3.4.2 Carrier Tracking ........................................ 24
3.5 Navigation data extraction ............................... 25
3.6 Position computation and geographic coordinate conversion . 25

4 Error Sources in GNSS .................................... 27
  4.1 Space segment-related errors .......................... 27
  4.2 Propagation channel-related errors ................. 29
     4.2.1 Ionosphere ........................................... 29
     4.2.2 Troposphere .......................................... 31
     4.2.3 Multipath propagation ............................ 32
  4.3 Receiver-related error sources ....................... 33

5 Methods for Mitigating Dominant Propagation Effects in GNSS-based Positioning .............................. 35
  5.1 Ionosphere Mitigation Techniques .................... 35
     5.1.1 Single-frequency approach ....................... 35
     5.1.2 Dual-frequency approach ......................... 37
  5.2 Multipath Mitigation Techniques ...................... 38
     5.2.1 Signal tracking methods ......................... 38
     5.2.2 C/N0 estimation methods ....................... 41

6 Overview of Publications .................................. 43
  6.1 Description of Publications ......................... 43
  6.2 Author’s Contribution ................................ 47

7 Conclusions .................................................. 49

Bibliography .................................................. 53

Publications .................................................. 67
List of Publications

This thesis consists of nine publications, which in the text are referred to as Publications [P1]-[P9]. The first three publications ([P1]-[P3]) are related to the research on ionospheric effects, the next four publications deal with the multipath propagation effects ([P4]-[P7]) and the last two publications focus on the estimation of the carrier to noise ratio spectral density ([P8],[P9]). Publications [P1], [P7] and [P9] are articles published in a journal or magazine, while the rest are conference publications.


## List of Abbreviations

This is a list of the most important and recurrently appearing abbreviations in this thesis.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>2D</td>
<td>Two-Dimensional</td>
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<tr>
<td>3D</td>
<td>Three-Dimensional</td>
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<tr>
<td>Acf</td>
<td>AutoCorrelation Function</td>
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<td>ADC</td>
<td>Analogue-to-Digital Converter</td>
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<tr>
<td>BFC</td>
<td>Brute Force Constraint</td>
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<tr>
<td>BOC</td>
<td>Binary Offset Carrier</td>
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<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>C/A</td>
<td>Coarse/Acquisition</td>
</tr>
<tr>
<td>CBOC</td>
<td>Composite Binary Offset Carrier</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>CLS</td>
<td>Constrained Least Squares</td>
</tr>
<tr>
<td>CNSS</td>
<td>China Navigation Satellite System</td>
</tr>
<tr>
<td>CRLB</td>
<td>Cramer Rao Lower Bound</td>
</tr>
<tr>
<td>CS</td>
<td>Commercial Service</td>
</tr>
<tr>
<td>DLL</td>
<td>Delay Locked Loop</td>
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<tr>
<td>E</td>
<td>Early</td>
</tr>
<tr>
<td>EC</td>
<td>European Commission</td>
</tr>
<tr>
<td>ECEF</td>
<td>Earth Centered Earth Fixed</td>
</tr>
<tr>
<td>EML</td>
<td>Early Minus Late</td>
</tr>
<tr>
<td>ESA</td>
<td>European Space Agency</td>
</tr>
<tr>
<td>FEC</td>
<td>Forward Error Correction</td>
</tr>
<tr>
<td>FLL</td>
<td>Frequency Locked Loop</td>
</tr>
<tr>
<td>FOC</td>
<td>Full Operational Capability</td>
</tr>
<tr>
<td>GCS</td>
<td>Ground Control Segment</td>
</tr>
<tr>
<td>GDOP</td>
<td>Geometric Dilution of Precision</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>--------------</td>
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</tr>
<tr>
<td>GEO</td>
<td>Geostationary Equatorial Orbit</td>
</tr>
<tr>
<td>GLONASS</td>
<td>Global Orbiting NAVigation Satellite System</td>
</tr>
<tr>
<td>GMS</td>
<td>Galileo Mission Segment</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<tr>
<td>GSA</td>
<td>Galileo Supervisory Authority</td>
</tr>
<tr>
<td>GST</td>
<td>Galileo System Time</td>
</tr>
<tr>
<td>GTRF</td>
<td>Galileo Terrestrial Reference Frame</td>
</tr>
<tr>
<td>ICA</td>
<td>Ionospheric Correction Algorithm</td>
</tr>
<tr>
<td>ICD</td>
<td>Interface Control Document</td>
</tr>
<tr>
<td>IF</td>
<td>Intermediate Frequency</td>
</tr>
<tr>
<td>IOV</td>
<td>Initial Operational Capability</td>
</tr>
<tr>
<td>IRI</td>
<td>International Reference Ionosphere</td>
</tr>
<tr>
<td>ITRF</td>
<td>International Terrestrial Reference Frame</td>
</tr>
<tr>
<td>L</td>
<td>Late</td>
</tr>
<tr>
<td>LBS</td>
<td>Location-Based Service</td>
</tr>
<tr>
<td>LCR</td>
<td>Level Crossing Rate</td>
</tr>
<tr>
<td>LCRE</td>
<td>Level Crossing Rate Estimation</td>
</tr>
<tr>
<td>LORAN</td>
<td>LOng-range Aid to Navigation</td>
</tr>
<tr>
<td>LOS</td>
<td>Line-Of-Sight</td>
</tr>
<tr>
<td>LS</td>
<td>Least Squares</td>
</tr>
<tr>
<td>MBOC</td>
<td>Multiplexed Binary Offset Carrier</td>
</tr>
<tr>
<td>MCS</td>
<td>Master Control Station</td>
</tr>
<tr>
<td>MEE</td>
<td>Multipath Error Envelope</td>
</tr>
<tr>
<td>MEO</td>
<td>Medium Earth Orbit</td>
</tr>
<tr>
<td>MF</td>
<td>Matched Filter</td>
</tr>
<tr>
<td>MGD</td>
<td>Multiple Gate Delay</td>
</tr>
<tr>
<td>mPOCS</td>
<td>modified Projection Onto Convex Space</td>
</tr>
<tr>
<td>MTLL</td>
<td>Mean Time To Lose Lock</td>
</tr>
<tr>
<td>NEML</td>
<td>Narrow Early Minus Late</td>
</tr>
<tr>
<td>NLOS</td>
<td>Non-Line-Of-Sight</td>
</tr>
<tr>
<td>OS</td>
<td>Open Service</td>
</tr>
<tr>
<td>P</td>
<td>Prompt</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase Locked Loop</td>
</tr>
<tr>
<td>POCS</td>
<td>Projection Onto Convex Space</td>
</tr>
<tr>
<td>PPS</td>
<td>Precise Positioning Service</td>
</tr>
<tr>
<td>PRN</td>
<td>Pseudo-Random Noise</td>
</tr>
<tr>
<td>PVT</td>
<td>Position Velocity Time</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
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<tr>
<td>---------</td>
<td>-----------------------------------------------</td>
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<tr>
<td>QZSS</td>
<td>Quasi Zenith Satellite System</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>RHCP</td>
<td>Right Hand Circular Polarization</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root Mean Square Error</td>
</tr>
<tr>
<td>SA</td>
<td>Selective availability</td>
</tr>
<tr>
<td>SAR</td>
<td>Search And Rescue</td>
</tr>
<tr>
<td>SoL</td>
<td>Safety-of-Life</td>
</tr>
<tr>
<td>SPS</td>
<td>Standard Positioning Service</td>
</tr>
<tr>
<td>TEC</td>
<td>Total Electron Content</td>
</tr>
<tr>
<td>TECU</td>
<td>Total Electron Content Unit</td>
</tr>
<tr>
<td>TMBOC</td>
<td>Time Multiplexed Binary Offset Carrier</td>
</tr>
<tr>
<td>TOA</td>
<td>Time Of Arrival</td>
</tr>
<tr>
<td>TTC</td>
<td>Telemetry, Tracking and Control Stations</td>
</tr>
<tr>
<td>UTC</td>
<td>Coordinated Universal Time</td>
</tr>
<tr>
<td>WCDMA</td>
<td>Wideband Code Division Multiple Access</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Background and Motivation

In 1991, Mark Weiser stated that "The most profound technologies are those that disappear. They weave themselves into the fabric of everyday life until they are indistinguishable from it." [1]. This statement depicted his personal vision of ubiquitous computing, where the influence of technology is all-pervasive. According to the author of this thesis, this metaphorical statement successfully illuminates the most profound technologies but not only them per se. It also describes the most critical technologies; those that people depend on and whose abnormal or interrupted operation has extensive impact. Every time a disappearing technology fails, it unveils itself and spreads confusion. The author believes that satellite-based navigation is fast becoming such a profound disappearing technology.

A Global Navigation Satellite System (GNSS) is a combination of different technologies into a complex platform that (1) provides precise timing information and (2) enables users to compute their location on the Earth. When the first GNSS was developed, known as Global Positioning System (GPS), its purpose was to augment U.S. military weaponry in times of war. During the sixteen years passed after the GPS became operational, the GNSS landscape has changed significantly. In particular, the year 2000 was a decisive milestone in GNSS history when President Bill Clinton ordered Selective Availability (SA) to be turned off. SA was a feature that allowed GPS operatives to degrade the quality of the GPS civil signal and limit the horizontal positioning accuracy to approximately 100 meters (in 95% of the cases). This signal degradation was employed as a measure to protect the security interests of the
U.S. and its allies by globally denying the full accuracy of the civil system to potential adversaries [2]. When SA was deactivated, the positioning accuracy increased by one magnitude order and a new era begun where GNSS-based positioning was useful not only for the military but also for civilians.

The earliest mass-market applications, those developed while SA was active, were mainly related to positioning in sea or directed to hikers. After SA was turned off, road vehicle applications became possible. In these, two major tasks are performed: (1) positioning (where am I?) and (2) navigation (how do I reach my destination?). Later on, the integration of Assisted-GPS chipsets into mobile phones allowed the expansion of the Location-based Services (LBS) market by reaching out to new user segments and innovative applications. According to the author’s opinion, vehicle and mobile applications are the ones that helped to cross the chasm between the early adopters and the early majority. Moreover, the establishment of GPS receiver as a standard feature of every smartphone is leading towards turning GNSS into a disappearing technology.

Nowadays, the variety of GNSS-based applications has greatly grown and positioning information is used not only for navigation but also for tracking objects or other people. Both end-user and professional markets are established and the sectors to which new applications are targeted include among others, entertainment, health and safety, security, agriculture, road-toll charging, maritime, etc. According to the 2010 GNSS Market Monitoring report published by The European GNSS Agency (GSA), the global market for GNSS will grow significantly over the next decade, reaching some 244 billion Euros for the enabled GNSS market in 2020. Mobile location based services (LBS) and road will be the market sectors with the highest revenue generation.

The figures, forecasts and trends drawn in the various media outlets indicate that GNSS-based positioning is potentially a disappearing technology. It will enable positioning information to be attached in a multitude of everyday activities and will disappear from people’s attention. Nonetheless, there is a number of error sources that affect the satellite signals and degrade their quality. This degradation in some cases may be large enough to cause high positioning errors or to prohibit positioning. When a user depends on GNSS technology for positioning, and GNSS fails, the user experiences the limitations of the technology and the extent of the effects varies according to the degree the technology in use has penetrated its life. This unwanted situation is what has motivated the author and the work presented in this thesis.
1.2 Research Objectives and Outcomes

Among the various error sources that affect the satellite signals, ionosphere and multipath cause the largest propagation-related errors in the estimation of the satellite-receiver range \[3\]. Ionosphere is part of the atmosphere and is located between 50 and 1000 km above the earth surface. It consists of charged particles, the content of which varies according to various temporal and spatial parameters, such as time of the day, season, height, geomagnetic latitude, etc. When the satellite signal propagates through ionosphere, it slows down. In addition, ionosphere is a dispersive medium and as such, signals with different carrier frequencies are affected differently.

Multipath is an environment-dependent phenomenon in which the received signal is a superposition of two or more paths due to e.g. reflection from buildings. Multipath propagation effects are more complex than ionospheric ones due to the fact that the dependency on time and space is stronger. Both ionosphere and multipath propagation effects may introduce large delays in the received signal. The research target of the author has been to develop methods to cope with these propagation effects that may significantly degrade the quality of the signal and consequently of the positioning.

The GNSS of interest is Galileo, which is funded by European Commission (EC) and is expected to be fully operable by 2019. More precisely, the signals of interest are the future Galileo Open Service (OS) signals which will be made freely available to the public. This research is focusing on mass-market Galileo receivers and the objectives are to (1) improve the range estimation accuracy of multi-frequency methods in the presence of ionospheric and multipath propagation errors and to (2) increase the tracking accuracy of single-frequency methods in the presence of multipath propagation errors.

The main outcomes of this dissertation include

(1) Development of a new dual-frequency method, called Brute Force Constraint (BFC), for estimating the satellite-receiver range in the presence of ionospheric and multipath propagation delays ([P1]).

(2) Design of new code and carrier tracking algorithms for mitigating multipath propagation effects. More precisely, the author developed (i) one code delay estimation algorithm, called DiscTracker, which combines features of feedback and feedforward techniques ([P5]) and (ii) one modified Projection Onto Convex Sets (mPOCS) algorithm which estimates jointly the carrier phase and code delay of the line-of-sight signal ([P6]).
Development of novel methods for Carrier-to-Noise-Ratio (CNR) determination which among other, can be used to improve the receiver’s tracking performance. Specifically, this dissertation proposes (i) a new method for distinguishing between indoor and outdoor environments ([P8]), as well as (ii) a very precise CNR estimator ([P9]). In addition to the new algorithms this dissertation contributes to the state of knowledge by providing

Impact analysis of ionospheric and multipath propagation delays on range estimation in single and multi-frequency receivers ([P1],[P2],[P3]).

Impact analysis of multipath propagation effects on the performance of state-of-the-art and proposed-by-the-author tracking algorithms ([P4],[P5],[P6],[P7]).

1.3 Thesis Outline

The remainder of this dissertation is organized as follows: Chapter 2 presents briefly the history of navigation from the early times of exploration until the development of the first GNSS and overviews current and future GNSS where particular focus is put on Galileo, Europe’s future own GNSS. Chapter 3 explains the principles of satellite-based positioning and describes the basic operations of a GNSS receiver with emphasis on the baseband functions, to which a large part of author’s work is devoted. Chapter 4 gives an overview of the various error sources that affect the satellite signals and degrade their quality. Chapter 5 discusses the various state-of-the-art methods for mitigating the dominant effects of ionosphere and multipath propagation and positions the algorithms proposed in this dissertation with respect to the state-of-the-art. Finally, Chapter 7 summarizes the main research outcomes and draws the conclusions.

A brief description of the author’s publications and her contribution to those is given in Chapter 6. The compilation of the publications included in this thesis can be found after the Bibliography.
Chapter 2

History of Navigation and GNSS

2.1 Before GNSS

Navigation is often defined as the process of going from a point A to a point B, assuming that the one who navigates lacks all the necessary information needed to reach destination and therefore, an external aid or source of information is required. This condensed definition deliberately omits the transportation mean since in modern times navigation is not restricted to a particular environment; instead, it is performed on land, sea, air or space and via different means of transport. In the earlier years of the human history, navigation was mostly exercised in the sea and this is reflected in the word etymology: navigation originates from the Latin word "navigus" which means ship. A brief look into the history is enough to reveal that navigation is closely connected with the will of human beings to explore unknown places, either by necessity (for example, in order to discover new trade routes) or by internal forces weaved deep into the fabric of human nature, such as curiosity. Especially, the period between the 15th and the 17th century is known as the Age of Discovery, characterized by major discoveries, such as those led by Christopher Columbus and Ferdinand Magellan.

One of the biggest breakthroughs in the history of navigation was set by the invention of the magnetic compass. While the opinion among historians as who invented compass varies, many believe that it was invented in China. Interestingly, Chinese used compass first not as a navigation tool but as a mean to bring harmony into their environments in accordance with the geomantic
principles of feng shui. Throughout ages, people also navigated by sight of landmarks or land characteristics complemented with a variety of methods developed through years.

Some of the most common navigational techniques used in the past are dead reckoning and celestial navigation. The main principle of dead reckoning (a.k.a. deduced reckoning) is that a destination can be reached if one starts from a known point on the map and holds the speed and direction according to the travel plan. In celestial navigation, the angle between the horizon and a celestial body (e.g., sun, moon or a star) is measured with the help of an appropriate instrument (e.g., quadrant, sextant, octant). Then, using the known projected geographical position of two celestial bodies (derived from tables), it is possible to compute one's own position.

With the advent of modern times and the introduction of wireless communication in the end of 19th century, new and more sophisticated navigational methods were developed. A major step forward for the navigation science was the establishment of radio beacons. More precisely, a radio beacon is essentially a transmitter placed at a known location (normally in the sea or in the shore) which broadcasts a continuous periodic signal at a fixed frequency and which is used to determine one’s position and navigate accordingly.

LOng-range Aid to Navigation (LORAN) was a single-pulse system developed by the MIT Radiation Lab [4]. LORAN was first established during World War II under a secret program to provide the Allies with a reliable and accurate means of navigation at sea in any weather [5]. Later, LORAN was operated by the Coast Guard for navigating ships at long distances. The position was determined by measuring the difference between the arrival times of pulses transmitted by several time-synchronized beacons and the accuracy depended on time of day, weather and relative geometry of transmitting beacons. Expansion of the original LORAN concept to meet operational requirements for greater accuracy and service coverage has resulted in the development of three related systems now designated as LORAN-A, LORAN-B, and LORAN-C [6]. LORAN-C was the latest and the most accurate version, serving the U.S Coast Guard for more than two decades. However, as a result of technological advancements LORAN-C became an antiquated system no longer required by the armed forces, the transportation sector or the nations security interests and its operation was ceased in February, 2010 [7].
2.2 GPS: The Beginning of GNSS Era

One of the great technological advancements that led to the disestablishment of LORAN-C is the Global Positioning System (GPS). In 1973, Navy and Air Force programs, directed by U.S. government, were combined to form the Navigation Technology Program which acted as the basis for the development of GPS. In April 1995, the U.S. Air Force Space Command formally declared the GPS as a system with Full Operational Capability (FOC). GPS is a Global Navigation Satellite System (GNSS) that comprises of three segments (see Fig. 2.1): (a) Space segment, (b) Ground segment and (c) User segment.

GPS space segment consists of 24 satellites, equally distributed in six orbital planes characterised by an inclination angle of 55 degrees. The ground segment includes the Master Control Station (MCS), five monitor stations and three ground antennas. Each station has several GPS receivers that continuously track the visible GPS satellites. The monitor stations passively track all satellites in view, accumulating ranging data which is processed at the MCS and used to determine satellite orbits and to update each satellite’s navigation message. The updated information is then transmitted to each satellite via the ground antennas. The user segment consists of the GPS receiver equipment that is used to compute user’s Position, Velocity and Time (PVT) [8].

GPS currently offers two types of services: a Standard Positioning Service (SPS) for public use and an encoded Precise Positioning Service (PPS), dedicated solely for military use. Each GPS satellite transmits two types of signals

Figure 2.1: GNSS segments.
and utilizes two frequency bands: Coarse/Acquisition or C/A signal is transmitted at L1 band with center frequency 1575.42 MHz and Precision signal or P(Y) is transmitted at both L1 and L2 bands, the latter with center frequency 1227.60 MHz. SPS is delivered via the L1 band while PPS via the L1 and L2 bands. GPS is a Code Division Multiple Access (CDMA) system where L1 and L2 signals are spread with a Pseudo Random (PRN) code of chip rate 1.023 MHz and 10.23 MHz, respectively and both signals are modulated with the Binary Phase Shift Keying (BPSK) scheme.

Although GPS was originally conceived as a military system, later the U.S. Department of Defense recognised the tremendous benefits for civilians and established it as a free-access utility. An important landmark in the history of GPS operation is when the Selective Availability (SA) feature was disabled in May 2000. The purpose of SA was to intentionally degrade the positioning performance by adding an error to the GPS L1 signal that was made available to non-military users. Removal of SA led to a significant increase in the positioning accuracy which enabled the development of GPS-based services such as standalone positioning and car navigation (when SA was on, positioning accuracy was in the order of hundred meters, while in off mode an accuracy of few meters could be achieved).

It is also important to emphasize that although GPS and in general GNSS technology is mostly known as a means for computing the three-dimensional position, it also provides a critical fourth dimension - time. Precise timing information and synchronisation are crucial in a variety of technical and financial operations such as in wired and wireless communication systems, electrical power grids, financial transactions, etc. For example, GPS time is used by the U.S. Federal Aviation Administration to synchronize reporting of hazardous weather from its weather radars and by wireless telephone and data networks to synchronize their base stations. Hollywood studios are also incorporating GPS time in their movie slates, allowing for unparalleled control of audio and video data, as well as multi-camera sequencing [9].

Since the time SA was turned off, the demand for GPS service was steadily growing as well as alternative GNSS systems were introduced. The growing demand for GNSS services and the need to remain competitive in the arena are two main reasons that recently initiated the GPS modernization program. A big part of program is dedicated to the design of new GPS signals with enhanced capabilities. Among others, the new signals will employ new modulation schemes, new structures, longer codes but also faster transmission rates, new data encoding, new navigation message formats and the possibil-
2.3 Current and Future Global Navigation Satellite Systems (GNSS)

While U.S. was the country to first develop a GNSS and although GPS has been the only fully operational system up to this moment, the landscape in the GNSS field has changed dramatically. More precisely, U.S. is not the only player. While the Soviet Union (later Russia) has been working on its own GNSS since the beginning of GPS, recently also other geopolitical entities, such as EU or China, understood the advantages of such a global system and initiated the development of their own systems as an attempt to enter the GNSS market and gain political independence.

2.3.1 GLONASS

The development of Global Navigation Satellite System (GLONASS) started in 1976, i.e. almost immediately after the GPS project was announced. Although by 1995 GLONASS was fully operational, the collapse of Soviet Union was a large step backwards in the system’s continuous operation. Between 1996 and 1998, due to the lack of funding, the GLONASS orbital constellation was not maintained and as a result, the number of operational satellites significantly declined, even with an increased lifetime [11]. With the advent of the 21st century and under the presidency of Vladimir Putin, the restoration of GLONASS system became one of the top priorities of the Russian government.

GLONASS space segment will consist of 24 satellites out of which 21 will be operational and three will be used as active backups. The satellites are placed in three orbital planes which have a greater inclination than in GPS and as a result GLONASS offers better coverage at higher latitudes [12]. In 2003, the Russian Department of Defence and the Russian Space Agency which are responsible for GLONASS operation and maintenance decided to upgrade the space segment with the introduction of a new satellite series called GLONASS-M (second generation). Compared to the first generation, GLONASS-M satellites have a number of new features such as improved navigation performance, longer lifetime, improved navigation signals and navigation message [13]. Each GLONASS-M satellite transmits two signals, one in L1 and one in L2 band,
enabling similarly with GPS two types of positioning services: one standard service meant for civilian use and one military service with improved positioning accuracy. Unlike GPS signals, the signals transmitted by the first and second generation satellites are based on the frequency division multiple access principles. GLONASS signals are spread with the same PRN code but each satellite transmits in different carrier frequencies. So, the satellites are distinguished from each other at the receiver side based on the frequencies assigned to them, instead of the PRN codes as in the case of GPS.

In 2011, Russia launched successfully the first GLONASS-K satellite (third generation) which is much smaller in size and weight than GLONASS M and it), has improved clock stability and a longer design life [14]. The third generation satellites will transmit additional navigation signals in order to improve the system’s performance. More precisely, each GLONASS K satellite will transmit five navigation signals; civil and military signals in each of the GLONASS L1 and L2 bands and a new CDMA signal for civilian applications in the L3 band [15]. At the time of this writing, GLONASS has 23 operational satellites in orbit, one in commissioning phase and three in maintenance [16].

The ground segment consists of the System Control Center, located in Moscow region, and several Telemetry, Tracking, and Control stations (TTC) distributed throughout the Russian territory which control, track, and upload ephemeris, timing information, and other data to the satellites [11]. GLONASS user segment is relatively small and mostly concentrated in Russia. Although, Russian industry has developed numerous types of GLONASS and GPS-GLONASS receivers and listed these for sale on the Russian Internet since the mid-1990s, the Russian press reported a lack of demand in early 2000 [13]. As an attempt to promote GLONASS receivers, in February 2011, the Russian government decided that all vehicles transporting passengers, large volumes, or dangerous materials will be required to use GLONASS-supported navigators starting from July 2011 [17]. Furthermore, in July 2011, Russian Deputy Prime Minister Sergei Ivanov announced that customs duties on imported GPS-enabled devices not receiving GLONASS signals might be introduced starting from January 2012.

2.3.2 Galileo

Overview

Galileo is Europe’s initiative for a state-of-the-art global navigation satellite system, that would provide a highly accurate, guaranteed global positioning
service under civilian control. The development of Galileo has been orchestrated by the European Commission (EC) and the European Space Agency (ESA) and it is expected to be fully operational in 2019 – 2020 [18].

The space segment of Galileo will consist of 30 satellites, equally distributed in three orbital planes. Each plane has an inclination of 56 degrees and contains nine operational satellites and one spare to replace any of the operational satellites in case of failures (all satellites are equally spaced in each plane, 40 degrees apart). The core of the Galileo ground segment will be two control centres which will manage "control" and "mission" functions, supported by dedicated Ground Control Segment (GCS) and Ground Mission Segment (GMS), respectively. The GCS will use a global network of five TTC stations to communicate with each satellite, while GMS will use a global network of thirty Galileo Sensor Stations to monitor the navigation signals of all satellites on a continuous basis [19]. The Galileo user segment translates the signals into services for the final users and it is composed by technologies (e.g., receiver technologies), added-value services (combined with communication, mapping, pricing services) and user applications [20].

Galileo will provide worldwide and independently from other systems the following services [19]:

- **Open Service (OS).** OS makes use of the open signals, based on which the user of a Galileo receiver can obtain location information free of charge. The positioning accuracy in OS mode is expected to be comparable or in some cases even higher than the one offered by C/A Global Positioning System (GPS) signals (e.g., the signal used to bear OS is expected to be more robust in environments prone to heavy multipath propagation such as urban canyons).

- **Safety of Life service (SoL).** SoL service offers better performance than the one offered by OS through the provision of timely warning to the user whenever the position solution falls outside the acceptable margins. SoL is mainly meant for safety-critical applications, such as maritime, aviation and rail, where guaranteed accuracy is essential [21].

- **Commercial Service (CS).** CS utilizes encrypted signals and is meant for commercial applications. Within CS, users will be offered data access via an authentication mechanism (yet to be defined), higher data rate throughput (i.e., the average rate of successfully received data), higher accuracy compared to OS, and service guarantee (i.e., on the liability of
the service). Services within CS will be developed by service providers, which will buy the right to use the commercial signals from the Galileo Operating Company (GOC) and then charge the users for accessing these services [21]. CS is considered to be the main source of revenues for the GOC.

- **Public Regulated Service (PRS)**. PRS is addressed to limited to a specific user segment, which requires high continuity of service and controlled access (e.g., meant for police, security services, firefighting, etc.). Moreover, a major advantage of PRS is that it is designed to be robust against jamming and spoofing.

- **Search And Rescue Service (SAR)**. SAR is designed to serve humanitarian purposes and reduce the response time to an emergency or catastrophe.

Two important landmarks in Galileo’s operation are the Initial Operational Capability (IOV) and Full Operational Capability (FOC). When Galileo reaches IOV, a constellation of 18 satellites will be available and early services for OS, SAR and PRS will be offered. In FOC, the constellation will be complete and all services will be available. IOV and FOC are expected in 2014 – 2015 and 2019 – 2020, respectively [18].

**Galileo Signals**

Galileo will provide ten navigation signals in Right Hand Circular Polarization (RHCP) in the frequency ranges 1164 – 1215 MHz (E5a and E5b), 1215 – 1300 MHz (E6) and 1559 – 1592 MHz (E1), which are part of the Radio Navigation Satellite Service allocation. The frequency bands used in Galileo are depicted in Fig. 2.2.

For a specific service and frequency, all Galileo satellites will share the same nominal frequency, making use of CDMA compatible with the GPS approach. Six signals, including three data-less channels, so-called pilot tones (ranging codes not modulated by data), are accessible to all Galileo users on the E5a, E5b and E1 carrier frequencies for OS and SoL service. Two signals on E6 with encrypted ranging codes, including one data-less channel, are accessible only to some dedicated users that gain access through a given CS provider. Finally, two signals (one in E6 band and one in E1 band) with encrypted ranging codes and data, are accessible to authorized users of the PRS [23].
A major difference between Galileo and GPS (at its current status) lies in the signal design in which modulation is one of the main structural characteristics. The choice of modulation is an important task because, among other reasons, it determines the signal’s spectrum and autocorrelation properties. The former is a critical characteristic due to the limited bandwidth and the potential interference with existing signals, while the latter affects the receiver’s performance.

Towards the end of the 1990’s, a new modulation technique, called Binary Offset Carrier (BOC), was recommended for future GNSS signals. BOC modulation is a square sub-carrier modulation, where a signal is multiplied by a rectangular sub-carrier of frequency $f_{sc}$, which splits the spectrum of the signal into two parts. The main advantage of BOC is that it enables sufficient spectral separation with existing GPS signals [24]. Moreover, because the width of the main lobe in the envelope of the AutoCorrelation function (Acf) is narrower than the one in Binary Phase Shift Key (BPSK) modulated signals (i.e., used in GPS C/A signal), improved tracking accuracy could be achieved.

There have been several variants of BOC suggested in the literature for different signal types included in the GPS modernisation plans and Galileo specifications [25]. Often these variants are combinations of sine BOC or cosine BOC implementations of different orders. Typically, the sine and cosine BOC modulations are defined via two parameters $BOC(m, n)$ [24], where
m = f_{sc}/1.023 and n = f_{c}/1.023, f_{c} is the chip rate and 1.023 MHz is the reference frequency (both f_{sc} and f_{c} are expressed in MHz here). From the point of view of the equivalent baseband signal, the sine BOC and cosine BOC modulations can be defined via the so-called BOC modulation order \( N_B = 2m/n = 2f_{sc}/f_{c} \) \[26\]. Among the various BOC implementations, Sine-BOC(1,1) was initially used in the standards for the L1 Open Service (OS) Galileo signals but afterwards Multiplexed BOC (MBOC) was selected \[22\]. MBOC is a weighted combination of sine-BOC(1,1) and sine-BOC(6,1) components and it is defined as a common spectrum to be matched by both the Galileo and the GPS L1/E1 OS signals. Because the MBOC signal is defined in the frequency domain (via the overall power spectral densities of data and pilot signals), it can be realized in the time-domain with many different approaches and the two chosen for GPS and Galileo are (1) Time Multiplexed BOC (TMBOC) and (2) Composite BOC (CBOC), respectively \[27,28\].

In the first implementation, the whole signal is divided into blocks of \( N \) code symbols and \( M < N \) of \( N \) code symbols are sine-BOC(1,1) modulated, while \( N - M \) code symbols are sine-BOC(6,1) modulated. In the CBOC implementation we have a weighted combination of Sine-BOC(1,1) and Sine-BOC(6,1) modulated code symbols \[29\]. When the combination is an addition of the two components we have the so-called CBOC(+) and when we subtract the sine-BOC(6,1) part from the sine-BOC(1,1) part, we have the so-called CBOC(−) type of modulation. The CBOC(+) scheme is used in the implementation of the Galileo OS data channel, while CBOC(−) is used in the pilot channel \[22\]. The waveforms of TMBOC, CBOC(+) and CBOC(−) modulated signals, as well as their normalized autocorrelation functions can be seen in the left and right plot of Fig. 2.3, respectively.

### 2.3.3 CNSS and QZSS

As a GNSS is offered at the discretion of the operating entity, more and more governments are willing to gain political independence by developing their own augmentation system or GNSS. China, the world’s second largest economy, is on its course to complete a 12-satellite regional version of its Compass Navigation Satellite System (CNSS) (in Chinese known as BeiDou-2) by 2012, with funding assured through 2020 to complete and operate a full constellation \[30\]. According to \[31\], the ninth Compass satellite has been already launched in July 2011.

The space segment of CNSS will consist of 5 Geostationary Earth Orbit
Figure 2.3: Waveforms of TMBOC, CBOC(+) and CBOC(−) modulated signals (left plot) and their normalized Acs for single-path channel, no noise, infinite bandwidth (right plot).

(GEO) and 30 Medium Earth Orbit (MEO) satellites. The ground segment consists of a Master Control Station, an Upload Station, and a Monitor Station. The user segment consists of Compass user terminals, which should be compatible with GPS, GLONASS, and Galileo. CNSS will offer two kinds of services: (1) an open service that will be free and open to users and (2) an authorized service which will offer more reliable positioning, velocity, timing and communications services as well as integrity information [32]. Unlike GPS or Galileo, no Interface Control Document (ICD) is publicly available for CNSS.

In addition to Chinese, Japanese are also developing a satellite navigation system, named Quasi-Zenith Satellite System (QZSS). QZSS is a Regional Navigation Satellite System that consists of multiple quasi-zenith satellites that fly in the orbit passing through the near zenith over Japan. According to a presentation given by the Japanese Secretariat of Strategic Headquarters for Space Policy, Satoshi Fujiwara, there are several QZSS satellite constellation plans in which the total number of satellites varies between 4 and 7 including quasi-zenith orbit and geostationary orbit satellites [33]. In order to have at least one quasi-zenith satellite always flying near Japan’s zenith, at least three satellites are necessary. QZSS also has slightly elliptical and highly inclined orbits in different orbital planes [34]. The purpose of QZSS is to augment GPS in order to enable positioning in areas where standalone GPS is not
sufficient, such as in mountainous and urban regions in Japan. In QZSS, signals have complete compatibility and interoperability with existing and future modernized GPS signals [35].
Chapter 3

Fundamentals of GNSS-based Positioning and GNSS Receiver Operation

3.1 Trilateration Principle

Satellite-based positioning is based on the trilateration technique which can be two-dimensional (2D) or three-dimensional (3D). In the simplest form (2D), if the positions of two points are known, it is possible to compute the unknown position of a third point by measuring the distance of it from each of the other two known positions. This is opposed to the more commonly understood method of triangulation, where an unknown position is determined by measuring the angles to it from two known points at either end of a fixed baseline [36]. In many positioning applications, it is often desired to know the position of a user, U, in terms of latitude \((x_u)\), longitude \((y_u)\) and altitude \((z_u)\), therefore, a 3D trilateration technique is employed. In satellite-based positioning, the position of the satellites (known as ephemeris) is part of the data transmitted to the user terminal. In order to compute the user location in all three dimensions, the distance from three satellites (also known as geometric range or range) has to be measured (see Fig. 3.1). Then, we can build the following system of non-linear equations:

\[
\begin{align*}
\rho_1' &= \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} + c\delta t + E_{alt} \\
\rho_2' &= \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} + c\delta t + E_{alt} \\
\rho_3' &= \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} + c\delta t + E_{alt}
\end{align*}
\]
where $\rho_i'$ and $(x_i, y_i, z_i)$ for $i = 1, 2, 3$ are the satellite-receiver pseudoranges and the 3-D Cartesian positions of the satellites, respectively and $(x_u, y_u, z_u)$ is the unknown user’s location, also in the Cartesian domain. The free of errors satellite-receiver distance ($\rho_i$) is equal to $\sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2}$ for $i = 1, 2, 3$ and it is called range (in contrast to pseudorange). The terms $\delta t$ and $E_{all}$ refer to the receiver clock bias and the errors that contaminate the received signal, respectively, and which are described in Chapter 4. Assuming that the satellite and receiver clocks are synchronized and that there are no other errors (i.e., $\delta t = 0$ and $E_{all} = 0$), it is possible to solve the above system by linearizing the three equations of it. The result is a system of three linear equations and three unknown variables which can be solved with the help of linear algebra. \[13\]

For the measurement of the satellite-receiver range, the Time-Of-Arrival (TOA) concept is utilized. In particular, a GNSS receiver computes the difference between the transmission time of the satellite signal and the reception time which corresponds to the signal’s propagation time. Because the satellite signals are electromagnetic signals that travel with the known speed of light, it is possible to compute the range simply by multiplying the signal’s propagation time with the speed of light. The transmission time is part of the signal’s data message and it is derived from the highly accurate atomic clocks placed on board. The time of arrival is obtained from the receiver clock which is by far less accurate than the satellite clock. This difference results in the clock bias term, $\delta t$, that appears in Eq. (3.1) and as it is explained in Chapter 4, this difference is the reason that practically four pseudoranges are required for
3.2 Receiver front-end and ADC

In order to gain a deeper understanding on how user’s position is computed, the basic operations of a GNSS receiver need to be described. As can be seen from Fig. 3.2, the main blocks of a GNSS receiver are (1) receiver front-end and Analogue-to-Digital-Conversion (ADC), (2) signal acquisition, (3) signal tracking which consists of code tracking and carrier tracking, (4) navigation data extraction and (5) computation of Position, Velocity and Time (PVT) and geographic coordinate conversion.

3.2 Receiver front-end and ADC

A diagram with the main blocks of a receiver front-end is depicted in Fig. 3.3. When the signal arrives at the receiver, the first hardware component it reaches is the antenna which typically is not considered part of the receiver’s front-end. Nonetheless, it plays a major role in the receiver’s overall performance [37]. Three fundamental parameters in the GNSS front-end design are: (1) frequency/bandwidth, (2) polarization and (3) gain pattern [38]. The antenna frequency is chosen based on the carrier frequencies of the signals that it is designed to process. The receivers antenna/front-end bandwidth is directly proportional to the accuracy required for the specific application of the receiver. In other words, the larger the receiver bandwidth is, the more frequency content of the received satellite signal is processed and ideally, the better the accuracy performance will be [13]. However, in real life one has also to take into consideration the amount of interference that is potentially added by increasing the receiver bandwidth. The polarization parameter denotes the
orientation of the electric field from the radio frequency transmission. GNSS systems usually employ Right-Hand Circularly Polarization (RHCP), and the antenna should be designed based on this characteristic. The main motivation for choosing circular polarization instead of linear, which is easier to implement, is that circular polarized signals are more tolerant to physical orientation mismatches [39]. The gain pattern describes the directivity of the antenna. In the case of GNSS, it is desirable for the gain pattern to be hemispherical and to receive signals from only positive elevation angles and from all azimuth directions [38]. Some of the most popular GNSS L1 antenna implementations are the inverted-F, patch and helix approaches but there are other designs available in the market as well. Moreover, antennas can be distinguished into three categories based on the application they are intended for: (1) geodetic, (2) rover and (3) handheld [40].

After the antenna, the next element is a bandpass filter whose task is to remove the interference in Radio Frequency (RF) and Intermediate Frequency (IF) bands and to select the desired RF or IF frequencies. Typical antennas have fairly poor frequency selectivity and that is why one cannot rely solely on the antenna for this task [38]. After the bandpass filter, an amplifier is needed in order to amplify the extremely weak satellite signal. Ideally, an amplifier would increase only the magnitude of the signal amplitude but in practice, noise is also amplified. One way to reduce the noise amplification is to place the amplifier as early as possible in the hardware chain in order to avoid amplifying the noise caused by the noisy analogue components that are placed in later stages (e.g. mixers and baseband amplifiers). Another approach is to place the amplifier within the antenna itself. This implementation is according to [38] the norm in many GNSS antennas, and such a design is known as an active antenna.

Next step in the front-end chain is the down-conversion of the signal. Three possible implementations for the front-end design are: (1) tuned radio frequency, (2) superheterodyne and (3) direct conversion. In the first design, no frequency conversion is implemented. Instead, the signal is converted to the digital domain directly on the carrier frequency [41]. In the superheterodyne design, the received signal is converted to a fixed intermediate frequency [42], while in the direct conversion design, the receiver directly converts the received signal to the baseband [43]. A comparison of these designs can be found in [44]. The last task in the receiver front-end is to convert the analog signal into digital with the help of an Analog-to-Digital-Converter (ADC). There are plenty of ADCs available in the market and the key parameters to consider when


3.3 Signal Acquisition

After the analog signal has been converted to digital, a series of baseband operations takes place, among which signal acquisition is the first one. The purpose of acquisition is to identify all the satellites visible to the receiver and to provide coarse estimates of the signal’s synchronization parameters. Essentially, signal acquisition is a three-dimensional search process where the corresponding satellite and the signal’s code delay and frequency parameters are estimated. Recall that in CDMA-based GNSS, each satellite is characterized by a unique PRN code. The search process is done by cross-correlating the received signal with each of the satellite’s PRN code. A high enough peak in the cross-correlation output with a certain PRN will indicate whether the corresponding satellite is visible and will also provide coarse estimates for the frequency and the code delay of the received signal. The estimates of the signal’s synchronization parameters are then supplied to the tracking block. It is important to know the frequency of the signal in order to be able to generate a local carrier signal which is used to remove the incoming carrier from the signal. Similarly important is to know the code phase of the received signal so as to be able to despread it [38]. The frequency band to be searched is defined as $+/-$ maximum Doppler shift from the carrier frequency. The Doppler shift is introduced due to the satellite and/or the receiver movement and depends on their relative speed.
In acquisition, the searching space is divided into frequency bins and time bins, respectively. The combination of one code bin and one frequency bin forms a cell. Then, the problem is to find the cell which is most likely to contain the unknown pair of parameters. In order to realize the magnitude of the search process, consider for example the GPS system and the C/A signal. On Sunday 30th of October 2011, the number of healthy GPS satellites was 30. The C/A code is 1023 chips long. If a GPS receiver is used on a high-speed vehicle, then according to [46], it is reasonable to assume a maximum Doppler shift of $\pm 100$ kHz. If we also assume that we search the code and the frequency axis with a step of half chip and one kHz, accordingly then, the number of code-frequency bins to be searched is $2 \times 1023 \times 201 = 411246$ for each of the 30 satellites, or $30 \times 411246 = 12337380$ combinations in total. This number indicates how large the computational burden can be. According to [47], in practice, the receiver chooses a certain frequency setting and searches for all possible code bins. The search process terminates either when a satellite signal is detected or when all possible combinations have been tested. If the entire grid is searched and no satellite is detected, then another PRN code is chosen. In some search techniques, such as in hybrid search, the cells are grouped into windows and the search is done first on a window level and then on a cell level. This approach enables faster results.

There are several search methods for code acquisition which can be divided into three general classes: serial, parallel and combination of serial and parallel (or hybrid) acquisition methods [13]. Serial search is the simplest and most frequently use method. As its name implies, the search in the time and frequency space is used in a sequential manner [48]. Each frequency bin is usually of the order of few kHz while each time bin corresponds to a fraction of a chip. Clearly, the higher the resolution of the search step is, the higher the execution time is. In the parallel search, the main idea is to parallelize the search for one parameter. Typically, the Fourier Transform is utilized in order to perform a transformation from the time into the frequency domain which allows to parallelize the search in the frequency space [38, 49]. Towards the end of 1990’s, consumer receivers employing parallel correlator banks became available and nowadays, it is a commonly used approach. Such design significantly reduces the acquisition time, nonetheless, at the expense of higher power consumption [50]. In the hybrid search [51], the idea is combine techniques from the serial and parallel search strategies in order to achieve a better trade-off in the performance parameters.

The main trade-off parameters are the complexity and the execution time.
3.4 Signal Tracking

Serial methods are slower than parallel ones but less complex and hybrid methods are designed with the aim to achieve a better balance between execution time and complexity. The detection stage of the signal is a statistical process because in each cell, the correlation result contains the noise together with the signal or only the noise. Each of these two cases has its own probability density function [13]. In particular, the statistic is commonly based on the comparison of signal energy against a selected threshold. If the signal energy in a time bin or in a window of bins is greater than the predefined threshold, then this bin or window of bins is considered as the correct one, else it is considered as a wrong bin/window. One way to determine such threshold is based on the estimated noise statistics of the signal [52]. A discussion on how such threshold can be chosen is found in [53,54].

3.4 Signal Tracking

In order to be able to obtain the navigation message, we need to have perfectly aligned code and carrier replicas. These replicas can be tracked using two tracking loops: code and carrier tracking loops. These tracking loops work together in an interactive process, aiding each other, in order to acquire and track the satellite signals. The tracking stage is initiated only after the initial acquisition has taken place, where coarse code delay and frequency estimates have been produced.

3.4.1 Code Tracking

The general idea of code tracking is to estimate as accurately as possible the delay of the received signal. This is done by generating a replica code (reference code) and correlating it with the received signal. The correlation can be seen as a similarity measure. The more aligned the codes of the replica and the received signal are, the higher the value of the correlation function is. In order to align the codes and find the delay, the replica signal is shifted iteratively. One popular choice for the tracking module, that is widely used both for GPS and Galileo signals, is called Delay Locked Loop (DLL). Among the DLL variants that exist in the literature, the Early-Minus-Late (EML) structure is one of the most popular. In this structure, we have three correlators: one Early (E), one Prompt (P) and one Late (L). Each of these correlators compute the correlation between the received signal and the replica code which is delayed by $-\Delta/2$, 0 and $\Delta/2$ chips, respectively, the correlators are placed in such a
way that P correlator is at equal distance ($\Delta/2$) from E and L correlators. The target is to properly delay or advance the replica code so as the prompt correlator corresponds to the maximum of the cross-correlation output, which in ideal scenarios indicates the delay of the signal in chips (then this delay can be easily computed in meters, simply by dividing it with the signal’s chip rate and then by multiplying it with the speed of light). This process is illustrated in Fig. 3.4.

3.4.2 Carrier Tracking

After the correct code delay is acquired by the tracking loop, a phase and/or frequency locked loop (PLL/FLL) can be employed to track the phase and the carrier frequency, respectively. PLLs wipe off the carrier by replicating the exact phase and frequency of the received IF signal. FLLs perform the carrier wipeoff process by replicating the approximate frequency, and they typically permit the phase to rotate with respect to the incoming carrier signal [13]. A very commonly used phase locked loop is known as Costas loop [55] and it is preferred because its performance does not depend on the phase shifts caused by the data bits [56]. In general, for a given signal power level, Costas PLL loops provide the most error-free data demodulation in comparison with
3.5 Navigation data extraction

The last task performed in the baseband domain is to demodulate the signal in order to extract the navigation data which consist of time-tagged data bits indicating the satellite transmission time of each subframe. More precisely, the steps that are typically required for this task are (1) bit synchronization, (2) frame synchronization and (3) data decoding [38]. In the first step the target is to determine the boundaries of each bit. If the bit boundaries are known, then we can despread the received signal by stripping off the PRN code and exposing the bit sequence. The main challenge in this step is that, although the beginning of each PRN code period is known, it is not known where the navigation data bits are located, since each data bit is composed of several PRN code periods. If the assumed databit offset does not correspond to the actual one, then we will not be able to despread the signal successfully and the navigation data will be misread [58]. Similarly, in the second step, the target is to determine the boundaries of the navigation data frames (each frame consists of a certain number of subframes). Finally, in the last step, the target is to extract those parameters from the transmitted binary bit stream which are used for the final position calculation (such as ephemeris and various pseudorange correction parameters).

3.6 Position computation and geographic coordinate conversion

The main outputs of the baseband processing block are the code/phase measurements and the navigation data streams. In order to compute the user’s position, the system of non-linear equations given in Eq. (3.1) has to be solved. In general, there are three main approaches for deriving its solution: (1) closed-form methods, (2) iterative techniques based on linearization, such as Least Squares (LS) and (3) Kalman filtering [13]. The specifics of such techniques are outside the scope of this manuscript and therefore will not be discussed further.
After the system in Eq. (3.1) is solved, the next task is to convert the computed coordinates into some "understandable" coordinates. More precise, the position computation is based on a Cartesian coordinated system, known as Earth-Centered, Earth-Fixed (ECEF) where the center of the Earth is the center of the gravity mass and the three axes are fixed with respect to the Earth. Although the x-y-z representation is useful in terms of mathematical calculations, it is not very useful in the real world [59], especially because the shape of the Earth and its movement are complex. Therefore, the ECEF coordinates are translated into geodetic coordinates, i.e. longitude, latitude and altitude [60]. Since 1994, GPS used the WGS84 geodetic reference frame [61] but Galileo’s reference frame will be based on stations and clocks different from those of GPS [13]. In particular, the Galileo Terrestrial Reference Frame (GTRF) will be linked to International Terrestrial Reference Frame (ITRF) to which the GPS reference frame is also linked [62]. Nonetheless, the differences between WGS84 and the GTRF are expected to be on the order of a few centimeters [13].
Chapter 4

Error Sources in GNSS

In ideal conditions, a GNSS receiver can very accurately estimate its position in the three-dimensional domain by measuring the distance to three satellites. However, in realistic scenarios, there is a large number of error sources that affect the positioning accuracy and degrade the receiver’s performance. These error sources are distinguished according to the area of their occurrence and they are grouped into three major categories: (1) space segment-related, (2) propagation channel-related and (3) receiver-related.

4.1 Space segment-related errors

The space segment of a GNSS refers to the satellite-constellation. Within this segment, errors may occur due to (1) hardware limitations of the satellites (2) natural forces and (3) satellite geometry.

One of the main limitation when it comes to satellite operation is the clock drifts. As it was mentioned in Chapter 3, the receiver calculates the distance from a satellite based on the propagation time which is defined as the difference between the transmission and the reception time. Each satellite is equipped with atomic clocks which are significantly more accurate than the receiver clock and also extremely stable. The latter is a very important aspect for the system’s good performance since all on-board timing operations, including broadcast signal generation, depend on it [13]. The Galileo satellites will carry two types of clocks: rubidium atomic frequency standards and passive hydrogen masers. In [19], it is stated that these clocks would lose three seconds in one million years and one second in three million years, respectively. Although satellite clocks are very stable, a drift of few nanoseconds can cause
several meters of error. Therefore, it is important to monitor the satellite’s clock stability. This task is performed by the Master Control Station of the ground segment which is responsible for detecting potential drifts and transmitting the clock correction parameters to the satellites for rebroadcast in the navigation message.

Besides the errors originating from the satellite itself, there are sources outside the satellite body which affect its orbit position. Recall that based on the trilateration principle the satellites’ position (known also as ephemeris information) shall be known at the receiver in order to compute its position (see Section 3.1), therefore, ephemeris information is part of satellite signal’s message. The forces present in the space segment are distinguished between gravitational and non-gravitational forces. The earth gravitational forces introduce perturbation which cause satellites to deviate from their Keplerian orbits. Therefore, these perturbations should be continually determined through the analysis of tracking data [63]. The non-gravitational forces are a significant contribution to the total force on a GPS satellite but their modelling is tedious whereas the gravitational forces are well modelled [64]. One of the most important non-gravitational force acting on the GNSS satellites is the one caused by solar radiation [65, 66]. More precisely, if the surface of the satellite is exposed to the sun light, the solar radiation pressure imposes perturbation on the orbit of the satellite [67].

When the principle of GNSS-based positioning was described in Chapter 3, it was mentioned that in order to locate a GNSS receiver, the distance to three satellites is required. In other words, it appears that the only critical aspect in the ability of a receiver to compute its position resides in the number of visible satellites. However, this is partially true. In practice, if the minimum number of satellites needed for positioning is met then it is possible to compute the receiver’s position but the confidence of the estimated position depends on satellite geometry which constitutes the third source of space-related errors. The effect of the satellite geometry on the position error is called Geometric Dilution of Precision (GDOP). More precisely, when the satellites are close to each other, the GDOP is poor while the further apart each satellite is, the better the geometry is and the higher confidence level is. Fig. 4.1 illustrates two scenarios of GDOP. In the case of bad geometry the satellites are close to each other (plot a1) and the projected intersection of the satellite transmission coverage (plot a2) is larger, thus the confidence level is low. When the satellites are better distributed on the sky (plot b1), relative to the receiver, then the projected intersection is much smaller and the confidence of estimation higher.
(plot b2). We notice that good satellite geometry does not guarantee Line-Of-Sight (LOS) signal reception. For example, if the receiver is situated in a difficult reception environment, such as one where it is surrounded by high buildings, satellite signals are likely to be blocked.

4.2 Propagation channel-related errors

As in any wireless communication channel, the transmitted signal is affected by a variety of error sources associated with the propagation environment (both natural and man-made). More precisely, these error sources are (1) troposphere, (2) ionosphere and (3) multipath propagation effects.

4.2.1 Ionosphere

Ionosphere is part of the atmosphere, located between 50 km and 1000 km from the Earth’s surface. Unlike the lower atmospheric layers (e.g. troposphere, stratosphere, etc.), the ionosphere contains charged particles, electrons and ions, the content of which depends on various spatial and temporal parameters (e.g. altitude, season, time of the day, etc.), as well as on the occurrence of natural phenomena (e.g. electromagnetic storms and travelling ionospheric disturbances). The presence of the charged particles makes ionosphere a dis-
persive medium, thus, signals transmitted at different carrier frequencies have different phase advances and time delays.

When the signal propagates through ionosphere, its velocity changes due to the interaction with particles present in it. As a result, the signal’s code is delayed and its phase is advanced. In particular, the signal is delayed almost by as much as the carrier phase is advanced, thus, it is sufficient to estimate one of the two parameters. In the case where the higher order and the bending effects are ignored, the values of code delay and carrier phase advance are exactly the same \[68\]. The interested reader may refer to \[13,68–70\] for a detailed description of the ionospheric effects. In order to mitigate the refraction effects, the knowledge of the involved refractive indices and signal’s frequency is required. However, because ionosphere is a heterogeneous medium, meaning that the density of the ionised particles within is not uniform, its refractive index is defined by the density of the particles. Appleton and Lassen have derived a formula for computing the ionospheric refractive index \[71\], with which the ionospheric delay can be defined as the sum of first, second, third order and bending effects \[68\]. These effects are a function of the Total Electron Content (TEC), which is a space-time varying parameter that needs to be estimated in order to remove the ionospheric delay from the range estimate. It can be shown that for GPS L1 and Galileo E1 signals, the second and third order terms are much less than 1% of the first-order term \[3,68\] (we remark, that the contribution of these two effects is similar for the rest of GNSS signals). Therefore, when mass-market receivers are considered (where the target accuracy is not so high as for high-end receivers), it suffices to consider only the first-order effect \[13\]. The first-order ionospheric delay is proportional to TEC and inversely proportional to the square of frequency. More precisely, it is defined as \[13\]

\[
I_i = \frac{40.3}{f_i^2} TEC
\]

where TEC is the total electron content measured in TEC Units (TECUs) with \(1\) TECU=\(10^{16}\) electrons/m\(^2\) and \(f_i\) is the \(i\)-th frequency for \(i = 1, 2\).

As it was mentioned earlier, typical values of TEC vary according to the time and the location. In \[72\], the authors claimed that a value of 100 TECU is typical in mid-latitude areas during daytime. During night-time, typical TEC values are between 0 and 30 TECUs \[72\]. In low latitudes and during a solar max, TEC can reach up to 140 TECU \[73\]. According to \[74\] TEC can be even as high as 220 TECU when the solar wind activity is very high. In \[75\], it was written that a value of 1000 TECU or higher is unrealistic.
Fig. 4.2 shows the first-order ionospheric delay as a function of TEC for different Galileo signals. Based on the typical TEC numbers given above, one can see that, for example, for E1 signal the nighttime ionospheric delay can be as low as few centimeters but in daytime, the typical ionospheric delay would be in the range of 10 to 40 meters depending on the latitude and the solar activity. We notice that according to [72], the Galileo E5 signal experiences greater ionospheric delay in reality than the one derived by Eq. (4.1) and plotted in Fig. 4.2. It is because the formula in Eq. (4.1) is effective only for narrowband signals, such as the GPS L1 C/A. As the frequency band gets wider, the ionospheric delay variation within the band becomes larger. Furthermore, the ionosphere causes additional effects on the wideband signals, such as power loss of the auto-correlation peak [72]. Thus, Galileo E5 signal will experience worse ionospheric effects than the other Galileo signals.

4.2.2 Troposphere

The troposphere is the lower part of the atmosphere and it varies between 7 and 14 km, depending on the observation point. Troposphere is non-dispersive for frequencies up to 15 GHz [76]. Because of this characteristic, the measurements at all GNSS signal frequencies for code and carrier experience the same delay [70].
The delay introduced by the propagation of the satellite signal through troposphere is a function of four parameters: (1) local temperature, (2) pressure, (3) relative humidity and (4) satellite elevation [59]. Moreover, the tropospheric delay can be separated into a hydrostatic and a wet component. The hydrostatic component in zenith direction is called Zenith Hydrostatic Delay. It can be precisely determined by surface pressure measurements. The Zenith Wet Delay, however, cannot be sufficiently modelled by surface measurements due to the irregular distribution of water vapour in the atmosphere [77]. Typically, the hydrostatic component of the delay in the zenith direction represents about 90% of the total delay while the non-hydrostatic delay is highly variable and can only be estimated with predictive models to an accuracy of a few centimetres in the zenith direction [78]. According to [79], the delay caused by the troposphere in zenith direction is about 2 m but it can reach up to 20 m for low elevation angles. Compared to the ionospheric delay, the tropospheric delay is typically much smaller at L-band frequencies, where GNSS signals fall into. However, direct estimation of the tropospheric contribution is difficult and usually it is necessary to provide the receiver with a correction algorithm [80].

4.2.3 Multipath propagation

Multipath propagation describes the phenomenon where the satellite signal arrives at the receiver via an indirect path due to reflection or scattering. The signals that arrive straight to the receiver are called LOS signals and those that arrive through longer, indirect routes, Non-LOS signals. In real scenarios and depending on the environment, it is possible to have only a LOS signal at the receiver (ideal case), LOS and NLOS signal(s) or only NLOS signal(s). Some of the most common local sources of multipath are buildings, trees, road vehicles or terrain (see Figure 4.3).

Unlike the other error sources, multipath is normally uncorrelated between antenna locations. Hence, the base and remote receivers experience different multipath interference and as a result differencing between them will not cancel the errors [81]. For the characterisation of the multipath channel the following parameters are important: (1) number of paths, (2) path amplitude, (2) path delay and (4) path phase. When the environment is subject to heavy multipath propagation effects (such as in densely-built areas and urban canyons), multiple phased and attenuated replicas of the main signal arrive at the receiver at different delays causing constructive and destructive interference and
eventually degrading the signal quality [82]. The receiver operations that are significantly affected by the multipath propagation effects are the code and carrier phase tracking functions. However, the multipath effects on code and carrier phase measurements differ considerably, i.e., the effects are much stronger in code than in phase measurements [83]. According to [84], when the multipath delay is less than one chip, the effects on the code measurements are typically up to a few of meters, a couple of hundred at most. For example, for GPS L1 and Galileo E1 signals a delay error of 0.1 chips is approximately 30 meters.

4.3 Receiver-related error sources

The accuracy and stability of receiver clocks are significantly smaller than of expensive satellite clocks, mainly due to the hard requirement in their size and price. As a matter of fact, the difference in the accuracy between the satellite and receiver clocks is so large that causes a very large error in the computation of the propagation time and consequently in the geometric range estimation. Therefore, in real scenarios the timing difference due to the less accurate receiver clock is compensated by introducing one temporal parameter, thus one more range measurement in the range system presented in Chapter 3. In other words, in order to estimate the receiver’s position in the three-dimensional domain, we actually need four satellites. Assuming that there are no other errors present apart from the clock bias, we have

$$\rho_i = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2 + c\delta t} \quad (4.2)$$
where $i = 1, 2, 3, 4$, $\delta t$ is the receiver clock bias. Then, we form a system that is solved in a similar way as in the case of three satellite-receiver geometric ranges, only that now one more equation has been added to compensate for the additional unknown parameter.

Besides the error introduced due to the less accurate receiver clock, there are also other receiver-related errors that affect the positioning performance. These errors are caused mainly due to the thermal noise jitter and interference effects that appear in the code and carrier tracking loops. Errors caused by other hardware components (such as antenna delay) are often ignored because they are relatively small compared to other error sources, especially when cancellation is considered [13].
Chapter 5

Methods for Mitigating Dominant Propagation Effects in GNSS-based Positioning

In Chapter 4, it was shown that among the propagation channel-related errors sources, ionosphere and multipath propagation effects can introduce the largest delays. In this chapter, we overview various state-of-the-art methods for mitigating these dominant propagation effects. In addition, we position the methods proposed in this dissertation with respect to the state-of-the-art.

5.1 Ionosphere Mitigation Techniques

Among the various error sources, ionosphere accounts for the biggest part of signal’s total delay [13, 85], therefore, it is essential to alleviate its effects. The methods employed to remove the ionospheric delay from the pseudorange measurements can be grouped into two main categories, depending on the receiver type: single-frequency and multi-frequency.

5.1.1 Single-frequency approach

In single-frequency category, the GNSS receiver is able to process satellite signals transmitted in one carrier frequency and this is the category where currently most of the mass market receivers currently fall into. The unknown Total Electron Content (TEC) is estimated with the help of empirical models whose accuracy is typically counterbalanced by their complexity.
One of the most well-known models is the Bent model that was introduced in 1972 by Bent and it uses an empirical worldwide algorithm for estimating the electron density and the delay caused to the satellite by refraction [86]. The model belongs to the computationally intensive ones but is able to correct up to 80% of the total ionospheric effect [87]. The most widely used model is the one named also after its designer, Klobuchar [88]. In this model, it assumed that all free electrons of the ionosphere are densely distributed in a shell located at a fixed altitude of 350 km above Earth's surface [89]. Klobuchar model (also known as Ionospheric Correction Algorithm - ICA) uses a cosine model for modelling the daily ionospheric variation, which is described with the help of eight coefficients that are included in the GPS navigation message. Typically, a 50% Root Mean Square Error (RMS) ionospheric range error correction can be achieved with this model [69] which is a severely truncated version of a more complex empirical model developed by Bent [90]. The Klobuchar model performs well only under stable ionospheric conditions and its performance deteriorates significantly in the presence of ionosphere disturbances [3, 91]. Nonetheless, it is a common designer choice because of its compromise between accuracy and complexity [92]. For example, a local Klobuchar model may come into use in the coming years in the mass-market Assisted-GNSS mobile phones according to [93]. In such a model, the user can carry together with a local Klobuchar model the parameters on its applicability area and period. This enables tailoring a model for users on an hourly basis, depending on their coarse location estimate.

Other world-wide ionospheric electron density profile models, from which average TEC can be obtained, are the International Reference Ionospheric (IRI) model [94], IRI90 and IRI95 models [95] and the Global Assimilative Ionospheric Model [96]. Several other models exist, but they are mostly used by ionospheric researchers and are not generally available to users [97]. Also, a guide to ionosphere models, sponsored by the American Institute of Aeronautics and Astronautics, can be found in [98].

The vast majority of the GNSS-based ionospheric models use GPS data, because GPS has been the only fully operational GNSS up to this time. With the advent of Galileo system, a new ionospheric model, called NeQuick, has been proposed for the Galileo-enabled receivers [99]. The NeQuick model is adopted by ITU-R and has been used by the ESA EGNOS project for assessment analysis [100]. Unlike Klobuchar model, NeQuick makes use of only three broadcast coefficients it outputs the ionospheric electron density as a function of height, geographic coordinates and epoch in Universal Time.
or Local Time. Unlike Klobuchar model, NeQuick does not make use of the thin-shell assumption; instead, it is a three-dimensional model that exhibits higher degree of realism. According to \cite{101} NeQuick model has been able to correct better the ionospheric delay in mid- and high- latitudes regions when compared with Klobuchar state-of-the art model. In addition, unlike Klobuchar model, NeQuick is able to show day-to-day variations in range delay corrections. Moreover, under the assumptions given in \cite{102}, the residual ionospheric delay is considered not to exceed 20 TECU or equivalently 30\% of the total ionospheric error. However, a disadvantage of NeQuick is that it performs poorly in near-equatorial regions (i.e. where electron density is highly variable). More performance assessments of NeQuick model can be found in \cite{103–106}. Also, a new version of the model, NeQuick 2, has been introduced in \cite{107}. In the new version, several optimizations have been implemented for example, to improve its computational efficiency. A performance comparison of NeQuick 2 with ICA can be found in \cite{108}.

### 5.1.2 Dual-frequency approach

Unlike single-frequency receivers, no modelling of the ionosphere is needed when more than one carrier frequencies, and consequently pseudoranges, are available. It is because the signals transmitted from a single satellite undergo the same ionospheric effects and this fact can be exploited in order to mitigate the ionospheric effects \cite{13}. In dual-frequency receivers, a linear combination of the available pseudorange measurements allows the receiver to completely remove the first-order ionospheric delay, under the assumption that the pseudoranges are contaminated only by the ionosphere \cite{109}. This can be easily shown starting from the model presented in Eq. (5.1). More precisely, in a dual-frequency receiver we can model the available pseudoranges as

\begin{align}
\rho_1 &= \rho + \frac{40.3}{f_1^2} TEC + \varepsilon_1 \\
\rho_2 &= \rho + \frac{40.3}{f_2^2} TEC + \varepsilon_2
\end{align}

where $\rho_i$ for $i = 1, 2$ are the pseudoranges, $\rho$ is the true satellite-receiver range and $\varepsilon_i$ is the measurement error (e.g., due to multipath propagation effects). We notice that in this model we ignored the common error sources (such as errors due to non-synchronism of the satellite and receiver clocks, ephemerides and the tropospheric refraction) since their effects contaminate both frequencies in the same way and do not affect the validity of the results.
METHODS FOR MITIGATING DOMINANT PROPAGATION EFFECTS IN GNSS-BASED POSITIONING

[110, 111]. If there are no other error present in the pseudoranges besides first-order ionospheric delay (i.e. $\varepsilon_i = 0$ for $i = 1, 2$), we can accurately estimate TEC, and consequently the satellite-receive range, by solving the linear system of two equations and two unknowns. Although this traditional method provides ionosphere-free measurements, it also has the side-effect of increasing the noise [13]. Nonetheless, the simplicity and the accuracy of this method are two of the main advantages advertised for dual-frequency receivers over single-frequency ones. Other methods that can be used for estimating the ionospheric delay are the Least Squares (LS) and the Constrained Least Squares (CLS). Unlike LS, CLS imposes constraints on the estimated solution, for example, the constraint of TEC being positive, with the target of improving the estimation accuracy.

The ability of dual-frequency receivers to remove ionospheric delay resides solely on the assumption of zero measurement errors. In reality, we cannot ignore the potential presence of measurement errors (i.e., due to multipath propagation effects) because they degrade significantly the accuracy of ionospheric delay estimation. This was shown in this dissertation and specifically, in publication [P1]. Motivated by this unwanted effect, this dissertation proposed a new dual-frequency algorithm, Brute Force Constrained (BFC), which is also described in [P1]. Another aspect in the performance of dual-frequency receivers and their ability to mitigate ionospheric effects is the choice of the carrier frequencies. GPS-based dual frequency receivers utilize the L1 and L2 frequencies since these are the two signals currently transmitted from GPS satellites; however, with the advent of the new modernised signals the designers will have the flexibility to choose a better combination. Considering the future Galileo system, the research on dual-frequency receivers is in its infancy. Motivated by this, it was shown in this dissertation that the E1-E5a combination yields the best performance among all possible frequency pairs [P1]. This frequency choice was also found to be the best one in [112], however, their decision was based on different criteria such as acquisition time, power containment and compatibility with GPS.

5.2 Multipath Mitigation Techniques

5.2.1 Signal tracking methods

The distortion effects of multipath propagation have been known to the GNSS community for a long time and several efforts to mitigate them have taken
A large portion of these efforts has been focused on the tracking stage of a receiver where fine estimates of the Line-Of-Sight (LOS) code delay and carrier phase are required. One of the most commonly used code tracking structures are the so-called Delay Locked Loops (DLLs), which belong to the category of feedback estimators. Among the various DLL implementations that exist in the literature, the Early-Minus-Late (EML) structure is one of the most widely used (see Section 3.4.1 for the operation of EML). The block diagram of a non-coherent EML is depicted in Fig. 5.1. We see that the input signal $r(t)$ is split and it acts as the input to the early (E) and the late (L) correlators. In both the early and late correlation channels, the output of the correlator is coherently integrated over $N_c$ microseconds and then squared (non-coherent combining). The code detector error signal $S(\varepsilon)$ is the difference between the early and late correlation channel outputs and it is driven to zero by the DLL in normal tracking operation [113]. The resulting combination of early-late correlators is commonly known as discriminator function which essentially represents the expected value of the error signal as a function of the reference parameter error (i.e., the code phase error) [114]. Based on the EML structure, several other implementations exist, the most popular of which is the Narrow Correlator (NEML) [115]. NEML was among the first approaches to mitigate multipath effects by reducing the spacing between the early and the late code to less than 1 chip (see Fig. 5.1). The idea of spacing reduction was based on the assumption that the distortion of the cross-correlation function near its peak due to multipath is less severe than that at regions away from
the peak [115]. The discriminator function of NEML is expressed as:

\[
D_{\text{NEML}}(\tau) = E_1 - L_1 = |R(\tau + \frac{\Delta}{2})|^2 - |R(\tau - \frac{\Delta}{2})|^2
\]  

(5.2)

where \(E_1\) and \(L_1\) are the early and the late delayed versions of the reference code, respectively, \(R(\tau)\) is the averaged correlation function between the received signal and the stored replica, and \(\Delta\) is the early-late chip spacing. Other known DLL implementations are the Strobe and Edge correlators [116], High Resolution Correlator and Double Delta Correlator [117]. Another structure that can be seen as a generalization of Double Delta structure is the Multiple Gate Delay (MGD) structure which has a variable number of early-late correlator pairs [118,119]. Two MGD structures have also been introduced in this dissertation and compared with NEML and HRC methods [P4].

The main disadvantage of the feedback-based estimators is that they are generally sensitive to closely spaced path scenarios and potential acquisition errors. As an alternative solution, various feedforward approaches have been proposed in the literature [120–123]. Feedforward estimators make use of an open loop structure where the estimation is done in a single step, without employing any feedback information [121]. A general classification of open-loop solutions for Wide CDMA (WCDMA) applications can be found in [124,125].

One of the simplest feedforward delay estimators is the so called Matched Filter, where the choice of the LOS delay is based on a simple thresholding of the AutoCorrelation Function (Acf) [125–127]. First, the threshold \((MF_{\text{thresh}})\) is computed based on the sum of second highest peak of the ideal Acf and the estimated noise variance. Second, the LOS delay is estimated as the first local maximum of the averaged correlation function which is higher than \(MF_{\text{thresh}}\). While improving the delay estimation accuracy, feedforward methods are sensitive to the noise-dependent threshold choice and typically require more correlators than DLL-based ones. One way to combine the advantages of feedback and feedforward approaches is via a hybrid approach such the one presented in [128]. Also, the DiscTracker algorithm proposed in this dissertation [P5] belongs to this hybrid category.

In the carrier tracking stage, multipath mitigation is also a challenging problem. Carrier phase multipath has been commonly studied in 2-path channels using a phasor diagram that illustrates the relation between the phase of the LOS signal and the multipath [129–131]. In [132], a geometric perspective is employed that involves different configurations of the antenna-reflector(s) geometry. Other methods include the Ashtech Enhanced Strobe Correlator [133] and the Multipath Estimating Delay Lock Loop; the latter jointly estimates
the delay, relative amplitude, and phase parameters of the direct and multi-
path signals based on the maximum likelihood theory [134]. Both are advanced
techniques with improved performance in long delay multipath errors; how-
ever, they are heavily covered by patents. Deconvolution methods can also
be used for estimating the signal’s multipath parameters. Essentially, they
are means of inverse filtering and as such one major drawback of them is the
noise enhancement [135]. In order to alleviate this negative effect, constrained
inverse filtering methods are considered. Among these, the best known ones
are the LS techniques and the Projection Onto Convex Sets (POCS) algo-
rithm [136]. The mPOCS algorithm proposed in this dissertation falls also
under the category of deconvolution methods [P6]. It estimates jointly the
signal’s synchronisation parameters in a constrained iterative manner and is
suitable for closely-spaced path environments.

5.2.2 C/N0 estimation methods

In general, one way to mitigate multipath propagation effects is by develop-
ing new tracking algorithms or by improving existing ones. In both ways, the
knowledge of the carrier-to-noise-ratio spectral density (C/N0) can be of great
assistance in improving the tracking performance. For example, in addition
to determining the relative signal to noise strength, C/N0 measurements can
be used as a triggering mechanism for switching among tracking algorithms
which are known to perform better in certain ranges. Furthermore, C/N0
knowledge can be used to adjust the values of different parameters used by
tracking algorithms. For example, it is possible to adjust the threshold em-
ployed in feedforward or hybrid methods based on the C/N0 level. This is one
application of C/N0 that was adopted by the author in the tracking methods
proposed in this dissertation [P6], [P5].

Several methods exist in the literature for estimating C/N0. Typical C/N0
estimators for GNSS signals are based on the first and higher-order moments
of the complex envelope of the received signal, for example, second-order and
fourth-order moments [137], [138], [139] and they require rather heavy com-
putations at the receiver. The most encountered CNR estimators for GPS
signals are those based on the averaged cross-correlation function between the
incoming signal and the reference code at the receiver [140, 141]. These esti-
mators use some statistical models and they usually work well for moderate to
high C/N0, but they perform poorly at low C/N0 values. The LCRE method
proposed in this dissertation belongs also to the above group but, unlike other
methods, it has a very good performance at low C/N0 as well [P9]. LCRE makes use of the Level Crossing Rate (LCR) information which traditionally has been used in GNSS field only in connection with fading channel characterization and Doppler spread estimation [142,143]. The use of LCR information in distinguishing CNR ranges is suggested in this dissertation and was first introduced by the author in [P8].
Chapter 6

Overview of Publications

This chapter describes the author’s publications that are included in this dissertation, as well as her contribution to these.

6.1 Description of Publications

Figure 6.1: Relation of author’s publications.

Fig. 6.1 illustrates the nine publications that this dissertation includes. It can be seen that the publications can be divided into three thematic categories: publications that focus on (a) ionosphere, (b) multipath propagation and (c) C/N0-related research. The publications included in squares indicate journal or magazine articles and the publications within circles denote conference publications. The arrows are used to denote logical connections. For example,
the research done in [P1] led to the research in [P2] and [P3]. Similarly, the research presented in [P8] was used in [P5] and [P6] and motivated the work in [P9]. In order to facilitate a smooth description of the publications, we describe them according to the fashion they are presented in Fig. 6.1.

Publication [P1] studies the effect of tracking errors on the ionosphere-corrected range in dual-frequency receivers. The dual-frequency methods included in this study are Least Square (LS), Constrained LS (CLS) and Brute Force Constraint (BFC), the latter being introduced in this publication. In addition to the comparison of the above-mentioned methods, this publication investigates which dual frequency combination is better to use, assuming all four Galileo Open Service (OS) signals are available (i.e., E1, E5, E5a and E5b). The results showed that LS and CLS methods perform well only under the assumption of no tracking error, while BFC method has the best average performance. Moreover, we found that the E1-E5a frequency pair is the best combination for a dual-frequency receiver. Finally, this publication contains also a statistical analysis of the performance of LS method, and its comparison with the simulation-based performance showed a very good matching.

Based on the results obtained in [P1] and assuming the context of problem, the following two research questions were born: (a) how well do the dual-frequency methods perform? and (b) is there any benefit by introducing a third pseudorange measurement? These questions were addressed in [P2] and [P3], respectively. More precisely, publication [P2] studies the performance of single and dual frequency GPS and Galileo GNSS receivers in terms of satellite-receiver range estimation. The focus is put on the effects caused by ionosphere and multipath propagation. It is assumed that the available pseudoranges are contaminated with first-order ionospheric delay and measurement errors produced in the code tracking stage. The comparison is made among three dual-frequency methods, LS, CLS and BFC, and two ionospheric models, Klobuchar and NeQuick for GPS and Galileo receivers, respectively. The simulation results showed that in bad channel conditions used for testing, a dual-frequency receiver is superior to a single-frequency one, only when the standard deviation of the measurement error is small and when the correlation factor between the two available pseudoranges is higher than −0.4.

Publication [P3] examines the performance of dual- and triple-frequency code-based Galileo receivers in terms of first-order ionospheric correction and under the assumption that the pseudorange measurements are contaminated with multipath tracking errors. More precisely, this publication compares E1-E5a pair, which was found in [P1] to be the best dual frequency combination,
with all four triple frequency combinations (i.e., E1-E5-E5a, E1-E5-E5b, E1-E5a-E5b, E5-E5a-E5b). The results show that the E5-E5a-E5b triplet (in which the frequency separation is the smaller among other combinations) leads to the worst performance for all cases, except for the case when the correlation factor is between 0.5 and 1 and when the methods used to estimate the range are LS and CLS. The performance of the other triple frequency combinations is similar with the E1-E5a, which leads us to the conclusion that the availability of a third pseudorange measurement in Galileo code-based receivers does not bring any significant benefit with respect to the estimation accuracy, assuming no a-priori information about the “quality” of the measurements is available. However, a third frequency enables redundancy of observations which can be useful in case tracking of one the signals is interrupted or lost.

The first results from the research on the effects of the multipath propagation on the signal tracking were published in publication [P4]. More precisely, publication [P4] presents two tracking structures with multiple gate delays and compares their performance with Narrow Correlator and High Resolution Correlator. The performance criteria are based on the Multipath Error Envelope (MEE), the RMSE and the Mean Time to Lose Lock (MTLL) in multipath channels. It is shown that the traditional MEE criterion can be misleading with respect to the performance in fading multipath channels. It is also shown that the two new multiple delay gate structures have promising tracking performance in short delay multipath scenarios but they are very sensitive to noise.

Publication [P5] presents a new code delay estimator for BOC (used in Galileo) and BPSK (used in GPS) modulated signals, DiscTracker, which combines features from both feedback and feedforward approaches. The proposed estimator is evaluated in multipath static and in multipath fading channels and its performance is compared with a set of state-of-art estimators in terms of RMSE and MTLL. The simulation results show that DiscTracker is robust in closely-spaced multipath scenarios and resistant to non-zero initial code delay errors.

Publication [P6] deals with the problem of joint LOS code delay and carrier phase estimation of GNSS signals in a multipath environment. The problem is formulated into a linear system of equations in which the unknowns are the channel complex coefficients corresponding to each observed signal sample. This publication introduces a modified Projection Onto Convex Sets (mPOCS) algorithm that is optimized for both the new E1 Open Service and Coarse/Acquisition (C/A) signals employed by the future European Galileo
and the GPS, respectively. The performance of mPOCS is compared with other state-of-art deconvolution algorithms. The simulation results indicate that the proposed algorithm is better in closely-spaced multipath static channels both when LOS code delay and carrier phase estimation are concerned.

Publication [P7] studies the impact of multipath propagation effects from a theoretical point of view. More precisely, it includes derivations of the Cramer Rao Lower Bounds (CRLBs) where in one case the unknown parameter vector corresponds to any of the three multipath signal parameters of carrier phase, code delay and amplitude, and in the second case all possible combinations of joint parameter estimation are considered. This publication also examines how various channel parameters, such as carrier to noise ratio spectral density \((C/N_0)\), number of channel paths, path separation, affect the computed CRLBs. Then, the performance of three deconvolution methods, Least Squares, Minimum Mean Square error and mPOCS, is evaluated and compared to the theoretical bounds. The simulations results show that mPOCS performs better than the other deconvolution methods for \(C/N_0\) higher than 35-40 dB-Hz.

The last two publications are related to the estimation of the carrier-to-noise-ratio spectral density of GNSS signals. In particular, publication [P8] proposes a novel method for distinguishing between two \(C/N_0\) ranges, for example those characterising indoor and outdoor regions in satellite-based navigation applications. This \(C/N_0\) identifier is based on the Level Crossing Rate (LCR) of the non-coherently averaged correlation function between the incoming signal and the reference code. The \(C/N_0\) identifier is evaluated in static and fading, single and multipath channels for Galileo and GPS signals. It is shown that by using the proposed identifier it is possible to distinguish the indoor from the outdoor environments in up to 80% - 95% of situations. In addition, the functionality of distinguishing between low and higher CNR ranges was used in the DiscTracker and mPOCS algorithms in publications [P5] and [P6].

Publication [P9] continues the work done in [P8]. Precisely, this publication proposes a new \(C/N_0\) estimation technique called Level-Crossing-Rate Estimation (LCRE) which exploits the statistical characteristics of correlation samples. The results show that the proposed LCRE method performs considerably better than the other three methods under very low \(C/N_0\) conditions, ranging from 5 to 20 dBHz or higher, depending on the scenario. However, the improved performance of LCRE is counterbalanced by its higher computational complexity. Therefore, it is more suitable for commercial receivers.
6.2 Author’s Contribution

The research for this dissertation has been conducted mostly at the Dept. of Communications Engineering of Tampere University of Technology, Finland and has been carried out during several research projects. More precisely, the author has been involved in the following projects: "Advanced Techniques for Personal Navigation (ATENA)" and "Future GNSS Applications and Techniques (FUGAT)" projects, both funded by the Finnish Funding Agency for Technology and Innovation (Tekes), "Galileo Ready Advanced Mass Market Receiver (GRAMMAR)" project funded by EU FP7 and "Digital Signal Processing Algorithms for Indoor Positioning Systems" project funded by Academy of Finland. The work that led to [P7] and [P6] was done during the six-month research visit, between March 2008 and August 2008, at the Dept. of Electrical Engineering of University of California, Los Angeles. All the work presented in this dissertation has been supervised by Adj. Prof. Dr. Elena-Simona Lohan.

The author of this thesis is the main contributor to [P1]-[P8], and has decisively contributed to [P9]. Some of the ideas have originated from the discussions with the supervisor of the author and some of the simulations models have been built in cooperation with her and other members of the research team. Therefore, the author’s contribution cannot be completely separated from the contribution of the co-authors.

More precisely, in [P1], the author designed the BFC algorithm and performed the simulations. The simulation model and the writing of the article were done together with the co-author. In [P2] and [P3], the author performed the analysis and wrote the paper, while the simulation model was built in cooperation with Prof. Lohan. In [P4], the author developed two new multiple gate delay tracking structures. The criteria based on which the structure coefficients were optimized, were chosen after discussions with the co-author. In addition, the author performed the simulations and wrote the paper. The simulation model was built together with the co-author. In [P5], the author designed the algorithm, performed the simulations in cooperation with the co-author and wrote largest part of the paper. In [P6], the author introduced and implemented the modified POCS algorithm. Also, the author optimized POCS algorithm for the Galileo OS signals, performed the analysis and wrote most of the paper. The idea of using POCS was initially introduced by Prof. Lohan. In [P7], the author derived the theoretical bounds, performed the simulations and wrote the most of the article. In [P8], the author conceived
the idea of using LCR information for determining $C/N_0$, built the simulation model, and wrote a big part of the paper. Finally, in [P9], the author performed the simulations and wrote the article with the help of the co-author. The idea of using LCR information for determining $C/N_0$ was conceived by the author, but the theoretical model and the design of the LCR estimator was made in cooperation with Prof. Lohan.
Chapter 7

Conclusions

According to [144], the contributions of a research study can be defined by two constituents. First, the applicability of the study results to a business need in an appropriate environment, i.e., aid in solving relevant problems. Second, in addition to that, the contribution to the archival knowledge base for further research and practice. This dissertation contributes to both.

The increasing number of people using location-based services or applications has essentially increased their dependency on positioning information. Apart from the societal, political or privacy issues arising from such dependency, there is a growing demand for better positioning performance, especially under poor signal conditions. This need has brought GNSS in the center of attention in both the academic and business circles, since it has been the default positioning technology with global coverage. Specifically, a substantial amount of effort is put into mitigating the effects of various error sources that degrade the quality of the satellite signals.

The relevance of this dissertation is based on the fact that the author’s research activities were framed to address the above-mentioned problem. In particular, this dissertation treated the effects caused by the ionospheric layer and the multipath-prone environments, such as densely-built areas. As argued in Chapter 4 and throughout the dissertation, the delay errors experienced by the transmitted satellite signals, due to the layer of ionosphere and the multipath propagation, are potentially the largest among the random delays introduced in the wireless channel. These delays result in range estimation and tracking delay errors, respectively, which consequently have a negative effect on the position solution. Therefore, mitigating such errors is one way of improving the receiver’s operation which in turn affects the user’s experience.
With respect to this, the dissertation contributes by the development of new methods or algorithms.

A new dual-frequency method, called Brute Force Constraint (BFC), for estimating the satellite-receiver range in the presence of ionospheric delays is presented in [P1]. BFC is an iterative method which ensures that, in at most two iterations, the computed solution complies with the constraint of positive electron content. Unlike the traditional dual-frequency method that linearly combines the available pseudoranges \([13]\) and other iterative solutions, such as linear Least Squares and Constrained Least Squares, BFC performs better in the presence of errors other than ionospheric, such as those caused by multipath propagation. In addition, the author suggests BFC because it is a method easy to implement and can be used in future mass-market dual-frequency receivers in situations of high multipath propagation errors.

Two new tracking algorithms for mitigating multipath propagation effects are introduced in [P5] and [P6]. More precisely, the author developed (i) a code delay estimation algorithm, called DiscTracker, which combines features of feedback and feedforward techniques [P5] and (ii) a deconvolution algorithm, called modified Projection Onto Convex Sets (mPOCS), which estimates jointly the carrier phase and code delay of the line-of-sight signal [P6]. The DiscTracker algorithm was shown to perform very well in cases where the acquisition stage results in high errors. However, DiscTracker requires a large processing window, which is a common requirement among feedforward estimators. Therefore, DiscTracker is more suitable for commercial receivers than mass-market ones. In turn, the mPOCS algorithm was shown to have the best performance among other deconvolution methods for mid and higher CNR values (i.e., higher than 40 dB-Hz). However, deconvolution methods, including mPOCS, require also a large processing window, therefore are more suitable for commercial receivers.

In addition to the above tracking algorithms, two novel methods for Carrier-to-Noise-Ratio (CNR) determination are developed which, among other uses, can be utilized to improve the receiver’s tracking performance. Specifically, this dissertation proposes (i) a new method for distinguishing between two CNR ranges [P8], as well as (ii) a CNR estimator, called Level Crossing Rate-based Estimator (LCRE) [P9]. The primary intended use for the first method was to distinguish between the CNR ranges characterizing indoor and outdoor environments; however, other CNR ranges can be used as well. The knowledge about the CNR range enables the adjustment of certain parameters within an algorithm (as done in [P6]) or the switching between two algorithms that are
known to perform better in certain CNR ranges. LCRE method was shown to perform significantly better than other CNR estimators, especially in low CNR environments ranging from 5 to 20 dB-Hz. However, the performance of LCRE is counterbalanced by a higher computational complexity. Therefore, the author suggests LCRE for commercial use where the performance requirements are stricter compared to mass-market and the complexity margins are lower.

Furthermore, this dissertation contributes to the knowledge base by advancing our understanding about the effects of ionosphere and multipath propagation in the performance of the range estimation and the signal tracking functions, respectively. In particular, this dissertation includes an impact analysis of first-order ionospheric and multipath propagation delays on range estimation in single and multi-frequency receivers. Precisely, this dissertation showed that, in contrast to common belief, existing dual-frequency methods can remove completely the ionospheric delays from the pseudorange estimates, only in the absence of additional errors [P1]. In cases where this condition does not hold, e.g., in the presence of multipath propagation error, existing methods fail to remove the ionospheric delays and their range estimation performance deteriorates greatly.

Moreover, this dissertation includes a performance comparison of single and multi-frequency methods. Specifically, in [P2] the author showed that dual-frequency methods are not always superior to single-frequency ones. Instead, their relative performance depends on parameters such as the statistical characteristics of the multipath tracking error and the correlation degree between the pseudorange measurements. Also, in [P3] it was shown that the availability of a third pseudorange measurement does not bring any significant benefit with respect to accuracy of the code-based range estimation, assuming only the first-order ionospheric delay and no prior information about the "quality" of the measurements. This dissertation also provides an impact analysis of multipath propagation effects on the performance of existing methods and methods designed by the author. More precisely, [P7] provided the theoretical limits in estimating the signal's synchronization parameters. These bounds can be used for theoretically assessing the performance of tracking estimators with varying channel parameters, such as CNR, path separation, and number of channel paths. In [P5] and [P6] the author compared the performance of a various tracking estimators. The results indicated that there is no single method that performs the best among all tested scenarios. Instead, the performance depends on various channel and signal parameters. In [P4] the author
showed that the multipath error envelopes, widely-used as metrics for the performance comparison of tracking algorithms, may be a misleading basis for multipath performance assessment. Furthermore, the developed methods also constitute contribution to knowledge, since they are themselves knowledge of "how to" type, and may become a basis for future research.

In conclusion, it is the author’s understanding that complex problems, such as those targeted in this dissertation, cannot always be treated with a single method approach. It is because the performance of a method is affected by numerous parameters which prohibit one-size-fits-all solutions. Therefore, the author believes that in the future the focus shall be placed on developing methods that sense the environment and evaluate the affecting parameters. In that way, receivers could switch among existing methods and achieve optimal performance. Moreover, despite the tremendous importance of GNSS, nowadays, satellite-based positioning is not the only available technology. For example, many smart phones use also cells and wireless local area networks for mobile-positioning as one way to deal with some of the inherent limitations of GNSS-based positioning, such as in city centers and indoor areas. Therefore, the author believes that in the future, it is also important to design devices that depending on the environment, switch to the best available positioning technology. These beliefs represent author’s view on what is important to address in the future research work in the GNSS and generally, in the mobile positioning field.
Bibliography


Publications

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D. Skournetou and E. S. Lohan, "Comparison of single and dual frequency GNSS receivers in the presence of ionospheric and multipath errors", in CD-ROM Proc. of International ICST Conference on Personal Satellite Services (PSATS), February 2011.

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Ionospheric Delay Corrections in Multi-Frequency Receivers: Are Three Frequencies Better than Two?

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Abstract—The amount of mobile applications that utilize positioning information is ever increasing. Together with the demand for more location-aware applications, there is a growing demand for a higher positioning accuracy. In order to increase the accuracy level and to improve the perceived quality of positioning, the different error sources have to be mitigated. Among those, ionosphere delay accounts for the largest delay of the satellite signal and therefore it has to be effectively corrected.

Dual-frequency receivers can effectively remove the first-order ionospheric delay, provided that there are no other errors present in the pseudorange measurements. However, recent work in this field showed that, when the measurements are contaminated also by multipath tracking errors, the performance of dual-frequency receivers degrades significantly. In this paper, we compare the range estimation performance of three algorithms and test the best dual-frequency pair, E1-E5a, together with all possible triple-frequency combinations in terms of ionospheric correction. The results indicate that the availability of a third measurement brings little benefit if any, and only when there is a very high positive correlation between the pseudorange measurements on the three frequencies.

I. INTRODUCTION

Year after year, the sales of mobile devices equipped with Global Navigation Satellite System (GNSS) chipsets is increasing. As positioning information is woven deeper into the fabric of our daily lives, the need for higher accuracy becomes more evident, and in some cases (such as in safety and security critical applications), even imperative. To meet this need, both academia and industry have directed their efforts towards the development of sophisticated hardware and software that would enable higher positioning accuracy.

One way to increase the accuracy level is to account for the various error sources which are present in the outdoor propagation channel and are responsible for delaying the satellite signals. Among those, the ionosphere is responsible for the signal’s biggest delay [4]. More precisely, when the satellite signal travels through the ionospheric layer (located 50-1000 km above the Earth’s surface) it is delayed due to the presence of charged particles (ions and electrons). The amount of the delay is a function of two parameters: the frequency of the signal which is constant and the Total Electron Content (TEC) which depends on various timing and spatial factors and can be modelled as a random variable [12].

In single-frequency receivers (currently, representing the vast majority of mass-market receivers) TEC is estimated with the help of mathematical models, among which Klobuchar model is the most widely used in Global Navigation Satellite (GPS) enabled devices [5] and NeQuick model is the one proposed for the future Galileo receivers [13]. While in single frequency receivers a better estimation of the ionospheric delay is synonymous with increasing the modelling complexity, in dual-frequency receivers no TEC modelling is required.

The availability of two signals which are transmitted from the same satellite but in two different frequencies allows to exploit the fact that they have undergone the same ionospheric effects. More precisely, in the absence of any measurement errors, first-order ionospheric delay can be fully removed via the linear combination of the available pseudorange measurements [4]. However, in recent works of the authors it was shown that the ability of a dual-frequency receiver to mitigate the first-order ionospheric delay errors may significantly degrade when the range measurements are in addition contaminated with other errors such as multipath propagation errors [15], [16]. And this earlier finding serves as the motivation for the work presented here.

In this paper, we study whether a triple-frequency receiver would allow more accurate code-based range estimation than a dual-frequency one in terms of ionospheric correction assuming the presence of multipath tracking errors. More specifically, we compare the best frequency pair found in [16] (i.e., E1-E5a) with all possible triple-frequency combinations in Galileo and we study the range estimation performance of three different algorithms: Least Squares (LS), Constrained Least Squares (CLS) and Brute Force Constraint (BFC) proposed by the authors in [16]. The results indicate that the benefit of the best triple-frequency combination is evident only in very limited cases (i.e., high correlation factor) which may not justify the additional complexity. Thus, in most of the cases, the dual-frequency ionospheric correction remains the best option.

The remainder of this manuscript is organised in the following manner: Section II includes the description of the
model used to analyse the effects of ionosphere and multipath propagation. Section III describes the simulation setup and presents the results. Finally, Section IV summarises the most important findings of our study.

II. System Model

In this section, we briefly describe the system model used in our study which is similar with the one introduced by the authors in [16], only that here we have extended it in a way that more than two frequencies can be considered.

Considering a multi-frequency GNSS receiver able to process up to n frequencies, it is possible to model the available pseudoranges into matricial form as

\[
\begin{bmatrix}
\rho_1 \\
\vdots \\
\rho_n \\
\end{bmatrix} = \begin{bmatrix}
1 & \frac{B}{2l_1} & \cdots & \frac{B}{2l_n} \\
\vdots & \vdots & \ddots & \vdots \\
1 & \frac{B}{2l_1} & \cdots & \frac{B}{2l_n} \\
\end{bmatrix} \begin{bmatrix}
\rho \\
TEC \\
\vdots \\
\end{bmatrix} + \begin{bmatrix}
e_1 \\
\vdots \\
e_n \\
\end{bmatrix}
\]

where \( B = \frac{e^2}{(4\pi^2 m_e e)} \approx 80.6 \text{ m}^3/\text{s}^2, \) \( e = 1.60218 \cdot 10^{-19} \text{ Coulomb} \) is the natural constants for the electron charge, \( m_e = 9.10939 \cdot 10^{-31} \text{ kg} \) is the electron mass and \( e_o = 8.85419 \cdot 10^{-12} \text{ Farad/meter} \) is the permittivity of free space [12]. The first order ionospheric delay is

\[ l_1 = \frac{B}{2f_1} \text{TEC} \]

We remind that here we consider only the first-order ionospheric effects since they account for 99% of the total delay [12] and because the effect of the higher order terms can be considered negligible for the accuracy requirements of mass-market code-based GNSS receivers. However, in the case of precise positioning (such as in carrier phase-based positioning) where the achievable accuracy is in the order of the carrier wavelength (i.e., in the order of few centimeters), higher order ionospheric effects should be taken into account as well and the benefit of a third frequency is evident [17].

For simplicity, in Eq. (1), we take into account only errors attributed to multipath propagation effects because we want to focus on the impact of multipath errors in the estimation of the satellite-receiver range which is characterised by ionospheric delay. However, Differential Code Biases (DCBs) in the satellites or receivers can affect the estimation of TEC [14] but their impact is outside the scope of this study. Finally, we notice that most of the errors sources affecting the transmitted signals from a single satellite are the same since they travel through the same medium. So, common errors (e.g., ephemeris, tropospheric and clock errors) can be easily removed by subtracting one of the two pseudoranges from the other [4].

Equivalently, Eq. (1) can be represented in a compact manner as

\[ \mathbf{r} = \mathbf{A} \mathbf{x} + \mathbf{e} \]

where \( \mathbf{r} \) is the observation vector that contains the pseudorange measurements, \( \mathbf{A} \) is a \( n \times 2 \) matrix, \( \mathbf{x} \) is the unknown parameter vector to be estimated and \( \mathbf{e} \) is the measurement error vector.

The measurement error represents the residue of the processing done in the code tracking stage. We notice that the code tracking error is different for different signals because it depends on signal-specific characteristics such as type (i.e., data or pilot), modulation, frequency, etc. and it represents mostly the effects of multipath propagation [16].

The unknown range and TEC parameters are estimated by three different methods: LS, CLS and BFC. The LS solution is computed as

\[ \mathbf{x}_{LS} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{r} \]

where \( \mathbf{T} \) denotes the operation of transposition. CLS minimises the squared difference between the observed data and \( \mathbf{Ax} \), subject to the linear inequality constraint \( \mathbf{Ax}_{CLS} \geq \mathbf{b} \), where \( \mathbf{b} = [0 \ 0] \). BFC offers also a constrained solution but at a lower complexity than CLS. Detailed description of the above-mentioned algorithms and explanation on the constraint choice can be found in [16].

III. Simulation Setup and Results

In this section, we compare the range estimation performance of dual- and triple-frequency receiver methods in terms of Root Mean Square Error (RMSE). The satellite system of interest is the future Galileo. In the case of dual-frequency receivers, we used the best pair found in [16], i.e., E1-E5a and for the triple-frequency case we examine all four possible combinations of E1, E5, E5a and E5b signals.

In order to compute the RMSE performance values, we generate 1000 random realisations of the signal and of the measurement errors. More precisely, we assume that the true range \( \rho \) is uniformly distributed between \( 18000 \text{ and } 25000 \text{ km} \) and the TEC is uniformly distributed either between \( 1 \) and \( 100 \) or between \( 1 \) and \( 200 \) TEC Units (TECU). The limits of the TEC parameter have been chosen in such a way that typical values encountered in various latitudes are included [2], [3], [9]. In our simulations, we do not employ a specific multipath channel profile. Instead, we model the measurement error as the tracking error attributed to the multipath propagation effects. Particularly, the error \( e \) is modelled as a random variable that follows the normal distribution (the assumption of normal distribution has been commonly encountered in the literature [6], [7], [10], [11] and is used here for simplicity).

More precisely, the errors are distributed according to \( e_i \sim \mathcal{N}(\mu_i, \sigma_i^2) \), where \( \mu_i \) is the mean error of the i-th frequency in chips and \( \sigma_i \) takes values from 0 to 0.1 chips with a step of 0.01 chip. We notice that the values of standard deviation were chosen in such a way that large multipath errors reported in the literature, such as in [1], [8], are considered (see Table I for chips-to-meters mapping). We remark that for the sake of simplicity, the measurement errors in all frequencies are assumed to follow the same distributions. Moreover, in the triple-frequency case, if for example, the error in \( f_1 \) frequency is correlated with the errors of the other two frequencies (\( f_2 \) and \( f_3 \)) then we assume that the correlation originates equally from the other two frequencies since the exact correlation factor cannot be determined beforehand.
In Fig. 1, we see the RMSE versus the standard deviation of error in the case of zero mean, uncorrelated measurements when TEC varies between 1 and 100 TECUs. We observe that all but one frequency combinations lead to similar performance for all three methods. More precisely, the E5-E5a-E5b triplet seems to yield worse RMSE values in the case of LS and CLS methods (middle and bottom plots). When the measurements are fully positively correlated (see Fig. 2), the E5-E5a-E5b combination is the worst only in the case of BFC (top plot) while the other combinations give the same performance for all three algorithms.

In Fig. 3, we see how RMSE is affected by different correlation factors, when the mean error remains zero and the standard deviation error is fixed to 0.01 chips (we notice that according to the author’s best effort, typical correlation values for certain environments were not found in the literature and that is why all possible correlation values are tested here). Interestingly, for high correlation factors (i.e., between 0.5 and 1), the E5-E5a-E5b combination leads to lower RMSE than E1-E5a and when LS or CLS methods are used. In Fig. 4, we see only the best triple frequency combination from Fig. 3) and compare it with the E1-E5a in the case 0.8 correlation factor and when the CLS method is used. In this case, we notice that the triple frequency combination is better for all standard deviation errors higher than zero. If we increase the standard deviation error to 0.05 chips, the relative performance of the various frequency combinations remains similar with the one in Fig. 3, only that now in the case of BFC E5-E5a-E5b gives better results (see Fig 5). We also see that BFC performs the best for all correlation factors.

Figs. 6 and 7 show the RMSE values versus standard deviation of error and correlation factor in the case where TEC varies between 100 and 200 TECUs. Interestingly, LS and CLS methods were not affected by the higher TEC, while the performance of BFC is somewhat deteriorated. Moreover, in the case of BFC, E5-E5a-E5b combination led to lower RMSE values than those seen in the case of lower TEC range.

Finally, in Figs. 8 and 9 we see what is the impact of non-zero mean error. More precisely, we notice that as expected the performance of all algorithms is degraded. Regarding the relative performance of the five frequency combination, E5-E5a-E5b remains the worst option, while all the rest lead to

| TABLE I
STANDARD DEVIATION OF MEASUREMENT RANGE ERROR: CHIPS-TO-METERS MAPPING. |
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<tr>
<td><strong>Signals</strong></td>
<td><strong>σ</strong></td>
<td></td>
<td>0.01</td>
<td>0.02</td>
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<td>0.04</td>
<td>0.05</td>
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<td>1.759</td>
<td>2.052</td>
<td>2.346</td>
<td>2.639</td>
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Fig. 1. RMSE vs. standard deviation of error (σ), TEC= [1 100], μ = 0 and λ = 0.

Fig. 2. RMSE vs. standard deviation of error (σ), TEC= [1 100], μ = 0 and λ = 1.
Fig. 3. RMSE vs. correlation factor ($\lambda$), TEC = [1 100], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 4. RMSE vs. standard deviation of error ($\sigma$) for CLS, TEC = [1 100], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 5. RMSE vs. correlation factor ($\lambda$), TEC = [1 100], $\mu = 0$ and $\sigma = 0.05$ chips.

Fig. 6. RMSE vs. standard deviation of error ($\sigma$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 7. RMSE vs. correlation factor ($\lambda$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.05$ chips.

Fig. 8. RMSE vs. standard deviation of error ($\sigma$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 9. RMSE vs. correlation factor ($\lambda$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.05$ chips.

Fig. 10. RMSE vs. standard deviation of error ($\sigma$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 11. RMSE vs. correlation factor ($\lambda$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.05$ chips.

Fig. 12. RMSE vs. standard deviation of error ($\sigma$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 13. RMSE vs. correlation factor ($\lambda$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.05$ chips.

Fig. 14. RMSE vs. standard deviation of error ($\sigma$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 15. RMSE vs. correlation factor ($\lambda$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.05$ chips.

Fig. 16. RMSE vs. standard deviation of error ($\sigma$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 17. RMSE vs. correlation factor ($\lambda$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.05$ chips.

Fig. 18. RMSE vs. standard deviation of error ($\sigma$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 19. RMSE vs. correlation factor ($\lambda$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.05$ chips.

Fig. 20. RMSE vs. standard deviation of error ($\sigma$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 21. RMSE vs. correlation factor ($\lambda$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.05$ chips.

Fig. 22. RMSE vs. standard deviation of error ($\sigma$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 23. RMSE vs. correlation factor ($\lambda$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.05$ chips.

Fig. 24. RMSE vs. standard deviation of error ($\sigma$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 25. RMSE vs. correlation factor ($\lambda$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.05$ chips.

Fig. 26. RMSE vs. standard deviation of error ($\sigma$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 27. RMSE vs. correlation factor ($\lambda$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.05$ chips.

Fig. 28. RMSE vs. standard deviation of error ($\sigma$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.01$ chips.

Fig. 29. RMSE vs. correlation factor ($\lambda$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.05$ chips.

Fig. 30. RMSE vs. standard deviation of error ($\sigma$), TEC = [100 200], $\mu = 0$ and $\sigma = 0.01$ chips.

IV. CONCLUSION

In this paper, we examined the performance of dual- and triple-frequency code-based receivers in terms of first-order ionospheric correction and under the assumption that the pseudorange measurements are contaminated with multipath tracking errors. More precisely, we tested E1-E5a pair, which was found by the authors to be the best dual frequency combination, with all four triple frequency combinations (i.e., E1-E5-E5a, E1-E5-E5b, E1-E5a-E5b, E5-E5a-E5b). The results showed that the E5-E5a-E5b triplet (in which the frequency

similar results.
Fig. 7. RMSE vs. correlation factor (J), TEC = [100 200], μ = 0 and σ = 0.05 chips.

Fig. 8. RMSE vs. standard deviation of error (σ), μ ≠ 0, TEC = [1 100] and λ = 0.

The performance of the other triple frequency combinations is similar with the E1-E5a, which leads us to the conclusion that the availability of a third pseudorange measurement does not bring any significant benefit with respect to the estimation accuracy (assuming no a-priori information about the "quality" of the measurements is available). However, a third frequency enables redundancy of observations which can be useful in case tracking of one of the signals is interrupted or lost.

We notice that our study does not cover precise positioning applications (such as carrier phase-based) where the impact of higher order ionospheric effects is important and in which case the availability of a third frequency has been proven to be beneficial. In addition, our simulations were done under the assumptions of Gaussian distributed multipath errors and absence of DCBs biases; more remains to be investigated about the possible advantage of a triple-frequency receiver over a dual-frequency one.

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REFERENCES


Publication P4


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Publication P5


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ABSTRACT

Undoubtedly, accurate Line-of-Sight (LOS) code delay estimation is encountered among the most challenging synchronization problems in the context of Global Navigation Satellite Systems (GNSSs). However, the way towards achieving high accuracy is seriously hindered by the various error sources affecting the transmitted satellite signal. Particularly, the destructive nature of multipath environments has posed several limitations in the tracking performance, thus increasing the need for estimators, resistant against multipath delays.

In this paper, we present a new code delay estimator for BOC and BPSK modulated signals, the so-called DiscTracker, which combines features from both feedback and feedforward approaches. The proposed estimator is tested in multipath static and in multipath fading channels and its performance is compared with a set of state-of-art estimators in terms of Root Mean Square Error (RMSE) and Mean Time To Lose Lock (MTLL). The simulation results show that DiscTracker is robust in closely-spaced multipath scenarios and resistant to non-zero initial code delay errors.

I. INTRODUCTION

The volume of reports and forecasts about the GNSS based service market is continuously increasing and the economic indexes show that doing business in the GNSS arena can be very lucrative. Moreover, the growing demand for Portable Navigation Devices (PNDs) has imposed stricter requirements on the positioning accuracy and thus, multiple efforts are put from the research community to meet the required standards.

Among the functions that a GNSS receiver performs in order to produce the positioning solution, code synchronization plays a crucial role in the achieved accuracy. After the satellite signal has been received, the acquisition stage takes place from which coarse estimates of the Doppler shift and the code delay estimate are produced. Code tracking is the next stage in the synchronization module and it is responsible for following the time changes of the code delay and producing fine estimates.

Improving the tracking capability of the receiver can lead to significant improvement in the performance of the receiver. However, there is a large number of error sources that affect the transmitted signal and distort it, thus making the tracking more difficult. One of the most challenging problems in this stage is the mitigation of the multipath effect due to wireless propagation channel characteristics (i.e. the received signal is a result of the superposition of two or more delayed versions of itself).

In general, the tracking method to be used might be restricted by the type of modulation. A new modulation technique, called Binary Offset Carrier (BOC), has been introduced for future GNSS signals, including several GALILEO signals as well as GPS M-code [1]. The main characteristic of BOC modulated signals is that the envelope of their autocorrelation function (Acf) contains several peaks. The number of these peaks depends on the order of the BOC modulation. Compared to the Binary Phase Shift Keying (BPSK) modulation used in GPS, with only one triangular-shaped peak in the Acf (if unlimited bandwidth is assumed), BOC modulation introduces more challenges due to the presence of multiple peaks (e.g. possibility to track a wrong peak is higher).

Undoubtedly, designing receivers capable of processing signals regardless of their modulation type is highly desirable. In our paper we deal with two modulation schemes: BPSK used in the GPS signals and Sine-BOC(1, 1) proposed for the Galileo Open Service (OS) signals. An example of the averaged correlation function for BPSK and Sine-BOC(1, 1) modulation for one path channel, no noise, zero residual Doppler error and unit bit energy is shown in Fig. 1. We notice that recent standardization documents specify Multiplexed BOC (MBOC) as the modulation type to be employed for OS signals. However, since MBOC is a combination of Sine-BOC(1, 1) and Sine-BOC(6, 1), the authors believe that the estimator presented here can be applicable for the MBOC case as well [2]. Nevertheless, further investigation is to be made for MBOC cases, and it is out the scope of this paper.

Several code tracking algorithms have been proposed in

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the literature with the target to keep track on the LOS path. The majority of them belong to the general group of feedback delay estimators which is based on a feedback loop. Among the most popular estimators belonging to the above group, we find the Delay Locked Loops (DLLs) where the main idea is to place a discrete amount of correlators along the signal’s correlation function and combine them in such a way that high tracking performance is achieved. When only 3 correlators are used (one early, one late and one in-prompt), we have the widely known Early-Minus-Late (EML) structure. Narrow Correlator was the first approach to reduce the chip spacing between a correlator pair, leading to a significant reduction of the multipath error [3]. Other state-of-art techniques include Double Delta Correlator (ΔΔ) which uses a linear combination of 4 correlators (2 early and 2 late correlators, plus the in-prompt correlator) [4], Strobe and Edge correlators [5] which combines two different narrow correlator discriminators, High Resolution Correlator (HRC) which is a particular case of Double Delta correlator [6] and other optimized Multiple Gate Delay (MGD) structures [7].

In general, the feedback-based estimators are sensitive to closely spaced path scenarios and potential initial code delay error from the acquisition stage. Moreover, their feedback nature can lead to accumulated errors and eventually to the loss of lock. As an alternative solution, various feedforward approaches have been proposed in the literature [8]. While improving the delay estimation accuracy, these approaches might require more correlators than EML approaches and are sensitive to the noise-dependent threshold choice. A new generation of hybrid delay estimators has been launched, based on the combination of both feedback and feedforward techniques with the aim of achieving robust estimation in adverse multipath environments [9]. This paper presents a novel hybrid delay estimator for efficient multipath mitigation and resistance to initial delay errors.

This manuscript is organized in the following manner: First, Section 2 introduces the signal model we employed in our research. Second, Section 3 presents the proposed code delay estimator, namely DiscTracker, and includes a detailed description of its procedure. Next, we introduce the estimators we used for performance comparison and describe briefly their concepts. Third, Section 4 contains the performance analysis of the chosen estimators as well as the simulation results. Last, Section 5 is dedicated to the conclusions of this work and future plans.

II. SIGNAL MODEL

The signals of interest here are GPS and Galileo civil signals. Both use a Direct-Sequence Code Division Multiple Access (DS-CDMA) technique, and either BPSK for C/A code of GPS, or Sine Binary-Offset-Carrier (BOC) modulation, for Galileo Open Services (OS) [10], [11]. Therefore, the transmitted signal \( x(t) \) can be written as the convolution between the modulating waveform \( s_{BOC}(t) \), the PseudoRaNdom (PRN) CDMA code and the data modulation[12]:

\[
x(t) = \sqrt{E_b}s_{BOC}(t) \ast \sum_{n=-\infty}^{+\infty} \sum_{k=1}^{S_F} b_n c_{k,n} \delta(t - nT_{sym}) - kT_c)\]

where \( E_b \) is the data bit energy, \( \ast \) is the convolution operator, \( b_n \) is the \( n \)-th complex data symbol (in case of a pilot channel, it is equal to 1), \( T_{sym} \) is the symbol period, \( c_{k,n} \) is the \( k \)-th complex symbol corresponding to the \( n \)-th symbol, \( T_c \) is the chip period, \( S_F \) is the spreading factor (\( S_F = T_{sym}/T_c \)) and \( \delta(t) \) is the Dirac pulse. Here, \( N_B \) is a modulation-related parameter, also named as BOC-modulation order (this was detailed in [12]). For example, for the most encountered GNSS modulations, namely BPSK and Sine-BOC(1,1) modulations, we have: \( N_B = 1 \) and \( N_B = 2 \), respectively. Above, \( s_{BOC}(t) \) stands for both BPSK and Sine-BOC modulated signals, and it can be expressed as in eq. (2) [12] (for Cosine-BOC modulation, the detailed expression of \( s_{BOC}(t) \) is given in [12]):

\[
s_{BOC}(t) = \text{sign} \left( \sin \left( \frac{N_B \pi t}{T_c} \right) \right), 0 \leq t \leq T_c \]

\[
= p_{TB}(t) \oplus \sum_{i=0}^{N_B-1} (-1)^i \delta(t - iT_B)\]

where \( p_{TB}(t) \) is the pulse shaping filter applied to pulses of duration \( T_B = T_c/N_B \). For instance, if infinite bandwidth is assumed, \( p_{TB}(t) \) will be a rectangular pulse of unit amplitude if \( 0 \leq t \leq T_B \) and 0 otherwise.

The signal \( x(t) \) is typically transmitted over a multipath static or multipath fading channel where all interference
sources (except for the multipaths) are lumped into a single additive Gaussian noise term \( \eta(t) \):

\[
r(t) = \sum_{p=1}^{P} \alpha_p x(t - \tau_p) e^{-j2\pi f_D t} + \eta(t),
\]

where \( r(t) \) is the received signal, \( \alpha_p \) is the complex coefficient of the \( p \)-th path, \( \tau_p \) is the channel delay introduced by the \( p \)-th path, \( P \) is the number of channel paths, \( f_D \) is the Doppler shift introduced by the channel, and \( \eta(t) \) is the additive Gaussian noise of zero mean and double-sided power spectral density \( N_0 \).

Both acquisition and delay tracking stages (i.e., code synchronization) are usually based on the code epoch-by-code epoch correlation \( \mathcal{R}(\cdot) \) between the incoming signal and the reference \( x_{ref}(\cdot) \) modulated PRN code, with a certain candidate Doppler frequency \( \hat{f}_D \) and delay \( \hat{\tau} \):

\[
\mathcal{R}(\hat{\tau}, \hat{f}_D, m) = E\left( \frac{1}{T_{sym}} \int_{m-1}^{m} e^{-j2\pi f_D t} r(t) x_{ref}(\hat{\tau}, \hat{f}_D) dt \right),
\]

where \( m \) is the code epoch index and \( E(\cdot) \) is the expectation operation, with respect to the PRN code, and

\[
x_{ref}(\hat{\tau}, \hat{f}_D) = \left( s_{BOC}(t - \hat{\tau}) + \sum_{n=-\infty}^{+\infty} \sum_{k=1}^{P} \hat{b}_n c_{k,n} \delta(t - nT_{sym} - kT_c - \hat{\tau}) e^{+j2\pi \hat{f}_D t},
\]

where \( \hat{b}_n \) are the estimated data bits. For Galileo signals, a separate pilot channel is transmitted, thus the data bits are known at the receiver [11]. In order to reduce the noise level, both coherent and non-coherent integration are typically used. The averaged non-coherent correlation function \( \bar{\mathcal{R}}(\hat{\tau}, \hat{f}_D) \) can be written as:

\[
\bar{\mathcal{R}}(\hat{\tau}, \hat{f}_D) = \frac{1}{N_{nc}} \sum_{n_{nc}} \left| \frac{1}{N_c} \sum_{m=1}^{N_c} \mathcal{R}(\hat{\tau}, \hat{f}_D, m) \right|^2
\]

where \( N_c \) is the coherent integration time (expressed in code epochs or ms for GPS/Galileo signals) and \( N_{nc} \) is the non-coherent integration time, expressed in blocks of length \( N_c \) ms.

### III. CODE DELAY ESTIMATORS

This section describes the set of code delay estimators which were used in our research. First, we introduce the concept of our main contribution, a newly proposed estimator, called DiscTracker, present its procedure and give a step by step description. Second, we overview the rest of the estimators, namely Early-Late-Slope (ELS), Matched Filter (MF), Peak Tracking using 2nd order Differential (PT(Diff2)), Teager Kaiser (TK), 2nd order Differential (Diff2), Narrow Early-Minus-Late (NEML) and High Resolution Correlator (HRC), employed here for comparative reasons.

![Fig. 2. Example of positive discontinuity points on the non-coherent Acf for a 3-path Nakagami-m fading channel.](image)

#### III-A. DiscTracker conceptual basis

DiscTracker encompasses features from both feedback and feedforward estimation approaches, as well as it introduces two new concepts. In particular, the feedback/feedforward combination allows for reducing the impact of initial code delay error \( (\hat{\tau}_{init}) \) in the LOS code delay estimate, as it will be shown in Section IV.

In addition to the above-mentioned characteristic, DiscTracker envisages the novel idea of positive Discontinuity Points \( (pDPs) \). We noticed that the existence of the LOS path appears as a Discontinuity Point (DP) in the Acf, in contrast with the previously implemented feedforward algorithms (i.e., MF, PT(Diff2)) which assume only peaks as candidates. Moreover, we empirically found that the majority of LOS paths appear as positive DPs (i.e., DPs that form convex angles) and thus will consider only p-DPs in what follows. In order to visualize better the above-mentioned we can see in Fig. 2 a snapshot of the Acf where the presence of the three paths (here closely spaced) is clearly indicated by three positive discontinuity points.

The second novel idea involved in DiscTracker procedure is based on the Level Crossing Rate (LCR) which is defined as the number of crossings (both from below and from above) of Acf at level \( l_j \). The potential of LCR information in GNSS context was first introduced by the authors in [13], where it was found that it can be used as a reliable indoor/outdoor CNR identifier. However, the LCR information here is employed with the target of thresholding the Acf, in such a way, that the most of its noisy part is left out. In what follows, we find a step-by-step description of the DiscTracker procedure.

#### III-B. DiscTracker procedure

This subsection contains a detailed description of the DiscTracker procedure which consists of five steps:
• Step 1
Find the set of positive discontinuity points \( (pDP) \) within a specified Acf window \( (W) \) according to:

\[
pDP = \{ w_i \in W \mid 0 \degree < \varphi_i < 170 \degree \} \tag{3}
\]

where \( w_i = Acf(s_i), s_i \in S \) is the \( i \)-th time sample of the time window \( S \) and \( \varphi_i = \angle w_{i-1}w_iw_{i+1} \), where \( \angle \) denotes the inner angle formed by a 3-point set \( (w_{i-1}, w_i \) and \( w_{i+1}) \). Each angle \( \varphi_i \) is simply computed as the tangent difference \( \tan(w_{i-1}w_i) - \tan(w_iw_{i+1}) \) of the 3-point set. The reason for keeping only the pDPs which result in angles smaller than \( 170 \degree \) is because we assume the LOS path to have a more evident presence in the Acf (e.g., we do not look for the LOS path in a sequence of time samples that form a straight or almost straight line).

• Step 2
Compute the parabolic threshold \( U \), which is of the form

\[ U(s) = as^2 + bs + c, \quad a > 0 \]

The vertex of the parabola \( (V_U) \) is defined as:

\[
V_U(v_x, v_y) = V_U\left( -\frac{b}{2a}, \frac{b^2}{4a} + c \right) \tag{4}
\]

\[
= V_U(s_{\text{max}}, 0.3 + l_{\text{max}}) \tag{5}
\]

where

\[
s_{\text{max}} = \{ s_i \in S \mid \arg \max_s Acf = Acf(s_i) \}
\]

\[
l_{\text{max}} = \{ l_j \in L \mid \arg \max_l LCR = LCR(l_j) \}
\]

The set of positive discontinuity points which are located above the threshold forms another set \( \overline{pDP} \) where \( \overline{pDP} \subset pDP \).

In difference with other feedforward estimators which use linear thresholding, DiscTracker employs a non-linear threshold with parabolic shape as it is shown in Fig. 3. The advantage of such threshold is that it allows us to avoid a bigger part of the noisy Acf and ideally excluding the secondary peaks coming form the BOC modulation type. More precisely, a parabola can be described by three characteristics: direction, opening and vertex. The direction is determined by the sign of \( a \) parameter and if it is positive then parabola points up (as in our case). The opening of the parabola is described by the value of \( a \) which here has been chosen based on the requirement to include the main Acf lobe and is equal to \( 10^{-2.5} \). The possibility of adopting a value for \( a \) which dynamically adjusts itself according to the channel profile is of our interest for future research. We remark that the other parameters, \( b \) and \( c \) of the parabola can be calculated by solving a system of two equations and two unknowns. This system is created by simply equating the representations of \( v_x \) and \( v_y \) from Eq. 4 and Eq. 5, respectively.

The last characteristic of a parabola is its vertex \( (V) \), meaning the global minimum \( (a > 0) \) or the global maximum \( (a < 0) \). The position of \( V \) in the time axis \( (v_x) \) is determined by the position of the Acf maximum. The rationale for such choice lies in the assumption that it suffices to search for the LOS path in an area around the maximum (for the scenarios where signal dominates noise). The position of \( V_x \) in the y-axis is defined in such a way that most of the floor noise is discarded. This is achieved by looking at which level of the Acf, \( l_j \in L \), the LCR is maximized or equivalently looking at the level that appears to be the most noisy. For instance, in Fig. 4 we see a snapshot of the Acf at CNR=30 dB-Hz where the LCR at level 0.2 is equal to 4 (i.e. number of crossing both from below and above). Further, we can observe, that at lower levels the LCR will be larger than in higher levels, thus indicating the presence of noise. Consequently, the level that results to the maximum LCR will reveal also the level in which Acf is the most noisy.

\[ \text{The vector } L, \text{ containing all levels the LCR is to be computed with, is given by } L = [2 \cdot 10^{-3} : 2 \cdot 10^{-3} : 8 \cdot 10^{-3}] \cup [10^{-2} : 10^{-2} : 1]. \]

The smaller step size at the low level was introduced in order to avoid wrong interpretations about noise when CNR is very high and the noise is below the level of 0.01. Computational time was the main reason for not increasing further the resolution. Lastly, the constant value 0.3 was empirically chosen with the target of avoiding the noisy area around \( l_{\text{max}} \) in the case of lower CNR. Moreover, when CNR is high and noise level is very low, such a value ensures that the vertex will be located above the Acf sidelobes (notice that for the case of Sine-BOC(1,1) modulated signals and 1-path channel, the maximum value of sidelobes is approximately 0.25 as can been seen at the top of Fig. 1). We also remark that in order to compute the LCR and the threshold \( U \), the time window \( S \) (measured in chips) was placed \( \pm \delta \) chips around the previous estimate \( \tilde{\tau}_{l-1} \), where \( \delta = \frac{S}{2} \).

• Step 3

![Fig. 3. Snapshot of Acf with DiscTracker threshold (CNR=30 dB-Hz).](image-url)
Find the positive discontinuity point $pDP_i \in pDP$ which is closest to the previous LOS estimate $\hat{\tau}_{p=1,t-1}$ ($pDP_i \in pDP$). This $pDP_i$ would be the first candidate ($C_1$) for the code delay estimate ($\hat{\tau}_t$) at time instance $t$ and is defined as $C_1 = \arg \min |\hat{\tau}_{t-1} - pDP_i|$. This candidate relies fully on the previous estimate of the LOS path ($\hat{\tau}_{t-1}$), thus justifying the feedback nature of the estimator.

- **Step 4**
  Find the first $pDP_j \in pDP$ which is higher than the threshold $U$. This $pDP_j$ would be the second candidate ($C_2$) for $\hat{\tau}_t$ and it is defined as: $C_2 = pDP(1)$. This candidate is found based only on the Acf, thus supporting the feedforward approach.

- **Step 5**
  If $C_1 = C_2$, then the code delay estimate is given by $\hat{\tau}_t = C_1 = C_2$. If $C_1 \in pDP$, we set $\hat{\tau}_t = C_1$ and if $C_1 \notin pDP$, then $\hat{\tau}_t = C_2$. In this last step of the procedure a conditional decision for estimating the LOS path takes place. The decision is based on the relation between $C_1$ and $C_2$, whereas the aim is to reduce the impact of erroneous feedback information by assigning more weight to the feedforward estimate. In the following section, we present the simulation results and a comparative analysis on the performance of the estimators in various channels profiles.

**III-C. Other estimators**

The set of code delay estimators we chose for performance comparison with DiscTracker include feedforward-based, feedback-based and hybrid (combination of feedforward and feedback approaches) estimators. Below, we find a brief description of their procedures:

- **ELS** estimator utilizes the feedback approach in which the estimate of the LOS delay at time instance $\hat{\tau}_t$ is a function of the previous estimate $\hat{\tau}_{t-1}$ [14].

The general idea of ELS is to determine the slope at both sides of the Acf central peak. Once both slopes are known and assuming that the central peak indicates the LOS presence, we can estimate the code delay. The ELS version we used in our simulations utilizes a threshold $U_{ELS}$ (in contrast to the version presented in [14]) for improving the estimation accuracy. In particular, $U_{ELS}$ is equal to the Acf level $l_{mean}$ which corresponds to the average LCR ($l_{mean} = \{l_j \in L \mid \text{mean}(LCR) = LCR(l_j)\}$). The choice of such an adaptively adjusted threshold improved the performance of the estimator. However, a more detailed description of the improved ELS procedure is not of our interest here.

- **MF** estimator is a representative of the feedforward approach in which the LOS code delay is estimated at a single step. More precisely, the delay estimate $\hat{\tau}_t$ is simply defined as the sample $w_i$ which corresponds to the first Acf peak that exceeds a predefined linear threshold $U_{MF}$. For the case of Sine-BOC(1,1) modulated signals the threshold is defined as $U_{MF} = 1 = SL_1 + \Delta n$, where $SL_1$ is the amplitude of the first sidelobe from the squared envelope of the ideal autocorrelation function (ACF) and $\Delta n$ is the estimated noise variance. In this context, the ideal Acf is the reference correlation function in the absence of multipath and which can be computed from the ideal codes. The noise variance is estimated from the noncoherent Acf with the use of first order statistics. In the case of BPSK-modulated signals, the threshold is computed as $U_{MF} = 0.2 + \Delta n^2$, where the noise variance is computed as in the case of Sine-BOC(1,1) signals and the constant value of 0.2 was empirically found to give the optimum support in the threshold computation.

- **PT(Diff2)** estimator is a hybrid approach, meaning that it combines both feedforward and feedback techniques. The main idea of this estimator is to create a set of competitive peaks for the LOS path and then to assign them with weights based on three predefined criteria (height, position and distance from the previous estimate). The search for the competitive peaks is done on the Acf after the operation of second order differentiation has taken place. The delay estimate $\hat{\tau}$ is simply the time sample that corresponds to the competitive peak that has the largest overall weighting. A detailed description of PT(Diff2) procedure can be found in [15].

- **TK** is a feedforward estimator that utilizes the nonlinear quadratic operator introduced by Teager and Kaiser for measuring the real physical energy of a system [16]. However, this operator has been also applied in various code division multiple access (CDMA) applications [17], [18], [15]. Here, the estimate of the LOS delay is defined as the time sample that corresponds to the first peak which exceeds a predefined threshold.
The path separation between successive paths when the path delays are expressed in samples. The coefficient, (assumed in the simulations to be equal to 1 of the chip. The oversampling factor (i.e., the number of samples per BOC interval) is equal to \( \alpha \) and the processing of Acf is done in a window \( t_0 \) distributed between \( 0.2 \) chips.

**IV. PERFORMANCE ANALYSIS**

The main target of the simulations was to compare the performance of the proposed DiscTracker with a set of estimators utilizing three general techniques used in code delay estimation: feedback-based, feedforward-based and combination of both. Furthermore, the estimators were tested for Sine-BOC(1, 1) and BPSK modulated signals. First, we give a detailed description of the channel profiles and parameters we used in our research. Second, we present the results and analyze the estimators performance in terms of Root Mean Square Error (RMSE) and Mean Time to Lose Lock (MTLL).

**IV-A. Simulation parameters**

In the simulations we considered static and fading channels. More precisely, the model we used employs a decaying Power Delay Profile (PDP), meaning that \( \overline{\sigma_p} = \overline{\sigma_1}e^{-\xi_{PDP}(\tau_p-\tau_1)} \), where \( \overline{\sigma_1} \) is the average amplitude of the 1-st path and \( \xi_{PDP} \) is the power decaying profile coefficient, (assumed in the simulations to be equal to 0.09 when the path delays are expressed in samples).

The number of channel paths was uniformly varying between 2 and 4 paths for both static and fading channel. The path separation between successive paths \( \tau_{p,t} + t \) at any time instance \( t \), was assumed to be uniformly distributed between 0 and 0.35 chips (i.e., closely-spaced paths, typical in indoor and densely populated urban scenarios), while the variation of the first path delay was also assumed uniformly distributed between \(-0.05\) and \(0.05\) chip. The oversampling factor (i.e., the number of samples per BOC interval) is equal to 10 and the processing of Acf is done in a window \( S \) of 8 chips length. The coherent integration time \( (N_c) \) was set to 20 ms, while the non-coherent integration \( (N_{nc}) \) was performed in 2 blocks of \( N_c \) length. The speed of the mobile receiver was set equal to 4 km/h and the speed of the satellite was assumed to be 8000 km/h. In the case where the receiver’s bandwidth is limited, we used \( B_M = 8 \) MHz and Butterworth type of filter with 0.1 dB loss in passband, 40 dB attenuation in stopband and transition bandwidth equal to \( \frac{B_M}{2} \). For the case of fading channel we used Nakagami-m type, where Nakagami m-factor was equal to 0.65.

It is further assumed that there is no residual Doppler error from the acquisition stage. The simulations are based on \( N_{rand} = 8000\) random realizations (of channel and signals), each realization having an observation interval of \( N_cN_{nc} \) ms. RMSE is computed over the total number of estimates (here 8000), while MTLL is defined as the number of estimates which resulted in errors equal or less than the width of the main lobe (approximately 0.35 chip and 1 chip for Sine-BOC(1, 1) and BPSK modulated signals, correspondingly).

**IV-B. Simulation results**

For better clarity, we present first the simulation results in which Sine-BOC(1, 1) modulated signals are used and then the results for BPSK signals. We start our analysis by observing the performance of the estimators in multipath static channel. In Figs. 5 and 6, we see how RMSE and MTLL vary according to different CNR levels. HRC seems to benefit the most in terms of RMSE when CNR increases and it is followed by DiscTracker. The RMSE of most of the estimators varies between 30 and 40 meters, while Diff2 appears to perform the worst. Regarding MTLL, we notice no significant differences among the estimators.

The results for the case of Nakagami-m channel and \( \hat{\tau}_{init} = 0 \) chip are shown in Figs. 7 and 8. We observe
that DiscTracker performs better in the fading scenario than in the static and that for higher CNRs it becomes slightly better than HRC. MTLL curves show that the relative performance of the estimators is similar to the RMSE-based one. Figs. 9 and 10 show the estimators’ performance for the case when the initial code delay error is increased to 1 chip. In comparison with Figs. 7 and 8, we notice that DiscTracker’s performance is affected the least. More precisely, the RMSE in DiscTracker increased only by a couple of meters for the higher CNRs, while in HRC RMSE was increased by more than 10 meters. We also notice the ELS was affected the most by this increase in the initial error. The RMSE of the other estimators is affected the least and lies within the range of 20 to 40 meters.

Concerning MTLL curves, we notice that DiscTracker performs the best for CNR higher than 30 dB-Hz, while ELS seems to be the most sensitive to the initial delay error.

It is also interesting to see how the estimators behave for different values of initial code delay error. For this purpose, we performed one more test in which the relation of RMSE and MTLL with an increasing initial delay error from 0 up to 2 chips is examined. Figs. 11 and 12 show that DiscTracker is the best both from RMSE and MTLL point of view and that its performance remains more or less stable in the range of initial delay errors. On the other hand, HRC and ELS appear to be the most sensitive to the increasing initial error.

In order to show that the proposed algorithm is working both for Galileo and GPS signals, the following simulation results are dedicated to the study of estimators’ performance in the case where BPSK modulated signals are used. Figs. 13 and 14 show how RMSE varies versus CNR in both multipath static and multipath Nakagami-m channel, and no initial delay error. We observe that although DiscTracker has the best performance in the case of multipath static channel, while when Nakagami-m channel is concerned, HRC is the best and DiscTracker closely follows. Moreover, Diff2 seems to be the least suitable for BPSK modulated signals. In Fig. 15 we see how MTLL varies in fading channel and when $\hat{\tau}_{\text{init}} = 0$. There are no differences among the estimators in terms of MTLL for $\text{CNR} \geq 30$ dB-Hz except for Diff2 which has the worst performance. The MTLL here is higher than in the case of Sine-BOC(1,1) modulated signals because it has been computed for those estimates which are less than or equal to 1 chip instead of 0.35 chip (which values in both cases correspond to half the width of the Acf main lobe). This also implies that the majority of estimation errors lie within the range of ±1 chip (i.e., there were
V. CONCLUSIONS

In this paper, we proposed a new code delay estimator, called DiscTracker, which searches for the Line-Of-Sight (LOS) path delay among the positive discontinuity points (pDPs) present in the autocorrelation function (Acf). DiscTracker represents a hybrid approach in which both feedback and feedforward techniques are utilized with target of performance improvement.

The Monte-Carlo simulations we used included both static and fading channels with a closely-spaced multipath scenario and the considered signals were both Sine-BOC(1, 1) and BPSK modulated. We examined the impact

...
of Carrier to Noise Ratio (CNR) in different initial code delay errors ($\hat{\tau}_{init}$), as well as the impact of the initial code delay error in the LOS delay estimation when CNR=30 dB-Hz.

Our assessment was based on the comparison between DiscTracker and a set of state-of-the-art estimators in terms of Root Mean Square Error (RMSE) and Mean Time To Lose Lock (MTLL) criteria.

The simulation results showed that DiscTracker is the most resistant in initial code delay errors up to 2 chips, both in terms of RMSE and MTLL, in the case of Sine-BOC(1,1) modulated signals and multipath Nakagami-m channel. Moreover, DiscTracker performs the best at moderate to high CNR levels. In the case of BPSK-modulated signals and Nakagami-m channel, High Resolution Correlator is the best when $\hat{\tau}_{init} = 0$ chip and closely followed by DiscTracker. When $\hat{\tau}_{init} = 1$ chip, Peak Tracking using 2nd order Differential (PT(Diff2)) is the best estimator for CNR greater than 30 dB-Hz, however, DiscTracker remains the best choice at high initial delay errors (e.g., higher than 1 chip), when the processing window is of few chips length.

In conclusion, the proposed code delay estimator DiscTracker appears to be a strong candidate when the errors from the acquisition stage are expected to be high (e.g., order of few chips). The tradeoff is the need for a higher processing window (few chips around the previous delay estimation), but this is a characteristic of all feedforward delay estimation algorithms. The continuation of our work includes further optimization of DiscTracker procedures and consideration of MBOC modulated signals.
VI. REFERENCES


Publication P6


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Research Article


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Multipath propagation is one of the most difficult error sources to compensate in global navigation satellite systems due to its environment-specific nature. In order to gain a better understanding of its impact on the received signal, the establishment of a theoretical performance limit can be of great assistance. In this paper, we derive the Cramer Rao lower bounds (CRLBs), where in one case, the unknown parameter vector corresponds to any of the three multipath signal parameters of carrier phase, code delay, and amplitude, and in the second case, all possible combinations of joint parameter estimation are considered. Furthermore, we study how various channel parameters affect the computed CRLBs, and we use these bounds to compare the performance of three deconvolution methods: least squares, minimum mean square error, and projection onto convex space. In all our simulations, we employ CBOC modulation, which is the one selected for future Galileo E1 signals.

1. Introduction

In order for a user to compute their three-dimensional position and to correct the clock offset, the distance between its GNSS receiver and at least four satellites is required. Mass market receivers of code division multiple access (CDMA-) based positioning compute the unknown distance (also known as pseudorange) by estimating the total code delay.

Apart from the propagation delay, the signal undergoes a variety of channel distortions (such as those caused by ionosphere and troposphere layers) which introduce further delays [1]. Multipath propagation is a major source of error in the range measurement, because it can significantly delay the signal and it cannot be mitigated with differential methods due to its site-specific nature [2]. Environments prone to multipath effects are densely built areas or areas with large obstacles, which are typically encountered in metropolitan areas, where the concentration of GNSS users is high. If the receiver does not estimate the multipath delay with sufficient accuracy, then it suffers a degradation in the accuracy of range estimation and an increase in the processing time [3].

The distortion effects of multipath propagation have been known to the GNSS community for a long time, and several efforts to mitigate them have taken place. A large portion of these efforts has been focused on the tracking stage of a receiver where fine estimates of the line-of-sight (LOS) code delay and carrier phase are required. One of the most commonly used code tracking structures are the so-called Delay locked loops (DLLs), which belong to the category of feedback estimators. Examples of such structures include the popular narrow correlator [4], double delta correlator [5], strobe and edge correlators [6], high-resolution correlator and other optimized multiple gate delay (MGD) structures [7]. However, the feedback-based estimators are generally sensitive to closely spaced path scenarios and potential acquisition errors. As an alternative solution, various feedforward approaches have been proposed in the literature [8]. While improving the delay estimation accuracy, these approaches typically require more correlators than DLL-based ones and are sensitive to the noise-dependent threshold choice. Various combinations of feedback and feedforward approaches aim at improved accuracy [9, 10].
In the carrier tracking stage, multipath mitigation has been a challenging problem as well. Carrier phase multipath has been commonly studied in 2-path channels using a phasor diagram that illustrates the relation between the phase of the LOS signal and the multipath [11–13]. In [14], a geometric perspective is employed that involves different configurations of the antenna-reflector(s) geometry. Other methods include the ashtech enhanced Strobe correlator [15] and the multipath estimating delay lock loop (MEDLL); the latter jointly estimates the delay, relative amplitude, and phase parameters of the direct and multipath signals based on the maximum likelihood theory [16]. Both are advanced techniques with improved performance in long delay multipath errors; however, they are heavily covered by patents.

1.1. BOC Modulation. Towards the end of the 1990s, a new modulation technique, called binary offset carrier (BOC), was recommended for future GNSS signals for achieving sufficient spectral separation with existing GPS signals [17]. Moreover, because the width of the main lobe in the envelope of the autocorrelation function (Acf) is narrower than the one in binary phase shift Key (BPSK) modulated signals (i.e., used in GPS C/A signal), improved tracking accuracy could be achieved. There have been several variants of BOC suggested in the literature for different signal types included in the GPS modernisation plans and Galileo specifications. Among those variants, Sine-BOC(1, 1) was initially used in the standards for the L1 open service (OS) Galileo signals, but afterwards multiplexed BOC (MBOC) was selected [18]. MBOC is a weighted combination of sine-BOC(1, 1) and sine-BOC(6, 1) components (we notice that in the notation BOC(m, n), m is the ratio of the sub-carrier frequency over the reference frequency, of 1.023 MHz, n is the ratio of the chip rate over the reference frequency and the ratio 2m/n describes the BOC order) and is defined as a common spectrum to be matched by both the Galileo and the GPS L1/E1 OS signals. The MBOC spectrum can be realized in the time domain with many different approaches and the two chosen for GPS and Galileo are (1) time multiplexed BOC (TMBOC) and (2) composite BOC (CBOC), respectively.

In the first implementation, the whole signal is divided into blocks of N code symbols, and M < N of N code symbols are sine-BOC(1, 1) modulated, while N-M code symbols are sine-BOC(6, 1) modulated. In the CBOC implementation, we have a weighted combination of Sine-BOC(1, 1) and Sine-BOC(6, 1) modulated code symbols. When the combination is an addition of the two components, we have the so-called CBOC (“+”), and when we subtract the sine-BOC(6, 1) part from the sine-BOC(1, 1) part, we have the so-called CBOC (“−”) type of modulation. The CBOC (“+”) scheme is used in the implementation of the Galileo OS data channel, while CBOC (“−”) is used in the pilot channel [18]. The normalized envelope of autocorrelation function of a CBOC (“−”) modulated signal can be seen in the left plot of Figure 1.

While BOC modulation improves the tracking accuracy, it introduces an extra challenge in the tracking stage. More precisely, the additional peaks in the Acf, the number of which depends on the BOC type, increase the probability of tracking the wrong peak. In the right plot of Figure 1, we can see how the Acf is distorted due to the presence of a second path (located on the right side of the first path, 7 samples far from it). If the tracking module is falsely locked in the second peak, then code delay error is produced. One can now envision how more complex the Acf would look like in the presence of more paths and in the case of fading channel.

1.2. Motivation and Contribution. While there is an ample number of scientific works related to tracking of BPSK-modulated signals, the amount of studies focusing on CBOC modulated signals is significantly smaller, mostly because it was relatively recently selected for the Galileo OS. Examples of existing work include [19, 20] which compare the tracking performance of various discriminators for the
new modulation schemes, such as CBOC and TMBOC. In [21], the authors study the impact of the new modulation schemes on carrier tracking loop and in [22] the effects of heavy multipath propagation in combination with CBOC modulation are examined.

A thorough literature review reveals that the majority of existing work that adopts CBOC modulation studies the performance of mainly state-of-the-art code discriminators, such as early-minus-late, dot product, and Strobe correlator, while carrier phase estimation has been much less studied. In this paper, we are interested in the performance of less popular algorithms but which are used to estimate all three parameters of CBOC-modulated signals (i.e., carrier phase, code delay, and amplitude). In particular, we study the performance of deconvolution methods which are means of inverse filtering. One of the adverse effects of inverse filtering, when noise is present, is the noise enhancement. The noise enhancement effect can be reduced by using the so-called constrained inverse filtering methods. These methods are constrained in the sense that they do not allow the output values to lie outside some predefined set or in the sense that the inverse operator is never completely formed but only approximated iteratively. Among the constrained inverse filtering methods, the best known ones are the least-squares (LS) techniques and the projection onto convex sets (POCS) algorithm [23, 24].

**Figure 2**: RMSE of single and joint CRLB versus path separation ($\tau_2 - \tau_1$) for LOS carrier phase (a), code (b) delay and amplitude (c), $L = 2$ and $C/N_0 = 45$ dB-Hz.
A commonly used method for assessing the performance of an estimator is to compare its error variance with the theoretically minimum attainable, the latter of which is known as the Cramer Rao lower bound (CRLB) [25]. While the methodology for deriving the CRLB is straightforward and has been reported in [25], it always has to be tailored to the estimation problem in question (i.e., different estimation problems are encountered in different research areas; therefore depending, on the environment, each estimation problem is positioned, the assumptions made, and the parameters of interest may differ).

Based on the above discussion, the objectives of this paper are formed as follows: first, to provide a theoretical model that leads to the CRLB for the signal’s unknown parameter vectors of carrier phase, code delay, and amplitude by taking into account the multipath effects and the correlated noise at the output of the correlators and receiver filters. More precisely, we present two types of bounds. The first one, called single CRLB (sCRLB), represents the CRLB for a single parameter vector (i.e., a vector containing the unknown parameter for each path), where we assumed that the remaining parameters are known or perfectly estimated. The second type, called joint CRLB (jCRLB), reveals the theoretical limits given that a set of parameter vectors is jointly estimated. The reason for distinguishing between single and joint CRLB is that by comparing them, we can gain meaningful information related to the importance of each parameter in the estimation accuracy of the other set of parameters. The computations assume a static multipath channel with arbitrary number of paths and additive white Gaussian noise (AWGN). We use static channels, because we want to examine the minimum achievable performance and because modeling the phases in fading channels introduces additional errors. However, the model can be easily adapted for fading channels by taking into account the statistical characteristics of the profile at hand.

The second objective is to analyze theoretically the impact of different channel parameters such as C/N0, path separation, and number of channel paths on the estimator accuracy bounds. Finally, we provide performance comparisons between the derived theoretical limits and a set of deconvolution estimators, namely, least squares (LS), minimum mean square error (MMSE) and a POCS algorithm proposed earlier by the authors [24].

The remainder of this paper is organized as follows: Section 2 presents the system model. Section 4 describes the simulation setup and includes the results and the discussion. Finally, Section 5 summarizes the findings of this paper (the detailed derivation of the CRLBs can be found in the appendix).

2. System Model

The satellite transmitted signal, $x(t)$, can be modeled as the convolution between the modulating waveform $s_{\text{CBOC}}(t)$, the pseudorandom (PRN) CDMA code, and the modulated data as [26]

$$ x(t) = s_{\text{CBOC}}(t) \ast \sum_{n=-\infty}^{+\infty} \sum_{g=1}^{S_F} b_n c_g \delta(t - nT - gT_s), $$

where $\ast$ is the convolution operator, $b_n$ is the $n$th complex data symbol (in case of a pilot channel, it is equal to 1), $T$ is the symbol period, $c_g$ is the $g$th chip corresponding to the $n$th symbol, $T_s$ is the chip period, $S_F$ is the spreading factor ($S_F = T_{\text{sym}}/T_c$), $\delta(t)$ is the Dirac pulse, and $s_{\text{CBOC}}(t)$ stands for the CBOC modulated signal.

After the signal is transferred to the passband, it is transmitted over a multipath static or multipath fading channel, where all interference sources (except for the multipaths) are lumped into a single additive Gaussian noise term. At the receiver side, the signal is downconverted to the baseband, and it can be written as

$$ r_x(t) = \sqrt{E_b} \sum_{l=1}^{L} a_l x(t - \tau_l) e^{j(2\pi f_{D0} t + \phi_l)} + \eta(t), $$

where $E_b$ is the data bit energy, $a_l$, $\tau_l$, and $\phi_l$ are the amplitude, code delay, and carrier phase offset of the $l$th path, respectively, $L$ is the number of channel paths, $f_{D0}$ is the Doppler shift introduced by the channel, and $\eta(t)$ is the additive Gaussian noise of zero mean and double-sided power spectral density $N_0$ W/Hz.

After downconverting the received signal and correlating it with the reference modulated PRN code (stored in the receiver), we get [24]

$$ y_R(t) = \sqrt{E_b} \sum_{l=1}^{L} a_l R_m(t_l, \tau_g) e^{-j(2\pi \Delta f_0 t + \phi_l)} + \nu(t), $$

where $\Delta f_0$ is the Doppler shift error (i.e., a residue of the acquisition stage), $\nu(t)$ is the complex colored Gaussian noise of the despread signal with zero mean and covariance matrix $C_\nu = \sigma^2\mathbf{H}$, where $\sigma^2$ is the variance per each correlator, equal to the two-sided power spectral density of $N_0/2$ W/Hz, and $\mathbf{H}$ is the correlation matrix given in (6) (i.e., independent of the unknown parameters) [27]. Moreover, $R_m(t_l, \tau_g)$ is the ideal continuous autocorrelation function of the modulated code at delays $t_l$ and $\tau_g$, which is expressed as

$$ R_m(t_l, \tau_g) = E \left[ \frac{1}{T_{\text{sym}}} \int_{(m-1)T_{\text{sym}}}^{mT_{\text{sym}}} x(t - t_l) x(t - \tau_g) \right], $$

where $E(\cdot)$ is the expectation operation, $m$ is the code epoch index, and $T_{\text{sym}}$ is the symbol duration.

Assuming that the Doppler shift has been successfully removed (i.e., $\Delta f_0 = 0$), we can transform (3) into [24]

$$ y = Hw(t, \alpha) + \nu, $$

where $H = \mathbf{H}w(t, \alpha)$.
Figure 3: RMSE of Single and Joint CRLB versus $C/N_0$ for LOS carrier phase (a), code (b) delay and amplitude (c).

where $y$ is the data column vector that contains the complex correlation output sampled at rate $N_s$ and $H$ is the pulse shape deconvolution matrix of size $K \times K$ given by

$$H = \begin{bmatrix}
h(\tau_1, \tau_1) & \cdots & h(\tau_1, \tau_K) \\
\vdots & \ddots & \vdots \\
h(\tau_K, \tau_1) & \cdots & h(\tau_K, \tau_K)
\end{bmatrix}, \quad (6)$$

where $h(\tau_i, \tau_q)$ is the output of the ideal discrete autocorrelation function at code delays $\tau_i$ and $\tau_q$ for $1 \leq i \leq K$ and $1 \leq q \leq K$, respectively. The mathematical expression can be found from (4), by substituting the integral with a summation and the continuous time $t$ with the discrete sampled time instances). In addition, the term $\tau_K$ is the maximum delay spread of the channel (i.e., $\tau_K = \tau_{\text{max}} T_s$, where $T_s$ is the sampling period and $\tau_{\text{max}}$ is the maximum delay). The noise vector $v$ contains the complex colored Gaussian noise terms of the despread signal with zero mean and covariance matrix $C_v$. The unknown vector, $w$, is a function of the signal parameter vectors $\tau$, $\alpha$, and $\phi$ that we want to estimate. Specifically, the elements of the unknown vector $w$ have the following interpretation: ideally (i.e., in noise-free conditions), if a path is present at delay $\tau_k$, then $w_k$ would be equal to $a_k e^{i\phi_k}$; otherwise, $w_k$ is zero. In
other words, the positions of the nonzero elements in $w$ correspond to the delay of the channel paths and the value of the nonzero element contains the amplitude and phase information. Thus, in order to find the unknown signal parameters, we first need to locate the nonzero elements of the $w$.

The above formulation of the model transforms the problem into solving a system of linear equations. Several methods have been proposed for solving such a system (see Section 1). In what follows, this model constitutes the basis for deriving the theoretically achievable limits of the parameters of interest.

3. Theoretical Estimation Limits

A commonly used method for assessing the performance of an unbiased estimator is to compare its error variance with the Cramér-Rao lower bound (CRLB) [25]. The computation of CRLB requires that the probability density function (pdf) of the observed data is known. However, because in our case the observed data are contaminated by colored Gaussian noise, we perform a whitening process so as to transform the noise into additive white Gaussian noise (AWGN) whose pdf is known.

In [27], the covariance matrix $C_s$ is found to be equal to $\sigma^2 H$, which is independent of the unknown parameters. Furthermore, it can be shown that $C_s$ is positive definite; therefore, it can be factored as $C_s^{-1} = (1/\sigma^2)H^{-1} = DD^*$, where $D$ is a lower triangular, $K \times K$ invertible matrix, and $D^*$ denotes the conjugate of $D$. According to [25], the matrix $D$ can act as a whitening transformation when applied to $v$. Multiplying the terms of (5) leftwise with $D$ gives

$$Dy = DHw(r, a, \phi) + Dv,$$

$$\tilde{y} = \tilde{H}w(r, a, \phi) + \tilde{v} = s(r, a, \phi) + \tilde{v}, \quad (7)$$

where $\tilde{v}$ is now AWGN with zero mean and unit variance [25] and $s(r, a, \phi)$ is

$$s(r, a, \phi) = \left[ \sum_{l=1}^{L} \tilde{h}_{1,1} \alpha_{l} e^{j\theta_{l}} \cdots \sum_{l=1}^{L} \tilde{h}_{K,1} \alpha_{l} e^{j\theta_{l}} \right]^T, \quad (8)$$

where $\tilde{h}_i$ is used to describe the elements of the matrix $\tilde{H}$. Assuming that the unknown vector parameter to be estimated is denoted by $\theta$, we can write the pdf as [25]

$$p(\tilde{y}; \theta) = \frac{1}{(2\pi\sigma^2)^{K/2}} \exp\left\{-\frac{1}{2\sigma^2} \sum_{k=1}^{K} | \tilde{y}(k) - s(\theta, k) |^2 \right\}, \quad (9)$$

where in the case of single CRLB, $\theta$ is equal to one of the three parameter vectors, $r$, $a$, or $\phi$, and in the case of joint CRLB, $\theta$ is equal to any of the four possible combinations of the three parameter vectors (i.e., $\theta = [r; a]$, $\theta = [\phi; \theta]$, $\theta = [r; a]$, or $\theta = [\phi; r; a]$). Because (9) shows the dependency of the pdf upon the unknown parameter, it is termed the likelihood function [25].

If we assume that the estimator $\hat{\theta}$ is unbiased and that the pdf satisfies the regularity condition, the single and joint CRLBs can be derived in a straightforward manner (see the appendix for the detailed computations and Table 1 for the CRLBs notations used in this paper).

### Table 1: Single and Joint CRLBs notations.

<table>
<thead>
<tr>
<th>CRLB Type</th>
<th>Phase</th>
<th>Delay</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single CRLB</td>
<td>$\text{var}(\phi)$</td>
<td>$\text{var}(\tau)$</td>
<td>$\text{var}(a)$</td>
</tr>
<tr>
<td>Joint CRLB— Case 1</td>
<td>$\text{var}_{1,\text{joint}}(\phi)$</td>
<td>$\text{var}_{1,\text{joint}}(\tau)$</td>
<td>$\text{var}_{1,\text{joint}}(a)$</td>
</tr>
<tr>
<td>Joint CRLB— Case 2</td>
<td>$\text{var}_{2,\text{joint}}(\phi)$</td>
<td>$\text{var}_{2,\text{joint}}(\tau)$</td>
<td>$\text{var}_{2,\text{joint}}(a)$</td>
</tr>
<tr>
<td>Joint CRLB— Case 3</td>
<td>$\text{var}_{3,\text{joint}}(\phi)$</td>
<td>$\text{var}_{3,\text{joint}}(\tau)$</td>
<td>$\text{var}_{3,\text{joint}}(a)$</td>
</tr>
<tr>
<td>Joint CRLB— Case 4</td>
<td>$\text{var}_{4,\text{joint}}(\phi)$</td>
<td>$\text{var}_{4,\text{joint}}(\tau)$</td>
<td>$\text{var}_{4,\text{joint}}(a)$</td>
</tr>
</tbody>
</table>

4. Simulation Profiles and Results

In the first part of our simulations, we study the CRLB behavior of LOS signal versus various channel parameters. The signal was modulated using CBOC ("-" modulation (i.e., the modulation selected in the standards for future Galileo OS pilot signals [18]) and for the channel modeling we employ a decaying power delay profile (PDP) [28], meaning that $\bar{x} = \bar{x} e^{-\bar{\tau}(1-k)}$, where $\bar{x}$ is the average amplitude of the 1st path and $\bar{\tau}$ is the power decaying profile coefficient (assumed in the simulations to be equal to 0.09 when the path delays are expressed in samples). The carrier phase of each path was assumed to be uniformly distributed between $-\pi$ and $\pi$. At the receiver side, the bandwidth was assumed to be infinite, the sampling rate $f_s$ is equal to $N_c f_c$, where $N_c$ is oversampling factor and equal to 4 and $f_c$ is the chip rate (equal to 1.023 MHz for L1 OS signals). Also, the coherent integration time ($N_c f_c$) was 1 ms (the equivalent $C/N_0$ after coherent and noncoherent integration is $C/N_0 = C/N_0 + 10 \log_{10}(N_c \ast \sqrt{\text{N}(N_c)}$). For the computation of the CRLBs, we have assumed that the bit energy is 1 and the noise variance ($\sigma^2$) is then equal to $1/(N_c f_0 E_b/N_0)$, where $E_b/N_0$ is the energy per bit to noise spectral density ratio. Moreover, we consider only the case of static multipath channels, because we wanted to examine the maximum achievable performance and because modeling the phase changes in fading channels introduces additional errors. In cases where we deviate from these values or needed additional parameters, we note this in the title and/or caption of the figures. As the performance metric, we use the root mean square error (RMSE) which is computed of 5000 random channel realizations.

In Figure 2, we see how the CRLBs vary with increasing path separation (i.e., $\tau_2 - \tau_1$) and in the case when there are two paths and $C/N_0 = 45$ dB-Hz (we notice that all the $C/N_0$ values mentioned are the ones prior to any integration). For the case of carrier phase parameter (top left plot), we see that when the path separation is 0.3 chips the RMSE for single and joint CRLBs coincide. With respect to the code delay parameter, we see that the difference among single and joint CRLBs is minor, while in the case of amplitude, the differences are evident.

Figure 3 shows how the single and joint CRLBs behave with increasing $C/N_0$ in case of two-path channel. In this
case, the path separation was fixed to 0.3 chips, because according to Figure 2, this is the value when all CRLBs types behave the closest (thus, we can isolate the impact of $C/N_0$). From all three plots, we observe there are very slight differences between single and joint CRLBs. So, in two-path channels when, for example, we try to estimate all three synchronization parameters, we can achieve the same theoretical limit as when estimating only the LOS code delay and assuming the rest to be known.

Now, we are interested in studying the impact of the number of channel paths on the theoretically attainable bounds. Therefore, we used the same channel model with the previous scenario, only that now the $C/N_0$ was fixed to 45 dB-Hz and the path separation 0.3 chips. From the top left plot of Figure 4 we notice that when the number of paths is one or two, there is no difference among single and joint CRLBs. When the number of paths increases, estimating all three parameters leads to the same limit as in the case of estimating jointly the phase and the amplitude. Common behavior for $L > 2$ is also noticed for the case of single CRLB and the first case of joint. Regarding the code delay parameter, we notice that the similar performance between sCRLB and jCRLB—Case 1 and between jCRLB—Case 3 and jCRLB—Case 4 is evident for all number of channel paths.

**Figure 4:** RMSE of Single and Joint CRLB versus number of paths, $C/N_0 = 45$ dB-Hz.
In the second part of our simulation results, we compare the theoretical limits with the performance of a set of deconvolution algorithms: the least square (LS), the minimum mean square error (MMSE), and our proposed modified POCS algorithm (here, for simplicity, we refer to it as “mPOCS,” while in [24], it is denoted as “POCS2”). To briefly introduce it, mPOCS is an iterative deconvolution algorithm, which estimates jointly the LOS carrier phase and code delay and has been optimized for both Sine-BOC (1, 1) and BPSK modulated signals (the first modulation was the one initially proposed for the new Galileo OS signals, and the second modulation type is the one employed by the GPS coarse/acquisition (C/A) signals). Because both BOC and MBOC modulation types have been discussed in the context of GNSS specifications and since MBOC signals are also supposed to work with Sine-BOC receivers, the performance of mPOCS in the case of MBOC modulation will also show how flexible it is. Our POCS-based proposed algorithm is different from the previously proposed deconvolution approaches in two main ways: first, it incorporates some knowledge about the static multipath channel via estimated level crossing rates of receiver correlation function; second, it uses an adaptive threshold to reduce the various sources of interference (noise, multipath, and sidelobes in the autocorrelation function).
function of BOC-modulated signals). For brevity, we do not include a description of mPOCS algorithm. Instead, the interested reader is advised to refer to [24] for further details.

Because in this paper the estimation of all three synchronization parameters is considered, we incorporated into mPOCS the function of amplitude estimation as well (for the sake of completeness). More precisely, the amplitude of the LOS path is computed as \( \alpha_1 = \sqrt{|\hat{w}(\hat{\tau}_1)|} \), where \( \hat{w} \) is the estimate of \( w \) and \( \hat{\tau}_1 \) is the estimate of code delay of the LOS path, both of which are outputs of the mPOCS algorithm. Except for the incorporation of the amplitude estimation in mPOCS, we have also made the following modification in our model compared to the one presented in [24]: each row of the pulse shape deconvolution matrix, \( H \), has been normalized as \( H(k,:) = H(k,:)/\max(H(k,:)) \), that is, normalized to one. We did this normalization because we found out that it improves the performance of the deconvolution algorithms. We also emphasize that the normalization of \( H \) takes place only when it is used by the deconvolution algorithms and not when the CRLB is computed.

Regarding the channel setup, we used similar decaying PDP model with the previously described unless otherwise stated. The oversampling factor \( (N_s) \) was equal to 4, and the processing of \( ACF \) is done in a window \( (\tau_{\text{max}}) \) of 4 chips length.

Figure 6: LOS carrier phase (a), code delay (b), and amplitude (c) versus \( C/N_0 \) for 3-path static channel and CBOC signal.
with a resolution of \(1/(N_s N_B)\) chips (i.e., \(K = \tau_{\text{max}} N_s N_B\)). The coherent integration time (\(N_c\)) was set to 10 ms, while the noncoherent integration (\(N_{nc}\)) was performed in 1 blocks of \(N_c\) length. We remark that because mPOCS estimates jointly the three synchronization parameters, we have used the corresponding joint CRLB (i.e., Case 4).

In Figure 5, we see the performance of the estimators versus \(C/N_0\) in the case of two-path static channel. Among the deconvolution methods, mPOCS performs the best when \(C/N_0\) is higher than 40 dB-Hz for the phase and delay parameters, while for the amplitude parameter, mPOCs is better starting from 35 dB-Hz. We notice that in the case of delay parameter (top right plot), mPOCS is the one that converges faster than LS and MMSE towards CRLB. Figure 6 shows the results for the case of 3-path static channel. As in the case of 2-path scenario, LS has the worst average performance for all parameters, while mPOCS remains the best method for middle or higher \(C/N_0\) values. When we increase the noncoherent integration from 1 to 2, then the performance of the estimators is further improved (see Figure 7).

5. Conclusions

In this paper, we derived single and joint CRLBs for the unknown parameter vectors of carrier phase, code delay, and amplitude in multipath channel. Furthermore, we provided
a theoretical analysis of the impact of different channel parameters such as $C/N_0$, path separation, and number of channel paths on the estimator accuracy bounds. Finally, we compared the performance between the derived theoretical limits and a set of deconvolution estimators, among which a modified projection onto convex set (mPOCS) algorithm [24]. All our experiments assumed CBQC modulation, which is the one selected for future Galileo OS signals. The simulations results show that mPOCS has the best performance among the other deconvolution methods for $C/N_0$ higher than 35 or 40 dB-Hz, depending on the signal parameter and the channel profile.

Appendix

For the computation of CRLB, we assume that the pdf satisfies the regularity conditions; that is,

$$E \frac{\partial \ln p(\tilde{y}; \theta)}{\partial \theta} = 0.$$  \hspace{1cm} (A.1)

In what follows, we use the logarithm of the likelihood function for calculating the Fischer information matrix (FIM). If we assume that the estimator $\hat{\theta}$ is unbiased and that the pdf satisfies the regularity conditions, the CRLB for each of the four cases is found by inverting the corresponding FIM.

1. CRLB for Single Vector Parameter. When computing the single CRLB, we assume that the other signal parameters are known or equivalently that they have been perfectly estimated. This assumption is made in order to eliminate the impact of the other signal parameters on the estimation bound of the parameter at hand, thus, leading to a “stricter” bound.

(a) Carrier Phase. Starting with the case in which the unknowns to be estimated are the carrier phases of the channel paths, we set $\theta = \phi$. Differentiating once the log-likelihood function with respect to the unknown parameter vector $\phi$ gives

$$\frac{\partial \ln p(\tilde{y}; \phi)}{\partial \phi_i} = -\frac{1}{2\sigma^2} \sum_{k=1}^{K} \frac{\partial}{\partial \phi_i} | \tilde{y}(k) - s(\phi; k) |^2 = \frac{1}{\sigma^2} \sum_{k=1}^{K} \\Re \left\{ \frac{\partial s(\phi; k)}{\partial \phi_i} \tilde{y}^*(k) - s^*(\phi; k) \right\} ,$$  \hspace{1cm} (A.2)

where

$$\frac{\partial s(\phi; k)}{\partial \phi_i} = j \alpha_t \tilde{h}(\tau_i, \tau_k) e^{j\phi},$$  \hspace{1cm} (A.3)

$$s^*(\phi; k) = \sum_{l=1}^{L} \alpha_l \tilde{h}(\tau_i, \tau_k) e^{-j\phi},$$  \hspace{1cm} (A.4)

$$\tilde{y}^*(k) = \Re \{ \tilde{y}(k) \} - j \Im \{ \tilde{y}(k) \},$$  \hspace{1cm} (A.5)

and $\Re \{ \cdot \}$, $\Im \{ \cdot \}$ are used for denoting the real and the imaginary part, respectively. Substituting (A.3), (A.4), and (A.5) into (A.2) results in

$$\frac{\partial \ln p(\tilde{y}; \phi)}{\partial \phi_i} = \frac{\alpha_t}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \times \left[ -\Re \{ \tilde{y}(k) \} \sin(\phi_i) + \Im \{ \tilde{y}(k) \} \cos(\phi_i) + \sum_{l=1}^{L} \alpha_l \tilde{h}(\tau_i, \tau_k) \sin(\phi_i - \phi_l) \right].$$  \hspace{1cm} (A.6)

Then, we compute the second derivatives for $\phi_i = \phi_q$ and $\phi_i \neq \phi_q$, respectively, as

$$\frac{\partial^2 \ln p(\tilde{y}; \phi)}{\partial \phi_i^2} = \frac{\alpha_t}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \times \left[ -\Re \{ \tilde{y}(k) \} \cos(\phi_i) - \Im \{ \tilde{y}(k) \} \sin(\phi_i) + \sum_{l=1}^{L} \alpha_l \tilde{h}(\tau_i, \tau_k) \cos(\phi_i - \phi_l) \right],$$

$$\frac{\partial^2 \ln p(\tilde{y}; \phi)}{\partial \phi_i \partial \phi_q} = -\frac{\alpha_t}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \tilde{h}(\tau_i, \tau_q) \cos(\phi_i - \phi_q).$$  \hspace{1cm} (A.7)

Notice that because the first two terms in the square brackets of (A.6) do not depend on $\phi_i$ or $\phi_q$, they are set to zero. In order to populate the Fischer information matrix (FIM), we distinguish between the elements located in the main diagonal (denoted as $[I(\phi)]_{ii}$) and the elements located outside it (denoted as $[I(\phi)]_{ij}$ for $i \neq q$)

$$[I(\phi)]_{ii} = -E \left[ \frac{\partial^2 \ln p(\tilde{y}; \phi)}{\partial \phi_i^2} \right] = \frac{\alpha_t}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k)$$

$$[I(\phi)]_{ij} = -E \left[ \frac{\partial^2 \ln p(\tilde{y}; \phi)}{\partial \phi_i \partial \phi_j} \right] = -\frac{\alpha_t \alpha_q}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \tilde{h}(\tau_i, \tau_q) \cos(\phi_i - \phi_q).$$
We remark that the model up to here is valid for both fading and static channels. For subsequent derivations, in case of a fading channel, we would need to compute $E[\alpha_i \cos(\phi_i)]$, where $\alpha_i$ would be distributed according to the fading channel profile (e.g., Rayleigh or Nakagami distributed) and $\phi_i$ is uniformly distributed over $[-\pi, \pi]$. Similarly, for the elements outside the main diagonal, we have

\[ [I(\phi)]_{ij} = -E\left[ \frac{\partial^2 p(\tilde{y}; \phi)}{\partial \phi_i \partial \phi_j} \right] \]

\[ = \frac{\alpha_i \alpha_j}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \tilde{h}(\tau_j, \tau_k) \cos(\phi_i - \phi_j). \]  

Finally, the CRLB for the carrier phase of the $l$th path is given by the element located in the $l$th row and $l$th column of the inverse Fischer information matrix

\[ \text{var}(\hat{\phi}_l) = [I^{-1}(\phi)]_{ll}. \]

\[ (A.11) \]

(b) Code Delay. Here, we have $\theta = \tau$. Differentiating once the log-likelihood function with respect to the unknown code delay gives

\[ \frac{\partial \ln p(\tilde{y}; \tau)}{\partial \tau_i} = \frac{1}{\sigma^2} \sum_{k=1}^{K} \left[ \frac{\partial s(\tau;k)}{\partial \tau_i} \left\{ \tilde{y}^*(k) - s^*(\tau;k) \right\} \right], \]

\[ (A.12) \]

where

\[ \frac{\partial s(\tau;k)}{\partial \tau_i} = \alpha_i \tilde{h}'(\tau_i, \tau_k) e^{i\phi}. \]  

\[ (A.13) \]

Now, we substitute (A.4), (A.5), and (A.13) into (A.12), and we get

\[ \frac{\partial \ln p(\tilde{y}; \tau)}{\partial \tau_i} = \frac{\alpha_i}{\sigma^2} \sum_{k=1}^{K} \tilde{h}'(\tau_i, \tau_k) \left[ \mathcal{R} \{ \tilde{y}(k) \} \cos(\phi_i) + \mathcal{I} \{ \tilde{y}(k) \} \sin(\phi_i) \right] 
\]

\[ - \sum_{l=1}^{L} \alpha_l \tilde{h}(\tau_i, \tau_k) \cos(\phi_i - \phi_l). \]  

\[ (A.14) \]

Then, we calculate the second derivatives by distinguishing between differentiation with the same path or not. After some mathematical manipulation, we get

\[ \frac{\partial^2 \ln p(\tilde{y}; \tau)}{\partial \tau_i^2} = \frac{\alpha_i}{\sigma^2} \sum_{k=1}^{K} \tilde{h}''(\tau_i, \tau_k) \left[ \mathcal{R} \{ \tilde{y}(k) \} \cos(\phi_i) + \mathcal{I} \{ \tilde{y}(k) \} \sin(\phi_i) \right] 
\]

\[ - \sum_{l=1}^{L} \alpha_l \tilde{h}(\tau_i, \tau_k) \cos(\phi_i - \phi_l) \]

\[ - \frac{\alpha_i}{\sigma^2} \sum_{k=1}^{K} \tilde{h}'(\tau_i, \tau_k)^2. \]  

\[ (A.15) \]
Because the first two terms in the square brackets of (A.14) do not depend on the differentiating parameter, they are ignored, and we get

\[
\frac{\partial^2 \ln p(\hat{y} \mid \tau)}{\partial \tau_i \partial \tau_q} = -\frac{\alpha_i \alpha_q}{\sigma^2} \sum_{k=1}^{K} \tilde{h}''(\tau_i, \tau_k) \tilde{h}(\tau_q, \tau_k) \cos(\phi_i - \phi_q).
\]  

(A.16)

Using (A.15) and (A.16), we find that the FIM elements are

\[
[I(\tau)]_{ij} = -E \left[ \frac{\partial^2 \ln p(\hat{y} \mid \tau)}{\partial \tau_i \partial \tau_j} \right] = \frac{\alpha_i \alpha_j}{\sigma^2} \sum_{k=1}^{K} \tilde{h}''(\tau_i, \tau_k) \cdot \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \cos(\phi_i - \phi_j) \cdot \sum_{k=1}^{K} \tilde{h}'(\tau_i, \tau_k)^2.
\]

(A.17)

Substituting (A.4), (A.5), and (A.20) into (A.19), and after some mathematical manipulations, we get

\[
\frac{\partial \ln p(\hat{y} \mid \alpha)}{\partial \alpha_i} = \frac{1}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k)
\]

\[
\times \left[ \cos \phi_l \Re \{\tilde{y}(k)\} + \sin \phi_l \Im \{\tilde{y}(k)\} \right] \cdot \cos(\phi_i - \phi_l) \cdot \sum_{l=1}^{L} \tilde{h}(\tau_i, \tau_k) \cos(\phi_i - \phi_l).
\]

(A.21)

Because the first two terms inside the square brackets of (A.21) do not depend on the amplitude parameter, the differentiation of them leads to zero, and the final results for the second derivatives are

\[
\frac{\partial^2 \ln p(\hat{y} \mid \alpha)}{\partial \alpha_i \partial \alpha_j} = -\frac{1}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k),
\]

\[
\frac{\partial^2 \ln p(\hat{y} \mid \alpha)}{\partial \alpha_i \partial \alpha_q} = -\frac{1}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \tilde{h}(\tau_q, \tau_k) \cos(\phi_i - \phi_q).
\]

(A.22)

and the FIM elements

\[
[I(\alpha)]_{ii} = \frac{1}{\sigma^2} \sum_{k=1}^{K} \tilde{h}^2(\tau_i, \tau_k),
\]

\[
[I(\alpha)]_{iq} = \frac{1}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \tilde{h}(\tau_q, \tau_k) \cos(\phi_i - \phi_q).
\]

(A.23)

Finally, the CRLB for the amplitude of the \(i\)th path is

\[
\text{var}(\hat{\alpha}_i) = [I^{-1}(\alpha)]_{ii}.
\]

(A.24)

A.2. CRLB for Joint Vector Parameters. Here, we derive the joint CRLB for all possible combinations of the unknown parameter vectors. First, we consider the case of joint carrier phase and code delay estimation (from now on, this case will be referred to as Case 1). Second, we have the case of joint carrier phase and amplitude estimation (Case 2). Third, the case of joint code delay and amplitude (Case 3) and last the case of jointly estimating all three parameter vectors (Case 4). We also remind the reader that in all cases where two parameter vectors are jointly estimated, we assume that the third parameter vector is known or perfectly estimated.

Case 1 ((C1)—Carrier Phase and Code Delay). The 2 × 2 joint FIM is given by

\[
I_{1,\text{joint}}(\phi, \tau) = \begin{bmatrix} I(\phi) & I(\phi, \tau) \\ I(\phi, \tau) & I(\tau) \end{bmatrix}
\]

(A.25)

\[
\frac{\partial s(\alpha; \phi)}{\partial \alpha_i} = \tilde{h}(\tau_i, \tau_k) e^{i \phi_i}.
\]

(A.20)
where $I(\phi)$ and $I(\tau)$ can be constructed using (A.8), (A.10), and (A.17). Moreover, after some mathematical manipulations, we have

$$I(\phi, \tau)_{ij} = \frac{a_i a_j}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \tilde{h}(\tau_q, \tau_k) \sin(\phi_i - \phi_q),$$

$$I(\tau, \phi)_{ij} = \frac{a_i a_j}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \tilde{h}(\tau_q, \tau_k) \sin(\phi_i - \phi_q),$$  
(A.26)

where it can be proven that for $i = j$, we have $[I(\phi, \tau)]_{ii} = [I(\tau, \phi)]_{ii} = 0$. Now, we can obtain the CRLBs for each vector parameter from the diagonal elements of the inverse FIM as

$$\text{var}_{i,\text{joint}}(\phi) = \left[\text{diag}(I^{-1}_{i,\text{joint}}(\phi, \tau))\right]_{1:2}.$$  

$$\text{var}_{1,\text{joint}}(\tau) = \left[\text{diag}(I^{-1}_{1,\text{joint}}(\phi, \tau))\right]_{L+1:2L}.$$  
(A.27)

Case 2 ($(C2)$—Carrier Phase and Amplitude). Similarly with Case 1, the joint FIM is given by

$$I_{2,\text{joint}}(\phi, \alpha) = \begin{bmatrix} I(\phi) & I(\phi, \alpha) \\ I(\alpha, \phi) & I(\alpha) \end{bmatrix},$$  

(A.28)

where $I(\phi)$ and $I(\alpha)$ can be formed using (A.8), (A.10), and (A.23). Also, we have

$$I(\phi, \alpha)_{ij} = -\frac{a_i a_j}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \tilde{h}(\tau_q, \tau_k) \sin(\phi_i - \phi_q),$$

$$I(\alpha, \phi)_{ij} = \frac{a_i a_j}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \tilde{h}(\tau_q, \tau_k) \sin(\phi_i - \phi_q),$$  
(A.29)

where for $i = q$, we get $[I(\phi, \alpha)]_{ii} = [I(\alpha, \phi)]_{ii} = 0$. Similarly, the CRLBs can be found as

$$\text{var}_{2,\text{joint}}(\phi) = \left[\text{diag}(I^{-1}_{2,\text{joint}}(\phi, \alpha))\right]_{1:2},$$  

$$\text{var}_{2,\text{joint}}(\alpha) = \left[\text{diag}(I^{-1}_{2,\text{joint}}(\alpha, \phi))\right]_{1:2L}.$$  
(A.30)

Case 3 ($(C3)$—Code Delay and Amplitude). For this case, the FIM is formed as

$$I_{3,\text{joint}}(\tau, \alpha) = \begin{bmatrix} I(\tau) & I(\tau, \alpha) \\ I(\alpha, \tau) & I(\alpha) \end{bmatrix},$$  

(A.31)

where $I(\tau)$ and $I(\alpha)$ can be found using (A.8), (A.10), and (A.23). Furthermore, we have

$$I(\tau, \alpha)_{ij} = \frac{a_i a_j}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \tilde{h}(\tau_q, \tau_k),$$

$$I(\alpha, \tau)_{ij} = \frac{a_i a_j}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \tilde{h}(\tau_q, \tau_k) \cos(\phi_i - \phi_q),$$  
(A.32)

$$I(\alpha, \tau)_{ij} = \frac{a_i a_j}{\sigma^2} \sum_{k=1}^{K} \tilde{h}(\tau_i, \tau_k) \tilde{h}(\tau_q, \tau_k) \cos(\phi_i - \phi_q).$$

The CRLBs for each vector parameter are

$$\text{var}_{3,\text{joint}}(\phi) = \left[\text{diag}(I^{-1}_{3,\text{joint}}(\phi, \tau))\right]_{1:2},$$  

$$\text{var}_{3,\text{joint}}(\tau) = \left[\text{diag}(I^{-1}_{3,\text{joint}}(\phi, \tau))\right]_{L+1:2L}.$$  
(A.33)

$$\text{var}_{4,\text{joint}}(\phi) = \left[\text{diag}(I^{-1}_{4,\text{joint}}(\phi, \tau))\right]_{1:2},$$  

$$\text{var}_{4,\text{joint}}(\tau) = \left[\text{diag}(I^{-1}_{4,\text{joint}}(\phi, \tau))\right]_{L+1:2L},$$  
(A.35)

$$\text{var}_{4,\text{joint}}(\alpha) = \left[\text{diag}(I^{-1}_{4,\text{joint}}(\phi, \tau))\right]_{2L+1:3L}.$$  

(A.36)

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**References**


Publication P9


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