Multirate Filters: An Overview

Ljiljana D. Milić, Senior Member, IEEE, Tapio Saramäki, Fellow, IEEE, and Robert Bregović, Member, IEEE

Abstract—The multirate filtering techniques are widely used in sampling rate conversion systems, and for constructing filters with equal input and output rates. Various multirate design techniques provide that the overall filtering characteristic is shared between several simplified sub-filters that operate at the lowest possible sampling rates. Design constraints for sub-filters are relaxed if compared to a single rate overall filter. Hence, by using the multistage approach, the total number of coefficients is significantly reduced. As a consequence of the reduced design constraints, the effects of quantization (finite word-length effects) in sub-filters are decreased. Multirate filters provide a practical solution for digital filters with narrow spectral constraints that are very difficult to solve otherwise.

Index Terms—multirate filters, sampling rate conversion, decimation, interpolation, multistage systems, complementary multirate filters, halfband filters

I. INTRODUCTION

MULTIRATE filter can be defined as a digital filter in which the input data rate is changed in one or more intermediate points. Multirate filters are of essential importance for communications, image processing, digital audio, and multimedia.

Multirate filters have been developed during the past three decades for implementation of digital filters with stringent spectral constraints [1], [2], [3], [4], [5], [6], [9], [10], [11], [24], [26], [29], [38], [39], [40].

Multirate filtering is used whenever two digital systems with different sampling rates have to be connected. Filtering is used to suppress aliasing in decimation, and to remove imaging in interpolation. With the appropriate filter a digital signal of a specified sampling rate can be converted into another signal with a target sampling rate without destroying the signal components of interest.

Multirate techniques are used in filters for sampling rate conversion where the input and output rates are different, and also in constructing filters with equal input and output rates. Multirate filtering is one of the best approaches for solving complex filtering problems when a single filter operating at a fixed sampling rate is of a very high order. For multirate filters, FIR (finite impulse response) or IIR (infinite impulse response) transfer functions can be used. An FIR filter easily achieves a strictly linear phase response, but requires a larger number of operations per output sample when compared with an equal magnitude response IIR filter. Multirate techniques significantly improve the efficiency of FIR filters that makes them very desirable in practice.

The advantage of the multirate approach is the possibility to evaluate the computations in a discrete-time system at the lowest possible sampling rate, thus reducing significantly the processing workload. The second advantage is that with a lower sampling rate, the discrete time systems become easier to implement.

II. FILTERS IN SAMPLING RATE CONVERSION

Filters are used in decimation to suppress aliasing, and in interpolation to remove imaging. The performance of the system for sampling rate conversion is mainly determined by filter characteristics. Since an ideal frequency response cannot be achieved, the choice of an appropriate specification is the first step in filter design.

Reducing the sampling rate by a factor of M is achieved by omitting every M-1 samples, or equivalently, by keeping every Mth sample. This operation is called down-sampling. In order to avoid aliasing, a low-pass antialiasing filter before down-sampling is needed. Therefore, a decimator is a cascade of an antialiasing filter and a down-sampler, Fig. 1(a). To increase the sampling rate (interpolation by factor L), L-1 zeros are inserted between every two samples (up-sampling). An interpolation filter has to be used to prevent imaging in the frequency band above the low-pass cutoff frequency. An interpolator is a cascade of an up-sampler and an anti-imaging filter, Fig. 1(b). The role of filtering in sampling rate conversion is demonstrated in Fig. 2.

![Fig. 1: Decimator and interpolator](image-url)

The efficiency of FIR filters for sampling rate conversion is significantly improved using the polyphase realization. Filtering is embedded in the decimation/interpolation process and a polyphase structure is used to simultaneously achieve the
interpolation/decimation by a given factor but running at a low data rate.

The polyphase structure is obtained when an Nth order filter transfer function is decomposed into M (L) polyphase components, 
M.L<N. For FIR filters, polyphase decomposition is obtained simply by inspection of the transfer function [5], [9], [26], [29], [38]. Figure 3 shows the polyphase structures of M:1 decimator and 1:L interpolator.

Due to the polyphase multirate implementation, the number of arithmetic operations in a linear phase FIR filter is decreased by a factor M (or L). An effective method, which leads to high efficiency for a high-order FIR filter is proposed in [27].

For multirate IIR filters, several approaches to polyphase decomposition have been developed [2], [5], [8], [19], [31], [32]. For a rational conversion factor L/M an efficient decomposition of the Nth order IIR filter, based on the method given in [2] is proposed in [32].

Polyphase IIR filters require lower computation rates among the known decimators and interpolators [31]. If a strictly linear phase characteristic is not requested, an IIR filter is an adequate choice. Moreover, an IIR transfer function can be designed to approximate a linear phase in the pass-band [13], [20], [35]. An IIR decimator or interpolator is particularly useful in applications that cannot tolerate a considerably large delay of an adequate FIR decimator or interpolator. For a restricted class of filter specifications, an attractive solution based on all-pass sub-filters can be used leading to very efficient implementation [19], [31]. The most attractive solution is achieved for M.L=2 with the IIR half-band filter implemented as a parallel connection two all-pass sub-filters [15], [19], [23], [24], [31].

III. MULTISTAGE FILTERING

For decimation and interpolation filters, and for multirate narrowband filters, additional efficiency may be achieved by cascading several stages, each of them consisting of a sub-filter and down-sampler for decimation and an up-sampler and sub-filter for interpolation [9], [24], [26]. Design constraints for sub-filters are relaxed if compared to an overall filter. Hence, by using the multistage approach, the total number of coefficients is significantly reduced when compared with the single stage-design. The effects of finite word-length in sub-filters are low in comparison with the single-stage overall filter. When a decimation/interpolation factor is expressible as a power-of-two, the application of half-band filters improves the efficiency of the system. An example of multistage decimator is shown in Fig. 4.

IV. FILTERS WITH EQUAL INPUT AND OUTPUT RATES

Digital filters with sharp transition bands are difficult, sometimes impossible, to be implemented using conventional structures. A serious problem with a sharp FIR filter is its complexity. The FIR filter length is inversely proportional to transition-width and complexity becomes prohibitively high for sharp filters [5], [9], [26], [29], [33], [38]. In a very long FIR filter, the finite word-length effects produce a significant derogation of the filtering characteristics in fixed-point implementation [26]. IIR filters with sharp transition bands suffer from extremely high sensitivities of transfer function poles that make them inconvenient for fixed-point implementation [22]. In many practical cases, the multirate approach is the only solution that could be applied for the implementation of a sharp FIR or IIR filter. Thus to design a multirate narrowband FIR or IIR filter, a classical time-invariant filter is replaced with three stages consisting of: (1) a low-pass anti-aliasing filter and down-sampler, (2) a low-pass kernel filter, and (3) an up-sampler and low-pass anti-imaging filter [5], [9], [24], [26]. The total number of coefficients in a multirate solution is considerably lower than the number of coefficients of a single rate time invariant filter.

A. Multirate complementary filters

This method can be used in designing filters with any pass-band bandwidth. The multirate techniques are included to reduce the computational complexity. Using the complementary property, the multirate, narrow pass-band filter designs can be used to develop high-pass and low-pass filters with wide pass-bands [9], [26], [30]. When the output of a low-pass multirate filter is subtracted from the delayed replica of the input signal, the result is a wideband high-pass filter. The delay has to be selected to exactly equal the group delay.
of the multirate filter. For a low-pass wideband filter the multirate narrowband high-pass filter has to be used.

Efficient FIR filters with an arbitrary bandwidth can be designed using multirate and complementary filtering [9], [16] [30]. The overall design is evaluated by cascading complementary multirate filtering two-ports composed of two series branches and one parallel branch. The cascade is terminated with a simple kernel filter. One series branch of the cascade is a decimator (filter and down-sampler), while the other is an interpolator (up-sampler and filter). The parallel branch is a delay. The most efficient solution is obtained when half-band filters are used in the cascade.

Recently, the complementary filtering approach is extended to IIR filters [17]. The overall filter makes use of an IIR filter as a kernel filter, the periodic all-pass filters for constructing complementary pair, and linear phase FIR filters for the sampling rate alterations.

Figure 5 illustrates a solution of sharp wide band filter.

\[ \text{Magnitude response} \]

Fig. 5: Example of a sharp wideband filter.

### B. Pipelining/Interleaving techniques

The pipelining/interleaving (P/I) architectures when used in constructing multirate systems can significantly improve the hardware efficiency. The P/I technique developed in [14] is based on the repetitive use of the same filter with the clock rate two or three times higher than a data rate, and with interleaved feedback of the output samples. The resulting filtering is equivalent to the cascade of identical filters.

### V. HALF-BAND FILTERS

Half-band filters are basic building blocks in multirate systems. A half–band filter divides the basis band of a discrete-time system in two equal bands with symmetry properties. The FIR filters are most often used as half-band filters. For a linear phase FIR half-band filter, half of the constants are zero valued when the filter order is an even number [26], [33]. A half-band IIR filter can have fewer multipliers than the FIR filter for the same sharp cutoff specification. An IIR elliptic half-band filter when implemented as a parallel connection of two all-pass branches is an efficient solution [24]. The main disadvantage of elliptic IIR filters is their very nonlinear phase response. To overcome the phase distortion one can use optimization to design an IIR filter with an approximately linear phase response [35] or one can apply the double filtering with the block processing technique for real-time processing [21], [28].

For the appropriate usage of digital filter design software in half-band filter design, it is necessary to calculate the exact relations between the filter design parameters in advance [24].

The accurate FIR half-band filter design methods can be found in [33], [36], [39]. For the IIR half-band filter design see [24], [25], [34].

### VI. COMPLEMENTARY FILTER PAIRS

Complementary filter pairs are used to split the input signal in two adjacent bands, and also are of importance for constructing complex multirate systems and filter banks. A low-pass/high-pass filter pair can be designed to exhibit all-pass complementary, power complementary, or magnitude complementary properties [26]. In the most applications, half-band filter pairs are used to divide frequency band in two equal sub-bands. This solution benefits the possibility to implement a complementary filter pair at the cost of a single FIR (IIR) filter pair of a low-pass/high-pass linear phase FIR filter pair: filter order N=26 implemented with 7 multiplication constants, and 7 delays (memory save implementation); (b) low-pass/high-pass IIR filter pair: filter order N=7 implemented with only 3 multiplication constants and 3 double delays. When the sampling rate is to be changed by 2, all the arithmetic operations can be evaluated at the lower rate.

![Efficient half-band filter pairs](image)

As shown recently, an IIR filter pairs with the arbitrary crossover frequency can be easily obtained by simple transformation of the half-band filter prototypes [41], [42]. This new filter pair retains the complementary properties of the start-up half-band filter pair.

### VII. MULTIPLIERLESS SOLUTIONS

The efficiency of multirate filters is greatly improved by simplifying arithmetic operations. This is achieved by replacing a multiplier with a small number of shifters-and-adders. Generally, implementing multiplierless design techniques in sub-filters, at the cost of a slight derogation of filtering performances, increases the efficiency of the overall multirate filter. For instance, one can use the optimization technique [18], the multiplier block approach [7], or design based on EMQF (Elliptic Minimal Q-Factors) transfer functions [21], [23], [25]. A well-known solution for large
conversion factors in decimation is a cascaded integrated comb (CIC) filter, which performs multiplierless filtering [11], [12].

REFERENCES


