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An Efficient Implementation of Linear-Phase FIR Filters for a Rational Sampling Rate Conversion

Robert Bregović[+], Tapio Saramäki[+], Ya Jun Yu[*], and Yong Ching Lim[*]

[+] Institute of Signal Processing
Tampere University of Technology
P. O. Box 553, FIN-33101 Tampere, Finland
E-mail: {bregovic, ts}@cs.tut.fi

[*] School of Electrical and Electronic Engineering
Nanyang Technological University
Singapore 639798
E-mail: {eleyuyj, elelimyc}@pmail.ntu.edu.sg

***Abstract*—This paper considers how to efficiently implement linear-phase FIR filters for providing a sampling rate conversion by an arbitrary rational factor of *M*/*L*, where *L* (*M*) is the up-sampling (down-sampling) factor to be implemented before (after) the actual filter. In the proposed implementation, the coefficient symmetry of the linear-phase FIR filter is exploited as well as possible when taking into account the following facts. When increasing (decreasing) the sampling rate by the factor of *L* (*M*), only every *L*th input sample has a non-zero value (only every *M*th output sample has to be evaluated). The proposed implementation is, first, presented by two illustrative examples and, then, guidelines are given on how to efficiently implement a sampling rate converter having an arbitrary rational sampling rate factor *M*/*L*.**

I. Introduction

Many modern signal processing algorithms require more than one sampling rate in order to increase their efficiency [1]. Systems having multiple sampling rates are known as multirate systems. The basic building blocks of a multirate system are interpolators by an integer factor L and decimators by an integer factor M. Combining these two blocks results in a system that changes the sampling rate by a rational factor M/L. The block diagram for such a rational sampling rate converter is depicted in Figure 1. As shown in this figure, sampling rate conversion by the rational factor, as denoted by M/L, means that the signal is, first, up-sampled by the factor of L and, then, the filtered signal is down-sampled by the factor of M.

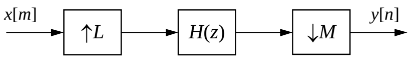

Figure 1. Rational sampling rate converter by factor M/L.

The filter with the transfer function $H(z)$, as shown in Figure 1, has the following two purposes. First, it should suppress the image spectra that are the results of up-sampling, and, second, it should prevent the aliasing effects that occur after down-sampling. For this purpose, in most cases due to its attractive properties, a linear-phase finite-impulse response (FIR) filter is used with transfer function defined as

$$H(z) = \sum_{n=0}^{N} h[n] z^{-n} , \qquad (1)$$

where $h[N-n] = h[n]$ for $n = 0, 1, ..., N$. [1]

[1] For simplifying the discussion in this paper, $h[n]$ is sometimes denoted by h_n, that is, $h_n \equiv h[n]$.

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In the past, efficient implementation structures have been proposed for implementing these filters for up-sampling and down-sampling with an integer sampling rate conversion factor [1]–[4]. When implementing these filters, in order to minimize the number of required multiplications per output (input) sample in the case of interpolators and decimators, the coefficient symmetry of the filters as well as the fact that the filters are preceded by an up-sampler by L or followed by a down-sampler by M are taken into consideration [4]. For a system that changes the sampling rate by a rational factor M/L, the proposed implementations utilize the fact that the filter is between an up-sampler (only every Lth sample is non-zero) and an down-sampler (only every Mth sample is taken as the output) [1]–[3]. However, according to the knowledge of the authors, the coefficient symmetry of this filter has not been exploited when implementing the filter.

This paper shows that by extending the approach proposed by Mou in [4] for exploiting the coefficient symmetry in the case of interpolators and decimators, structures can be proposed for implementing rational sampling rate converters having reduced implementation complexity compared with the approaches reported so far.

The outline of this paper is as follows: Section II gives two illustrative examples for the sampling rate converters by rational factors 3/2 and 5/3. Based on those examples, the proposed synthesis scheme is described in Section III.

II. Sampling Rate Convertors with Rational Sampling Factors – Examples

This section illustrates, by means of two examples, the proposed approach for implementing a sampling rate converter by a rational sampling factor M/L in such a manner that the filter coefficient symmetry is exploited.

A. Rational Sampling Factor 3/2

This subsection considers a rational sampling rate converter by factor M/L, as shown in Figure 1, with $L = 2$ and $M = 3$. Moreover, in order to make the example more illustrative, it is assumed that the filter order is $N = 11$.

For such a system, the relation between the output samples $y[n]$ and the input samples $x[m]$ is shown in Figure 2. As seen from this figure, for every three input samples, two new output samples are generated. The input-output relation for this system is expressible as

$$y[n] = [h_0 \quad h_2 \quad h_4 \quad h_6 \quad h_8 \quad h_{10}] \cdot \mathbf{x}_{m,m-5} \qquad (2a)$$

$$y[n+1] = [h_1 \quad h_3 \quad h_5 \quad h_7 \quad h_9 \quad h_{11}] \cdot \mathbf{x}_{m+1,m-4} , \qquad (2b)$$

where n is even,

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$$m = \frac{M}{L} n \,, \tag{3}$$

and [2]

$$\boldsymbol{x}_{k,l} = [x[k] \quad x[k-1] \quad x[k-2] \quad \ldots \quad x[l]]^T \,, \; l \le k. \tag{4}$$

Taking into account the filter coefficient symmetry, (2a) and (2b) can be rewritten into a matrix form as

$$\begin{bmatrix} y[n] \\ y[n+1] \end{bmatrix} = \begin{bmatrix} 0 & h_0 & h_2 & h_4 & h_5 & h_3 & h_1 \\ h_1 & h_3 & h_5 & h_4 & h_2 & h_0 & 0 \end{bmatrix} \cdot \boldsymbol{x}_{m+1,m-5} \,. \tag{5}$$

In order to obtain a form that is appropriate for implementation, the above equation is decomposed into three distinct parts as follows:

$$\begin{bmatrix} y[n] \\ y[n+1] \end{bmatrix} = h_1 \begin{bmatrix} x[m-5] \\ x[m+1] \end{bmatrix} + h_4 x[m-2] \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} h_0\, h_2\, h_5\, h_3 \\ h_3\, h_5\, h_2\, h_0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{m,m-1} \\ \boldsymbol{x}_{m-3,m-4} \end{bmatrix}. \tag{6}$$

The first two terms are very simple, thereby enabling one to implement them directly. The filter coefficient matrix in the third term is a so called centrosymmetric matrix [4], [5]. Hence, it can be efficiently implemented by using following decomposition:

$$\begin{bmatrix} h_0\, h_2\, h_5\, h_3 \\ h_3\, h_5\, h_2\, h_0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{m,m-1} \\ \boldsymbol{x}_{m-3,m-4} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} c_0 & c_1 & 0 & 0 \\ 0 & 0 & d_1 & d_0 \end{bmatrix} \boldsymbol{x}^{(1)}_{m,m-4} \,, \tag{7}$$

where

$$\begin{aligned} c_0 &= (h_0 + h_3)/2, \;\; c_1 = (h_2 + h_5)/2 \\ d_0 &= (h_0 - h_3)/2, \;\; d_1 = (h_2 - h_5)/2 \end{aligned} \tag{8}$$

and

$$\boldsymbol{x}^{(1)}_{m,m-4} = \begin{bmatrix} \boldsymbol{I}_2 & \boldsymbol{J}_2 \\ \boldsymbol{J}_2 & -\boldsymbol{I}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{m,m-1} \\ \boldsymbol{x}_{m-3,m-4} \end{bmatrix}. \tag{9}$$

The matrices $\boldsymbol{I_t}$ and $\boldsymbol{J_t}$ are of size t by t, where t is a positive integer, and are identity and counter-identity matrices, respectively. For $l < k$, the above term containing input data can be written compactly as

$$\boldsymbol{x}^{(r)}_{k,l} = \begin{bmatrix} x[k] + x[l] \\ x[k-1] + x[l+1] \\ \cdots \\ x[k-r] + x[l+r] \\ x[k-r] - x[l+r] \\ \cdots \\ x[k-1] - x[l+1] \\ x[k] - x[l] \end{bmatrix}. \tag{10}$$

The variables c_0, c_1, d_0, and d_1 depend only on filter coefficients and can be pre-calculated. The overall implementation structure is shown in Figure 3.

For generating two output samples, $y[n]$ and $y[n+1]$, this implementation requires 7 multiplications and 12 additions. This corresponds to 3.5 multiplications and 6 additions per output sample. The implementation of the same system but without utilizing the coefficient symmetry (e.g. as in [1]), requires 6 multiplications and 5 additions per output sample. Hence, even in this simple example, the computational savings are considerable.

[2] Throughout this paper it is assumed that $x[n] = 0$ for $n < 0$.

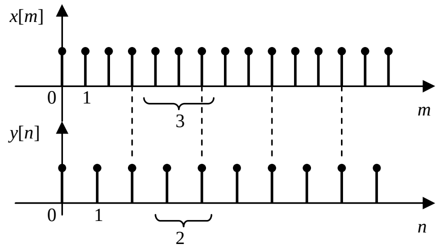

Figure 2. Input-output relation for a system with a rational sampling rate conversion by a factor of 3/2.

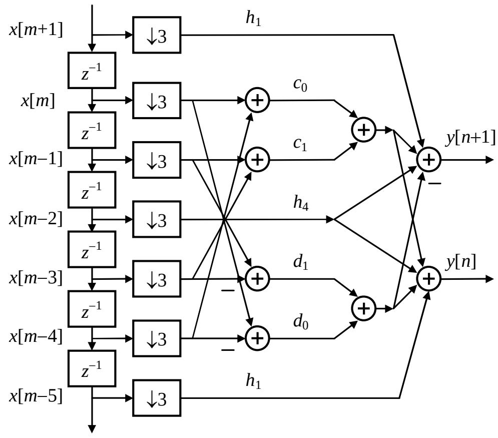

Figure 3. Implementation structure for rational sampling rate converter by 3/2 using a linear-phase FIR filter of order $N = 11$.

Following two remarks are in place: First, when exploiting the coefficient symmetry of an FIR filter, it is expected to halve the number of required multiplications. This is not true for rational sampling rate converter as can be seen in the above example. Here the achieved reduction is $6–3.5 = 2.5 < 6/2 = 3$. The reason for this lies in the fact that the coefficients with value h_1 are repeated twice in the structure of Figure 3. This repetition can be avoided by using additional delay elements in the structure (at the output sampling frequency, the sample $x[m–5]$ is a delayed version of sample $x[m+1]$ with the delay being equal to two). However, the number of required delay elements is proportional to the filter order. For large filter orders, this would considerably increase the memory consumption of the implementation. Moreover, as the filter order increases, the number of multiplications with respect to number of filter coefficients becomes smaller as there are more coefficients that are implemented efficiently, and, for all filter orders, only the coefficient h_1 is implemented non-efficiently.

Second, for filter orders $N = 4k–1$ for $k = 2, 3, 4, \ldots$, there is always a standalone term in the middle of the structure. For filter orders $N = 4k+1$ for $k = 1, 2, 3, \ldots$, there exist only c_l and d_l terms. In both cases, by increasing the filter order, the number of c_l and d_l terms grows.

B. Rational Sampling Factor 5/3

This subsection considers a rational sampling rate converter by factor M/L, as shown in Figure 1, with $L = 3$ and $M = 5$. The relation between the output samples $y[n]$ and the input samples $x[m]$ is shown in Figure 4. As seen from the figure, for every five input samples, three new output samples are generated. Assuming that the filter order is $N = 25$, and by following a similar strategy as in the previous section, three consequent output sample can be expressed in a matrix form as

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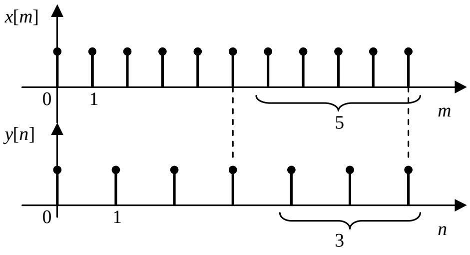

Figure 4. Input-output relation for a system with rational sampling rate conversion by 5/3.

$$\begin{bmatrix} y[n] \\ y[n+1] \\ y[n+2] \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & h_0 & h_3 & h_6 & h_9 & h_{12} & h_{10} & h_7 & h_4 & h_1 \\ 0 & 0 & h_2 & h_5 & h_8 & h_{11} & h_{11} & h_8 & h_5 & h_2 & 0 & 0 \\ h_1 & h_4 & h_7 & h_{10} & h_{12} & h_9 & h_6 & h_3 & h_0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{x}_{m+3,m-8}. \quad (11)$$

After some modifications, the above system can be transformed into the following form:

$$\begin{bmatrix} y[n] \\ y[n+1] \\ y[n+2] \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ h_1 & h_4 & h_7 \end{bmatrix} \boldsymbol{x}_{m+3,m+1} + \begin{bmatrix} h_7 & h_4 & h_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{x}_{m-6,m-8} +$$
$$\begin{bmatrix} 0 \\ h_2 \\ 0 \end{bmatrix} (x[m+1] + x[m-6]) + \quad (12)$$
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} c_0 & c_1 & c_2 & 0 & 0 & 0 \\ h_5 & h_8 & h_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & d_2 & d_1 & d_0 \end{bmatrix} \boldsymbol{x}^{(2)}_{m,m-5}$$

for $n = 0, 3, 6, \ldots$. The corresponding m is derived from (3) resulting in $m = (5/3)\,n$. For implementing (12), a structure similar to the one shown in Figure 3 can be derived.

For generating three output samples, this implementation requires 16 multiplications and 18 additions. This corresponds to 5.33 multiplications and 7.33 additions per output sample. In comparison, the implementation that does not utilize the coefficient symmetry requires 8.67 multiplications and 7.67 additions per output sample.

III. SAMPLING RATE CONVERTORS WITH RATIONAL SAMPLING FACTORS – GENERALIZATION

As seen from the examples in Section II, a slightly different implementation structures are achieved depending on the rational factors as well as the filter order. The main purpose of this section is to give guidelines on how to derive an efficient implementation structure for the given rational factor and filter order.

A. Filter Orders

In order to provide a unified approach for a given rational factor M/L, the filter orders in this paper are restricted to have the following values:

$$N = M(L-1) + 2kL \quad (13a)$$

$$N = M(L-1) + (2k+1)L \quad (13b)$$

for $k = 0, 1, 2, \ldots$ .Such selections of filter orders enable one to develop efficient implementations. The differences in the implementations for filters with orders satisfying (13a) or (13b) will be discussed in Section III.B.

It should be noted that filters with orders generated for an integer $k < 0$ are also acceptable as long as $N > 0$. However, due to a few coefficients, such filters are not of practical interest.

B. Matrix Decomposition

For a given rational factor M/L and a filter order satisfying either (13a) or (13b), the input-output function for a rational sampling converter given in Figure 1 can be expressed as follows:

$$\begin{bmatrix} y[n] \\ y[n+1] \\ \vdots \\ y[n+L-1] \end{bmatrix} = \boldsymbol{H}_{L\times(p+q+1)} \boldsymbol{x}_{m+p,m-q}. \quad (14a)$$

Here, $\boldsymbol{x}_{m+p,m-q}$ is a vector containing $p+q+1$ consequent input signal samples as defined by (4) and $\boldsymbol{H}_{L\times(p+q+1)}$ is an L by $p+q+1$ matrix containing the filter coefficients as given by (14b) with $q = \lfloor N/L \rfloor$, $p = \lfloor (L-1)M/L \rfloor$, and $h[n] = 0$ for $n < 0$ and $n > N$. It should be noted that the L rows of the matrix $\boldsymbol{H}_{L\times(p+q+1)}$ are shifted version of the L polyphase components of the filter transfer function $H(z)$.

In order to generate an efficient implementation, the matrix, as given by (14b), and consequently the input-output relation, as given by (14a), can be decomposed into two parts as

$$\begin{bmatrix} y[n] \\ y[n+1] \\ \vdots \\ y[n+L-1] \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_a & \boldsymbol{H}_c \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{m+p,m+1} \\ \boldsymbol{x}_{m+p-q-1,m-q} \end{bmatrix} + \boldsymbol{H}_b \boldsymbol{x}_{m,m-q+p}. \quad (15)$$

After taking into account the coefficient symmetry, the matrix $[\boldsymbol{H}_a\ \boldsymbol{H}_c]$ of size L by $2p$ becomes

$$\begin{bmatrix} \boldsymbol{H}_a & \boldsymbol{H}_c \end{bmatrix} = \begin{bmatrix} h_{-pL} & \cdots & h_{-L} & h_{(L-1)M-L} & \cdots & h_{qL} \\ h_{M-pL} & \cdots & h_{M-L} & h_{(L-2)M-L} & \cdots & h_{M+qL} \\ & & \cdots & \cdots & & \\ h_{M+qL} & \cdots & h_{(L-2)M-L} & h_{M-L} & \cdots & h_{M-pL} \\ h_{qL} & \cdots & h_{(L-1)M-L} & h_{-L} & \cdots & h_{-pL} \end{bmatrix}. \quad (16)$$

It should be noted that most elements in the matrix $[\boldsymbol{H}_a\ \boldsymbol{H}_c]$ are zero-valued. Therefore, this matrix can be further optimized when used for implementing the overall system. This was shown through the example in Section II.B, and will not be further discussed here due to the lack of space and the fact that the major savings are obtained by optimizing the implementation of the matrix $\boldsymbol{H}_b$.

$$\boldsymbol{H}_{L\times(p+q+1)} = \begin{bmatrix} \boldsymbol{H}_a & \boldsymbol{H}_b & \boldsymbol{H}_c \end{bmatrix} = \begin{bmatrix} h_{-pL} & \cdots & h_{-L} & h_0 & h_L & \cdots & h_{(q-p)L} & h_{(q-p+1)L} & \cdots & h_{qL} \\ h_{M-pL} & \cdots & h_{M-L} & h_M & h_{M+L} & \cdots & h_{M+(q-p)L} & h_{M+(q-p+1)L} & \cdots & h_{M+qL} \\ h_{2M-pL} & \cdots & h_{2M-L} & h_{2M} & h_{2M+L} & \cdots & h_{2M+(q-p)L} & h_{2M+(q-p+1)L} & \cdots & h_{2M+qL} \\ \cdots & & \cdots & \cdots & \cdots & & & & & \\ h_{(L-1)M-pL} & \cdots & h_{(L-1)M-L} & h_{(L-1)M} & h_{(L-1)M+L} & \cdots & h_{(L-1)M+(q-p)L} & h_{(L-1)M+(q-p+1)L} & \cdots & h_{(L-1)M+qL} \end{bmatrix} \quad (14b)$$
$$\quad \leftarrow \ \boldsymbol{H}_a \ \rightarrow \ \leftarrow \ \boldsymbol{H}_b \ \rightarrow \ \leftarrow \ \boldsymbol{H}_c \ \rightarrow$$

By taking into account the coefficient symmetry, the matrix $\boldsymbol{H}_b$ of size L by q+1–p can be written as

$$\boldsymbol{H}_b = \begin{bmatrix} h_0 & h_L & \cdots & h_{(L-1)M-L} & h_{(L-1)M} \\ h_M & h_{M+L} & \cdots & h_{(L-2)M-L} & h_{(L-2)M} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ h_{(L-2)M} & h_{(L-2)M-L} & \cdots & h_{M+L} & h_M \\ h_{(L-1)M} & h_{(L-1)M-L} & \cdots & h_L & h_0 \end{bmatrix}. \quad (17)$$

For q+1–p being even[3], the above matrix is a centrosymmetric matrix [4], [5] and, therefore, it can be decomposed into one of the following two forms:

$$\boldsymbol{H}_b = \begin{bmatrix} \boldsymbol{I}_{\lfloor L/2 \rfloor} & 0 & \boldsymbol{J}_{\lfloor L/2 \rfloor} \\ 0 & 1 & 0 \\ \boldsymbol{J}_{\lfloor L/2 \rfloor} & 0 & -\boldsymbol{I}_{\lfloor L/2 \rfloor} \end{bmatrix} \boldsymbol{H}_{b_1} \begin{bmatrix} \boldsymbol{I}_s & \boldsymbol{J}_s \\ \boldsymbol{J}_s & -\boldsymbol{I}_s \end{bmatrix} \quad (18a)$$

$$\boldsymbol{H}_b = \begin{bmatrix} \boldsymbol{I}_{L/2} & \boldsymbol{J}_{L/2} \\ \boldsymbol{J}_{L/2} & -\boldsymbol{I}_{L/2} \end{bmatrix} \boldsymbol{H}_{b_2} \begin{bmatrix} \boldsymbol{I}_s & \boldsymbol{J}_s \\ \boldsymbol{J}_s & -\boldsymbol{I}_s \end{bmatrix} \quad (18b)$$

with

$$s = (q+1-p)/2\,. \quad (18c)$$

Equations (18a) and (18b) correspond to the cases, where the parameter L is odd and even, respectively. The matrices $\boldsymbol{H}_{b1}$ and $\boldsymbol{H}_{b2}$ are defined as follows:

$$\boldsymbol{H}_{b_1} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,s} & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & & \cdots & \cdots & & \cdots & \cdots \\ c_{\lfloor L/2 \rfloor,1} & c_{\lfloor L/2 \rfloor,2} & \cdots & c_{\lfloor L/2 \rfloor,s} & 0 & \cdots & 0 & 0 \\ h_{\lfloor L/2 \rfloor M} & h_{\lfloor L/2 \rfloor M+L} & \cdots & h_{\lfloor L/2 \rfloor M+L(s-1)} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & d_{\lfloor L/2 \rfloor,s} & \cdots & d_{\lfloor L/2 \rfloor,2} & d_{\lfloor L/2 \rfloor,1} \\ \cdots & \cdots & & \cdots & \cdots & & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & d_{1,s} & \cdots & d_{1,2} & d_{1,1} \end{bmatrix} \quad (19a)$$

$$\boldsymbol{H}_{b_2} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,s} & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & & \cdots & \cdots & & \cdots & \cdots \\ c_{L/2,1} & c_{L/2,2} & \cdots & c_{L/2,s} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & d_{L/2,s} & \cdots & d_{L/2,2} & d_{L/2,1} \\ \cdots & \cdots & & \cdots & \cdots & & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & d_{1,s} & \cdots & d_{1,2} & d_{1,1} \end{bmatrix}. \quad (19b)$$

In both cases,

$$c_{k,l} = (h_{(k-1)M+(l-1)L} + h_{(L-k)M+(l-1)L})/2 \quad (20a)$$

$$d_{k,l} = (h_{(k-1)M+(l-1)L} - h_{(L-k)M+(l-1)L})/2 \quad (20b)$$

are calculated from the filter coefficient values and can thus be pre-calculated. Moreover, some parts of the expression in (18a) and (18b) can be combined with the vector of the input samples as

$$\boldsymbol{x}^{(s-1)}_{m,m-q+p} = \begin{bmatrix} \boldsymbol{I}_s & \boldsymbol{J}_s \\ \boldsymbol{J}_s & -\boldsymbol{I}_s \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{m,m-s+1} \\ \boldsymbol{x}_{m-q+p-s+1,m-q+p} \end{bmatrix}, \quad (21)$$

where $\boldsymbol{x}^{(s-1)}_{m,m-q+p}$ is defined by (10). Finally, an implementation structure can be derived in a manner similar to the one shown in Figure 3.

[3] For q+1–p being odd, similar equations to the ones given bellow (17) can be applied after removing the middle column of matrix $\boldsymbol{H}_b$ (cf. example in Section II.A). A detailed discussion about this case is omitted from this paper due to the lack of space.

C. Implementation Complexity Evaluation

By using the proposed technique, the achieved implementation complexity of the overall system is always smaller than in the case where the coefficient symmetry is not utilized. However, due to the structure of the matrices $\boldsymbol{H}_a$ and $\boldsymbol{H}_c$, the number of multiplications cannot, in general, be halved as in the case of 'normal' FIR filters (as was briefly discussed in Section II.A, this goal can be exactly achieved at the expense of a drastically increased number of delay elements). Nevertheless, the proposed implementation complexity approaches the desired goal as shown in TABLE I for the system in Section II.B. For order $N = 25$, the number of multiplications required by the proposed implementation is only 61.5% of that required by the direct implementation that does not exploit the coefficient symmetry, whereas for $N = 145$, this figure drops to 52.1%. Similar observations are also valid for other rational sampling factors.

TABLE I IMPLEMENTATION COMPLEXITY FOR RATIONAL SAMPLING RATE CONVERTERS BY 5/3 IN NUMBER OF MULTIPLICATIONS (C*) AND ADDITIONS (C^+) PER OUTPUT SAMPLE

	Dirrect		Proposed		Comparison	
N	C_d*	C_d^+	C_p*	C_p^+	C_p*/ C_d*	C_p^+/ C_d^+
25	8.7	8.3	5.3	7.3	0.615	0.880
37	12.7	12.3	7.3	10.0	0.579	0.865
49	16.7	16.3	9.3	14.0	0.560	0.857
61	20.7	20.3	11.3	17.3	0.548	0.852
73	24.7	24.3	13.3	20.7	0.541	0.849
109	36.7	36.3	19.3	30.7	0.527	0.844
145	48.7	48.3	25.3	40.7	0.521	0.841
205	68.7	68.3	35.3	57.3	0.515	0.839

In general, the implementation complexity of rational sampling rate converters discussed in this paper can be roughly estimated by[4]

$$C^*_p = C^*_{H_{ac}} + C^*_{H_b} \approx L + s \quad (22a)$$

$$C^+_p = C^+_{H_{ac}} + C^+_{H_b} \approx L + s + 2 + 2s/L\,, \quad (22b)$$

where s is given by (19c).[5] In the above equations C_p* and C_p^+ stand for the number of multiplications and the number of additions per output sample, respectively. Finally, as briefly mentioned in Section III.B, the implementation of the matrices $\boldsymbol{H}_a$ and $\boldsymbol{H}_c$ can be further optimized, particularly for large values of L.

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[4] Values in TABLE I are estimated more precisely.

[5] For filters with orders defined by (13a), $d_{k,s} = 0$ for $k = 0, 1, \ldots \lfloor L/2 \rfloor$; thereby reducing C* and C^+ by $\lfloor L/2 \rfloor$+1 and L, respectively.