Modeling of Hysteresis Losses in Ferromagnetic Laminations under Mechanical Stress

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A novel approach for predicting magnetic hysteresis loops and losses in ferromagnetic laminations under mechanical stress is presented. The model is based on combining a Helmholtz free energy-based anhysteretic magnetoelastic constitutive law to a vector Jiles-Atherton hysteresis model. This paper focuses only on unidirectional and parallel magnetic fields and stresses, albeit the model is developed in full 3-D configuration in order to account also for strains perpendicular to the loading direction. The model parameters are fitted to magnetization curve measurements under compressive and tensile stresses. Both the hysteresis loops and losses are modeled accurately for stresses ranging from –50 to 80 MPa.

Index Terms—Helmholtz free energy, magnetic hysteresis, magnetoelasticity, magnetostriction, strain, stress.

I. INTRODUCTION

Dependence of iron losses on mechanical stresses and strains remains a problem in accurate design and analysis of electrical machines. The increase of power losses due to mechanical processing of the core laminations [1] is generally associated to the plastic deformations and residual stresses caused by the process. In addition, temperature gradients and centrifugal forces give rise to additional mechanical loadings in the cores.

Different approaches for both theoretical [2]-[3] and experimental [4]-[5] formulations for coupled multiaxial magnetostructural field problems have been studied quite recently. However, modeling of the losses has received less attention, and the presented loss models have typically been based on large amounts of experimental data. For example in [1] and [5], measured iron-loss and magnetization curves were used in finite-element (FE) analysis to study the effects of shrink-fitting and punching in electrical machine stator cores. After the developments of [6], theoretical models for hysteresis effects under mechanical loadings have not received very much attention. Especially multiaxial modeling approaches for hysteresis losses have been starting to gain ground only during the recent years [7], [8].

In [7], an interesting approach was taken for coupling the single-valued (SV) constitutive law of [3] to the vector Jiles-Atherton (JA) hysteresis model. In this paper, we implement a similar extension for the energy-based SV constitutive law of [2]. The SV constitutive law is first derived by partial differentiation of a Helmholtz free energy density function.

\[ \psi = \psi(\mathbf{M}, \mathbf{B}, \mathbf{e}) \]

Index Terms—Helmholtz free energy, magnetic hysteresis, magnetoelasticity, magnetostriction, strain, stress.

II. MODELS

A. Single-Valued Constitutive Law

The 3-D single-valued material model is developed similarly to [2] and [9]. The flux density vector \( \mathbf{B} \) (of size 3×1) and the total strain tensor \( \mathbf{e} \) (3×3) are chosen as the independent state variables. The magnetization \( \mathbf{M} \) and magnetostrictive stress \( \sigma_{\text{me}} \) are expressed as partial derivatives of a Helmholtz free energy density \( \psi \) with respect to \( \mathbf{B} \) and \( \mathbf{e} \):

\[
\mathbf{M} = \frac{\partial \psi}{\partial \mathbf{B}}, \quad \sigma_{\text{me}} = \frac{\partial \psi}{\partial \mathbf{e}}.
\]

The magnetic field strength is \( \mathbf{H} = \nu_0 \mathbf{B} - \mathbf{M} \), and the total stress \( \sigma = \sigma_{\text{me}} + \sigma_{\text{mag}} \) also includes the purely electromagnetic contribution from the Maxwell stress tensor:

\[
\sigma_{\text{mag}} = \nu_0 \left( \frac{1}{2} \mathbf{B}^T \mathbf{B} - \frac{1}{2} (\mathbf{B} : \mathbf{B}) \mathbf{I} + (\mathbf{M} : \mathbf{B}) \mathbf{I} - \mathbf{B} \mathbf{M}^T \right),
\]

in which \( \nu_0 = 1/\mu_0 \) is the reluctivity of free space and \( \mathbf{I} \) is the 3×3 unit tensor.

The integrity basis of an isotropic scalar function \( \psi \) depending on one vector \( \mathbf{B} \) and one tensor \( \mathbf{e} \) includes six scalar invariants, which in this case are written as...
in which the functions have been derived so as to obtain isochoric magnetostriction under purely magnetic loading [9]. \( \lambda \) and \( \mu \) are the Lamé constants of the material, and \( a_i \), \( b_i \), and \( c_i \) are parameters determined by fitting. The first two terms in (6) yield Hooke’s law and account for the purely mechanical behavior, while the last three terms account for the magnetomechanical coupling. The summation term in the middle (with \( n_i > 1 \)) accounts for the nonlinear \( \mathbf{M}(B, \mathbf{e}) \) relationship. Finally, the quadratic dependence of invariant \( I_6 \) on \( \mathbf{e} \) allows modeling the decreasing permeability under both compressive and high tensile stresses.

B. Jiles-Atherton Model

The hysteretic magnetization behavior is modeled following the inverse vector JA hysteresis model described in [11]. The model is summarized with the following five equations:

\[
\mathbf{H}_{\text{eff}} = \mathbf{H} + \alpha \mathbf{M},
\]

\[
\mathbf{M}_{\text{an}} = f \left( |\mathbf{H}_{\text{eff}}| \right) \frac{\mathbf{H}_{\text{eff}}}{|\mathbf{H}_{\text{eff}}|},
\]

\[
d = \mathbf{M}_{\text{an}} - \mathbf{M}_{\text{irr}} \quad \text{and} \quad \delta = \frac{d\mathbf{B}}{dt} \cdot d,
\]

where \( \mathbf{H}_{\text{eff}} = 1 \) T. Invariants \( I_1-I_3 \) describe purely elastic behavior, and \( I_3 \) is not used here since linear elasticity is assumed. \( I_3 \) describes purely magnetic behavior. \( I_5 \) and \( I_6 \) describe the magnetoelastic coupling, and are written using the deviatoric strain

\[
\mathbf{e} = \frac{1}{3} (\mathbf{tr} \mathbf{e}) \mathbf{I}
\]

in order to eliminate the effect of hydrostatic pressure on the magnetization properties [10].

The problem of forming a coupled magnetomechanical model requires obtaining \( \mathbf{M}_{\text{an}} \) as a function of \( \mathbf{H}_{\text{eff}} \), but the input of the SV model is \( \mathbf{B} \), we iterate the anhysteretic magnetization for a given \( \mathbf{H}_{\text{eff}} \) from (1) using the Newton-Raphson (NR) method. We are searching for an equivalent flux density \( \mathbf{B}_{\text{an}} = \mu_0 \mathbf{H}_{\text{eff}} + \mathbf{M}_{\text{an}} \), which for a given stress \( \mathbf{e} \) satisfies \( \mathbf{M}(\mathbf{B}_{\text{an}}, \mathbf{e}) = \mathbf{M}_{\text{an}} \). We thus write a residual vector and a Jacobian matrix, respectively, as

\[
\mathbf{r}(\mathbf{B}_{\text{an}}) = \mathbf{M}(\mathbf{B}_{\text{an}}, \mathbf{e}) - (\nu \mathbf{B}_{\text{an}} - \mathbf{H}_{\text{eff}})
\]

\[
\mathbf{J}(\mathbf{B}_{\text{an}}) = \frac{d\mathbf{r}(\mathbf{B}_{\text{an}})}{d\mathbf{B}_{\text{an}}} = \frac{\partial \mathbf{M}(\mathbf{B}_{\text{an}}, \mathbf{e})}{\partial \mathbf{B}_{\text{an}}} - \nu \mathbf{J}.
\]

C. Stress-Dependent Coercive Field

The effect of stress on the coercive field strength is modeled by introducing stress-induced anisotropy to the JA model pinning parameter \( k \) in (11). The scalar \( k \) is replaced by a second-order tensor \( \mathbf{k} (3 \times 3) \), which is an isotropic function of the total stress \( \mathbf{\sigma} \), meaning that

\[
k \left( R \mathbf{\sigma} R^T \right) = R k(\mathbf{\sigma}) R^T,
\]

for any coordinate transformation matrix \( R \). The tensorial integrity basis of such an isotropic function is \( \{1, \mathbf{\sigma}, \mathbf{\sigma}^2\} \), which means that \( \mathbf{k} \) can be formed as a linear combination

\[
k(\mathbf{\sigma}) = k_0 \left( 1 + a \mathbf{\sigma} + b \mathbf{\sigma}^2 \right),
\]

in which \( k_0, a \) and \( b \) depend only on the scalar invariants of \( \mathbf{\sigma} \). As a first approximation, we assume that \( k_0, a \) and \( b \) are constant, and they are treated as fitting parameters. The tensor \( \mathbf{k} \) obtained from (16) is coaxial with \( \mathbf{\sigma} \), meaning that both tensors have the same principal axes.

III. MEASUREMENT SETUP

A custom-built setup was used to measure magnetization properties of electrical steel sheet samples under uniaxial
stress. A picture of the setup is shown in Fig. 1. A rectangular sample is stressed using a manual screw, and a spring is connected in series with the sample in order to allow accurate control of the force.

The sample is magnetized using two vertical cores. The average flux density is measured using a coil wound around the sample, and the surface field strength is measured using an H-coil. The signals are recorded using a National Instruments USB-6251 BNC data acquisition system. The average flux density is controlled to be sinusoidal by a feedback control system implemented using the Data Acquisition Toolbox of MATLAB. The control is implemented similarly to [12] and iteratively searches for an input voltage waveform which yields a sinusoidal flux density in the sample.

IV. RESULTS

The parameters \( \alpha, \beta, \) and \( \gamma \) of the SV model were first fitted by least-squares comparison to magnetization curve measurements for a 0.5-mm nonoriented Fe-Si sheet sample. The number of terms in the polynomials of (6) were \( n_b = 7, \) \( n_\sigma = 1, \) \( n_L = 2. \) The magnetization loops were measured at 1.7 T sinusoidal flux densities under nine different stresses \( \sigma_i \) ranging from 50 MPa compression (–) to 80 MPa tension (+). Fig. 2 shows the results of fitting the parameters to the \( H \)-averaged magnetization loops at four different stress values. The parameter values are given in Table I. For a given load \( \sigma_L \) and flux density \( B, \) the total strain \( \varepsilon \) has been iterated with the NR method by expressing the tensors using the Voigt notation, and writing the residual and Jacobian matrix as:

\[
\begin{align*}
\mathbf{r}(\varepsilon) &= \mathbf{\sigma}(\mathbf{B}, \varepsilon) - \mathbf{\sigma}_L, \\
\mathbf{J}(\varepsilon) &= \frac{\partial \mathbf{r}(\varepsilon)}{\partial \varepsilon} = \frac{\partial \mathbf{\sigma}(\mathbf{B}, \varepsilon)}{\partial \varepsilon}.
\end{align*}
\]

(17)

(18)

It is emphasized that despite the unidirectional flux density and stress, using the multiaxial model is essential since the perpendicular components of \( \varepsilon \) also become nonzero. The model fits reasonably well and is able to predict the quadratic dependency of the magnetization curves on the stress, so that both compression and high tension reduce the permeability from the zero-stress case. This effect is not observed with the energy definitions of [3] and [7], although the effect can be taken into account with an additional correction term [8].

We next fitted the pinning parameters \( k_0, a, b, \) as well as \( \alpha \) and \( c \) of the JA model. The fitting was done by comparing both the simulated ascending major-loop branch and the hysteresis losses to the measured ones in the least-squares sense. Fig. 3 shows the results of the fitting at the same four stresses as in the SV case. The parameter values are given in Table I. Both the loop shapes and the coercive fields are reasonably modeled. Finally, a good correspondence is observed in Fig. 4 between the measured and modeled hysteresis losses at the other five stress values used in the measurements. The errors between the measured and simulated field strengths vary between 3.2 % and 7.8 %.
The need for iterating the $M_{st}(H_{st})$ relationship using (13)-(14) is a drawback if the model is to be implemented in numerical calculations tools, for example in FE solvers. Although the NR method typically converges in 2-3 iterations, this significantly increases the computation time compared to an explicit material model. The iteration could be avoided by developing the SV model using the field strength $H$ as the variable instead of the flux density $B$. On the other hand, if hysteresis does not need to be considered during the solution, the $B$-based model is more comfortable with magnetic vector potential formulations. Since the mechanical state variable is the total strain, the model is directly suitable for solving displacement fields.

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