Model-based force and position tracking control of an asymmetric cylinder with a digital hydraulic valve

Matti Linjama, Mikko Huova, Kalevi Huhtala

Department of Intelligent Hydraulics and Automation, Tampere University of Technology, Tampere, Finland

P.O. Box 589, 33101 Tampere, Finland, matti.linjama@tut.fi

Matti Linjama is Adjunct Professor at the Tampere University of Technology. He leads the digital hydraulics research group at the Department of Intelligent Hydraulics and Automation.

Mikko Huova has just finished his PhD thesis about energy efficient digital hydraulic valve control and works as a senior researcher in the digital hydraulics research group.

Kalevi Huhtala is head of the Department of Intelligent Hydraulics and Automation. He is a professor of mobile machines.
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This paper presents a model-based control solution for large inertia systems controlled by a fast digital hydraulic valve. The solution is based on model-based force control and it is shown that the cylinder chamber pressures have first order dynamics with the proper parameter selection. The robust stability is analyzed under unknown load mass, bulk modulus, and delay, and it is shown that a simple cascaded P+PID controller results in good control performance and robustness. The simulated results show smooth and stable response with good tracking performance despite large variations in the load mass and bulk modulus.

Keywords: digital hydraulics; tracking control; force control; position control

1 Introduction

Electrohydraulic servo systems are used in many applications, such as aircraft actuators and paper mills, because of their high power-to-weight ratio. Position, velocity and force control are the basic functions of modern hydraulic actuators (Jelali and Kroll 2004). The compressibility of the fluid and the non-linear flow characteristics of the control valves make these systems difficult to control, and several different approaches have been suggested for controlling force, velocity, and position tracking. Kim et al. (2015) studied flatness based non-linear control and demonstrated an approximate 0.3 mm position tracking error with a peak velocity of 63 mm/s. Thus, the commonly used performance index—the maximum error divided by the peak velocity (Zhu and Vukovich 2011)—was about 0.005 s. Koivumäki and Mattila (2015) used virtual decomposition control in a construction crane and achieved a performance index of 0.0030 s. Won et al. (2015) used the high-gain disturbance observer in the control of 1-DOF hydraulic arm and experimental results demonstrated performance index of about 0.005 s.
Eyrilmaz and Wilson (2001) designed a controller using the singular perturbation theory and Lyapunov-type nonlinear feedback. The most interesting benefits of their solution are its robustness against variations in the bulk modulus and the absence of the need to differentiate velocity or pressures. The simulated results showed a performance index of 0.0008 s, but the load mass was only 12 kg.

Digital hydraulic valve control is a potential alternative for traditional hydraulic servo systems (Linjama and Vilenius 2005, Linjama et al. 2008). The basic layout of a digital hydraulic four-way valve is shown in Figure 1. It consists of four valve series, which are also known as a digital flow control unit (DFCU), to control each flow path independently. Each DFCU has $2^N$ opening combinations where $N$ is the number of parallel connected valves; thus, the four-way valve has $2^{4N}$ opening combinations. The typical value for $N$ is five, so the number of opening combinations is over one million. Another approach is to use mutually similar valves, which results in $N+1$ opening combinations with $N$ parallel connected valves (Linjama and Vilenius 2008). Model-based control has been used to determine the optimal opening combination (Linjama and Vilenius 2005). The steady state model has been used and the intrinsic assumption is that the system dynamics is fast in comparison to the sampling period of the controller. However, that assumption is not valid in systems with large inertia.

This paper presents a model-based control solution for large inertia systems controlled by a fast digital hydraulic valve. The solution is based on a model-based force control and the outcome shows that the cylinder chamber pressures have first order dynamics with the proper parameter selection. The robust stability was analyzed under unknown load mass, bulk modulus, and delay, and it was shown that a simple cascaded P+PID controller results in good control performance and robustness. The simulated results show smooth and stable response with good tracking performance.
despite large variations in the load mass. A performance index of 0.0032 s was achieved, which is the same level that was obtained using more complicated controllers as shown in the work of Kim et al. (2015), Koivumäki and Mattila (2015) and Won et al. (2015).

2 Model-based control of an asymmetric cylinder

2.1 System model

The dynamics of a hydraulic cylinder with inertia $m$ and external force $F_{\text{load}}$ can be expressed as:

$$\dot{p}_A = \frac{B_A}{A_AX + V_0A}(Q_{PA} - Q_{AT} - A_A\dot{x})$$

$$\dot{p}_B = \frac{B_B}{A_B(x_{\text{max}} - x) + V_{0B}}(Q_{PB} - Q_{BT} + A_B\dot{x})$$

$$m\ddot{x} = p_AA_A - p_BA_B - F_{\mu}(\dot{x}) - F_{\text{load}}$$

The flow rates of DFCUs are modelled using the generalized turbulent flow model with cavitation choking (Linjama et al. 2012):

$$Q_{XY} = \sum_{i=1}^{N} u_{XY}(i)Q_{XY,i}$$

where XY is either PA, AT, PB, or BT and $\bullet(i)$ refers to the $i$:th element of the vector.

Combining Equations 1 and 2 gives three non-linear differential equations:
\begin{align*}
\dot{p}_A &= f_A(p_A, x, \dot{x}, u_{PA}, u_{AT}, p_P, p_T) \\
\dot{p}_B &= f_B(p_B, x, \dot{x}, u_{PB}, u_{BT}, p_P, p_T) \\
\dot{x} &= f_x(p_A, p_B, \dot{x}, F_{load})
\end{align*}

The control inputs of the system are the control vectors of the DFCUs \( (u_{PA}, u_{AT}, u_{PB}, u_{BT}) \) and the disturbances are supply pressure \( p_P \), tank pressure \( p_T \), and load force \( F_{load} \). The state variables are chamber pressures \( p_A \) and \( p_B \) as well as piston position \( x \) and velocity \( \dot{x} \).

### 2.2 One-step ahead estimator

The assumption made in this paper is that the inertia \( m \) is so large that the velocity does not change significantly during one controller sampling period. This decouples the differential equations and makes it possible to estimate pressures using scalar integration methods without knowledge of the inertia. Heun’s method is used for estimating the pressure changes (Goode 2000, p. 86):

\begin{align*}
\tilde{p}_A(k+1) &= p_A(k) + T_s f_A(p_A(k), x(k), \dot{x}(k), u_{PA}(k), u_{AT}(k), p_P(k), p_T(k)) \\
\hat{p}_A(k+1) &= p_A(k) + \frac{T_s}{2} \left( f_A(p_A(k), x(k), \dot{x}(k), u_{PA}(k), u_{AT}(k), p_P(k), p_T(k)) + f_A(\tilde{p}_A(k+1), x(k), \dot{x}(k), u_{PA}(k), u_{AT}(k), p_P(k), p_T(k)) \right)
\end{align*}

The equations for the B-side pressure are similar.

### 2.3 Inner-loop force controller

The chamber pressures can be controlled by selecting the control signals so that the differences between the target pressures and the estimated pressures are minimized. The output of the hydraulic cylinder is piston force, which is given by:

\[ F = p_A A_A - p_B A_B - F_\mu \]
Thus, there are two control variables and one output. One possible option is that the
pressure differentials over the DFCUs are the same:

\[
 p_p - p_{A,\text{ref}} = p_{B,\text{ref}} - p_T \\
 F_{\text{ref}} = p_{A,\text{ref}} A_A - p_{B,\text{ref}} A_B = p_{A,\text{ref}} A_A - (p_p + p_T - p_{A,\text{ref}})A_B \\
\]

(6)

Equation 6 does not include friction force. If friction compensation is used, it can be
added to \( F_{\text{ref}} \). The primary objective of the force controller is to minimize the force error
while the secondary objective is to keep the pressures near the target values. This is
solved by first selecting the \( N_{\text{cand}} \) control candidates for the A-side and the B-side, so
that the estimated error in the chamber pressures is small, and then calculating the force
error for each combination of the control candidates. The control candidates are selected
according to the weighted sum of the estimated pressure error and the magnitude of the
openings of the DFCUs:

\[
 J_A = |p_{A,\text{ref}} - \hat{p}_A (k + 1)| + W_{u,p} b^T (u_{PA} + u_{AT}) \\
 J_B = |p_{B,\text{ref}} - \hat{p}_B (k + 1)| + W_{u,p} b^T (u_{PB} + u_{BT}) \\
\]

(7)

where \( W_{u,p} \) is the weight for the openings of the DFCUs and the vector \( b \) determines the
relative sizes of the valves; for example, \( b = [1, 2, 4, 8]^T \) for the binary coded DFCU
with four valves. The values of \( J_A \) and \( J_B \) are calculated for all possible control
combinations and the \( N_{\text{cand}} \) candidates that give the smallest value for \( J_A \) and \( J_B \) are
selected for further analysis. Thus, the total number of candidates is \( N_{\text{cand}}^2 \). The final
selection is made by minimizing the following penalty function:
\[
J = \left| F_{\text{ref}} - \hat{F}(k+1) \right| + W_{u,e} \mathbf{b}^T (u_A + u_{AT} + u_{PB} + u_{BT}) \\
\hat{F}(k+1) = \hat{p}_A(k+1)A_A - \hat{p}_B(k+1)A_B
\]

(8)

2.4 Pressure and force dynamics under an unknown bulk modulus

The inner-loop controller is a non-linear selection scheme that tries to minimize the error between the target pressure and the estimated pressure. In the ideal case, the controller finds the valve openings where the pressure error is small after one sampling period. In practice, error occurs due to the parameter uncertainty of the model and the finite resolution of the DFCUs. The flow rate of the on/off valve (Eq. 2) can be calculated reliably for the given pressures (Linjama et al. 2012). Much larger uncertainty comes from the uncertainty of the bulk modulus, because it depends on the pressure and the amount of entrapped air. Additional error for the pressure estimate comes from the inaccuracy of the estimator (Eq. 4) because simple fixed step integration is used. This error can be reduced by decreasing the sampling period of the controller or by using a multi-step approach. For simplicity, it is assumed that all uncertainty can be lumped into the bulk modulus. It is assumed that the controller works perfectly but the bulk modulus estimate is incorrect. Thus, the estimated pressure equals the target value after one sampling period:

\[
\hat{p}_A(k+1) = p_A(k) + \frac{T_s}{A_A x + V_{0A}} (Q_{PA} - Q_{AT} - A_A \dot{x}) dt = p_{A,\text{ref}}
\]

(9)

Now, the real pressure dynamics can be expressed as:

\[
p_A(k+1) = p_A(k) + \frac{T_s}{A_A x + V_{0A}} \frac{B_A}{B} (Q_{PA} - Q_{AT} - A_A \dot{x}) dt =
\]

\[
p_A(k) + \frac{B_A}{B} \int_0^{T_s} \frac{\hat{B}_A}{A_A x + V_{0A}} (Q_{PA} - Q_{AT} - A_A \dot{x}) dt
\]

(10)
Combining Equations 9 and 10 gives the following difference equation for the pressure dynamics:

\[ p_A(k + 1) = \left(1 - \frac{B_A}{B_A^*} \right) p_A(k) + \frac{B_A}{B_A^*} p_{A,ref} \]  \hspace{1cm} (11)

which is stable for \( \hat{B}_A > B_A/2 \). The optimal response is achieved with \( \hat{B}_A = B_A \), and for \( \hat{B}_A > B_A \) the response has the first order dynamics with the time constant:

\[ \tau_A = -\frac{T_s}{\ln\left(1 - B_A/\hat{B}_A\right)} \]  \hspace{1cm} (12)

The analysis shows it is safer to use too big value for the estimate of the bulk modulus. Too big value makes the pressure estimation only slower while too small value may yield instability. The tuning is made in such a way that the estimate of the bulk modulus is selected to be the highest possible value, i.e., the bulk modulus of the fluid at the maximum system pressure without any entrapped air. This guarantees that the pressure has the first order dynamics.

The B-side pressure dynamics can be derived in a similar way. The A-side and B-side pressure dynamics can have different time constants because the real bulk moduli can be different. The force dynamics in s-domain is:

\[ F(s) = A_p p_A(s) - A_B p_B(s) = \left(\frac{A_A}{\tau_A s + 1} p_{A,ref} - \frac{A_B}{\tau_B s + 1} p_{B,ref} \right) \] 
\[ \frac{\tau_B A_A + \tau_A A_B}{\tau_A s + 1}(\tau_A s + 1) F_{ref}^A + \frac{A_A A_B}{\tau_A s + 1}(\tau_B s + 1)(\tau_A s + 1) \left(\frac{(\tau_B - \tau_A)s}{\tau_B s + 1} \right) \left( p_p(s) + p_T(s) \right) = \]  \hspace{1cm} (13)

\[ G_F(s) F_{ref} + G_p(s) \left( p_p(s) + p_T(s) \right) \]
The second term vanishes if the supply and tank pressures are constant.

2.5 The outer-loop velocity controller

The analysis presented in Section 2.4 shows that, with proper selection of the estimate of the bulk modulus, the inner-loop controller has the dynamics $G_F(s)$ with unknown time constants $\tau_A$ and $\tau_B$. The upper limit of the time constants can be determined by estimating the lowest possible bulk modulus. For example, if it is assumed that the real bulk modulus is at least half of the estimated value and the sampling period is 10 ms, the time constant is 14.4 ms. The lower limit for the time constants is zero. In addition to the time constants, the system also has delay $d$ caused by the valves, the sampling, the computation, and the pipeline dynamics. The resonances caused by the pipeline dynamics are assumed to be in high frequency range and the controller is tuned to be robust against high frequency modelling errors. This assumption is valid if pipeline lengths are 2 meters at maximum. The lowest natural frequency of a 2 meters pipeline is about 1800 rad/s, and it will be shown later that the system is robust against such modelling error. The block diagram of the system is shown in Figure 2 in which $G_V(s)$ is the transfer function of the velocity controller. The transfer function of the nominal model is selected to be:

$$G_N(s) = \frac{1}{m_{\text{min}}s} e^{-d_{\text{max}}s}$$

(14)

where $m_{\text{min}}$ is the smallest possible load mass of the system and $d_{\text{min}}$ is the minimum delay of the valves. The true transfer function is:

$$G_F(s) = \frac{1}{m s} G_F(s) e^{-d s}$$

(15)
where \( m \) is the actual mass, \( G_T(s) \), as given by Equation 13, and \( d \) is the actual system delay caused by the sampling, the computation, the valve dynamics, and the pipeline dynamics. The multiplicative error caused by the parameter variations is:

\[
\Delta(s) = \frac{G_T(s) - G_N(s)}{G_N(s)}
\]  

(16)

Now, the system remains stable if \( G_T(s) \) and \( G_N(s) \) have the same number of unstable poles and the following holds (Green and Limebeer 1995):

\[
|\Delta(j\omega)| < \left| \frac{1 + G_N(j\omega)G_v(j\omega)}{G_N(j\omega)G_v(j\omega)} \right| \forall \omega > 0
\]  

(17)

Both \( G_T(s) \) and \( G_N(s) \) have one pole at the origin and the other poles are stable. Therefore, the stability criterion of Equation 17 can be used.

**Example 1.** Figure 3 shows the multiplicative modelling error when \( m_{\text{min}} \) is 250 kg, \( m \) is 250–500 kg, time constants \( \tau_A \) and \( \tau_B \) are 1–6.4 ms, and delay \( d \) is 1.5–7 ms. The piston diameter is 40 mm and the piston rod diameter is 20 mm.

Based on Example 1, it can be stated that the multiplicative modelling error is below curve:

\[
\Delta_M = K|\tau j\omega + 1|
\]  

(18)

where the time constant \( \tau \) is larger than the largest possible value of time constants \( \tau_A \) and \( \tau_B \), and \( K \) is a constant smaller than 1. A good property of \( \Delta_M \) is that it increases with frequency; thus, it takes into account unknown high frequency modelling errors such as pipeline dynamics. The robust controller tuning is such that:
\[
\frac{1 + G_N(j\omega)G_Y(j\omega)}{G_N(j\omega)G_Y(j\omega)} > \Delta_M \quad \forall \omega > 0
\]  

(19)

One possible controller is the P-controller:

\[G_Y(s) = K_{P,\text{vel}}\]  

(20)

**Example 2.** Figure 4 shows both sides of Equation 19 for the system presented in Example 1 when \(K_{P,\text{vel}}\) is 19000 Ns/m. The parameters of \(\Delta_M\) are selected to be \(K = 0.9\) and \(\tau = 13\) ms. It is seen that the P-controller yields almost an exact fit on \(\Delta_M\). Figure 5 shows the velocity step responses of the system with the parameter variations of Example 1. The maximum overshoot is 35\% and the worst case settling time is about 0.13 s.

### 2.6 Position tracking controller

The block diagram of the position controlled system is shown in Figure 6. The position loop design is similar to the velocity controller. The nominal open-loop transfer function is:

\[G_{N,\text{pos}} = \frac{G_N(s)G_Y(s)}{s(1 + G_N(s)G_Y(s))}\]  

(21)

and the true transfer function is:

\[G_{T,\text{pos}} = \frac{G_T(s)G_Y(s)}{s(1 + G_T(s)G_Y(s))}\]  

(22)

The multiplicative modelling error is:
\[ \Delta_{pos}(s) = \frac{G_{r,pos}(s) - G_{N,pos}(s)}{G_{N,pos}(s)} \]  

Analysis similar to Example 2 shows that the modified PID-controller is a good candidate for the position loop:

\[ G_p(s) = \frac{K_{P,pos}}{\tau_p s + 1} + \frac{K_{I,pos}}{s} + \frac{K_{D,pos}s}{\tau_D s + 1} \]  

**Example 3.** Figure 7 shows the multiplicative modelling error of Equation 23 for the system presented in Examples 1 and 2. The parameter variations are the same as those in Example 1 and the velocity controller is tuned according to Example 2. Figure 7 also shows the term:

\[ \left| \frac{1 + G_{N,pos}(j\omega)G_p(j\omega)}{G_{N,pos}(j\omega)G_p(j\omega)} \right| \]  

when the controller parameters are \( K_{P,pos} = 32 \text{ s}^{-1}, \tau_p = 15 \text{ ms}, K_{I,pos} = 120 \text{ s}^{-2}, K_{D,pos} = 0.4, \) and \( \tau_D = 2 \text{ ms}. \) Figure 8 shows the position step responses with different parameter values.

Example 3 shows that the integrator causes overshoot, but is essential in order to compensate for the force disturbances. The target is tracking control, and fast rise time and overall stability are more important than overshoot.

3 Simulation results

3.1 System model

The simulated system is a horizontal cylinder with a load mass ranging between 250 kg and 500 kg. The piston diameter is 40 mm, the piston rod diameter is 20 mm, and the
stroke is 500 mm. The supply pressure is 10 MPa and the tank pressure is zero. The
digital valve system consists of $4 \times 16$ miniaturized valves (Linjama et al. 2014). All the
valves have the same flow capacity of 1.3 l/min at a 3.5 MPa pressure differential. The
exponents are selected to be 0.53 and the cavitation choking parameters $b_{PA}(i)$, $b_{AT}(i)$,
$b_{PB}(i)$ and $b_{BT}(i)$ are 0.3 for all of the valves. The valve delay is assumed to be 2.5 ms
and the opening and closing happens with a linear motion profile within 1 ms. The
friction force is modelled using the following equation:

$$ F_{\mu} = \tanh\left(20000 \frac{m}{s} \cdot \dot{x}\right)
\left(280 N + 70 N \cdot e^{-10000 m^{2} \cdot x^2}\right) + 100 \frac{Nm}{s} \cdot \dot{x} \quad (26) $$

which is simplified version of the friction model presented by Makkar et al. (2005). The
dead volumes $V_{0A}$ and $V_{0B}$ are 0.001 m$^3$, and the nominal value of the bulk modulus is
1200 MPa.

### 3.2 Implementation of position and velocity measurement

Real velocity is normally not measured but it is obtained from the measured position by
differentiation. It is assumed that the position is measured by a pulse encoder with 0.01
mm resolution. The effect of quantization noise on velocity is reduced by using a filter
suggested by Harrison and Stoten (1995):

$$ \hat{x}(k) = \frac{5x(k) + 3x(k - 1) + x(k - 2) - x(k - 3) - 3x(k - 4) - 5x(k - 5)}{35T_s} \quad (27) $$

The filter runs with 1 ms sampling period.

### 3.3 Force control results

The outer-loop controller of Equations 20 and 24 is disabled when the force tracking
results are simulated. The sampling period of the controller is selected to be 4 ms, i.e.,
slightly more than the maximum response time of the valves. The weight factors $W_u,p$ and $W_{u,F}$ are selected such that the simultaneous opening of DFCUs PA and AT and DFCUs PB and BT is minimized, but the force tracking performance is still good. The values used are $W_{u,p} = 0.01 \text{ MPa}$ and $W_{u,F} = 2 \text{ N}$. The estimate of the bulk modulus is selected to be 1500 MPa, which is considered to be the largest possible value. The parameter $N_{cand}$ is selected to be six.

Figure 9 shows the simulated Bode diagram of the force controlled system when the load mass is 500 kg. The initial piston position is 0.2 m. The input signal is a sinusoidal force reference with 2 kN amplitude. The sinusoidal signal is fitted to the simulated force by using the least squares method and the magnitude ratio and phase shift are determined from the fitted curve. The $-90^\circ$ bandwidth is 180 rad/s for $B = 700$ MPa and 228 rad/s for $B = 1200$ MPa.

### 3.4 Position tracking results

The minimum load mass is assumed to be 250 kg. It is assumed that the true bulk modulus is at least 700 MPa, which results in a maximum time constant of 6.4 ms. The maximum system delay is assumed to be the 3 ms valve delay plus the sampling period, i.e., 7 ms. Thus, the parameters are the same as those found in Examples 1–3 and the resulting tuning is $K_{p,vel} = 19000 \text{ Ns/m}$, $K_{p,pos} = 32 \text{ s}^{-1}$, $\tau_p = 15 \text{ ms}$, $K_{I,pos} = 120 \text{ s}^{-2}$, and $K_{D,pos} = 0.4$. The derivative of the error signal is formed by equation 27. The integrator term has a ±0.3 mm dead zone in order to avoid the limit cycles. The error signal of the integrator term is calculated as follows:

$$e_I = \begin{cases} e, & |e| \geq \text{deadzone} \\ 0, & |e| < \text{deadzone} \end{cases}$$

(28)

The estimate of the friction force is added to the force reference and is calculated as
follows:

$$\hat{F}_\mu = 250 \text{ N} \cdot \tanh \left( 20000 \text{ m}^{-1} \text{s} \cdot \dot{x} \right)$$ (29)

The estimate is smaller than real friction in order to guarantee stability at zero velocity.

Figure 10 presents the simulated response with a 250 kg load mass. The position trajectory is the fifth order polynomial with 120 mm amplitude and 1 s movement time. The peak velocity is 225 mm/s and the maximum position tracking error is 0.52 mm. Thus, the performance index is 0.0023 s. The plots also show that the pressure and force tracking are reasonable.

Figure 11 depicts the simulated response when the load mass is increased to 500 kg. The tracking error increases to 0.73 mm and the performance index increases to 0.0032 s.

4 Conclusions

This paper has presented a new digital hydraulic tracking control solution for systems with big inertia. The parameter uncertainty is expressed as the unstructured multiplicative error and the low-order controllers are tuned in order to achieve robust stability. The core component of the controller is the model-based force controller, which allows force tracking under unknown bulk modulus. The suggested outer loop P + modified PID cascade controller results in good position tracking performance under unknown load mass, bulk modulus, and system delay. The simulated maximum position tracking error is 0.73 mm while the maximum piston velocity is 225 mm/s. The relative accuracy (tracking error divided by peak velocity) is comparable to more advanced control solutions. The suggested controller needs pressure, position, and velocity measurements but not derivatives of those. Moreover, when comparing the performance
against other controller solutions it must be noted that the solution presented in this paper does not require exact knowledge of inertia, delay, or bulk modulus.

**Nomenclature**

\( A_A \quad \text{Piston area, piston side [m}^2\text{]} \)

\( A_B \quad \text{Piston area, rod side [m}^2\text{]} \)

\( b \quad \text{Vector of relative sizes of valves of DFCUs} \)

\( b_{XY} \quad \text{Vector of critical pressure ratios of valve model of DFCU XY, where XY is either PA, AT, PB or BT [-]} \)

\( B_A, B_B \quad \text{Bulk moduli of A- and B-side chambers [Pa]} \)

\( d \quad \text{System delay [s]} \)

\( d_{min} \quad \text{Minimum system delay [s]} \)

\( e \quad \text{Position error [m]} \)

\( e_I \quad \text{Position error used by integrator term [m]} \)

\( F_{load} \quad \text{External force [N]} \)

\( F_{ref} \quad \text{Target force [N]} \)

\( F_\mu \quad \text{Friction force [N]} \)

\( G_F(s) \quad \text{Transfer function from } F_{ref} \text{ to force} \)

\( G_N(s) \quad \text{Nominal transfer function from force to velocity} \)

\( G_{N,pos}(s) \quad \text{Nominal transfer function from force to position} \)

\( G_p(s) \quad \text{Transfer function from supply and tank pressure to force} \)

\( G_{p}(s) \quad \text{Transfer function of the position controller} \)

\( G_T(s) \quad \text{True transfer function from force to velocity} \)

\( G_{T,pos}(s) \quad \text{True transfer function from force to position} \)

\( G_V(s) \quad \text{Transfer function of the velocity controller} \)

\( J \quad \text{Cost function for force error [N]} \)
\( J_A \) Cost function for A-side pressure error [Pa]
\( J_B \) Cost function for B-side pressure error [Pa]
\( K \) Parameter of multiplicative modelling error limit
\( K_{D,pos} \) Position controller gain, D-term [-]
\( K_{I,pos} \) Position controller gain, I-term \( [s^{-2}] \)
\( K_{P,pos} \) Position controller gain, P-term \( [s^{-1}] \)
\( K_{P,vel} \) Velocity controller gain \( [N \cdot m^{-1}s] \)
\( K_{v,XY} \) Vector of flow coefficients of valve model of DFCU XY, where XY is either PA, AT, PB or BT \( [m^3/s \cdot Pa^x] \)
\( m \) Load mass [kg]
\( m_{min} \) Minimum load mass [kg]
\( N \) Number of parallel connected valves
\( N_{cand} \) Number of control candidates for A- and B-side
\( p_A \) Pressure at A-chamber [Pa]
\( p_{A,ref} \) Target value for A-pressure [Pa]
\( p_B \) Pressure at B-chamber [Pa]
\( p_{B,ref} \) Target value for B-pressure [Pa]
\( p_P \) Supply pressure [Pa]
\( p_T \) Tank line pressure [Pa]
\( Q_{AT} \) Flow rate from A-chamber to tank line \( [m^3/s] \)
\( Q_{BT} \) Flow rate from B-chamber to tank line \( [m^3/s] \)
\( Q_{PA} \) Flow rate from supply line to A-chamber \( [m^3/s] \)
\( Q_{PB} \) Flow rate from supply line to B-chamber \( [m^3/s] \)
\( T_S \) Sampling period of the controller [s]
**u_{XY}** Vector of control signals of valves of DFCU XY, where XY is either PA, AT, PB or BT [-]

\( V_{0A} \) A-side dead volume \([\text{m}^3]\)

\( V_{0A} \) B-side dead volume \([\text{m}^3]\)

\( W_{u,F} \) Weight for openings of DFCUs [N]

\( W_{u,p} \) Weight for openings of DFCUs [Pa]

\( x \) Piston position [m]

\( x_{\text{max}} \) Maximum piston position [m]

**x_{XY}** Vector of exponents of valve model of DFCU XY, where XY is either PA, AT, PB or BT [-]

\( \Delta \) Multiplicative modelling error, velocity loop

\( \Delta_{M} \) Limit for multiplicative modelling error

\( \Delta_{\text{pos}} \) Multiplicative modelling error, position loop

\( \tau \) Parameter of multiplicative modelling error limit

\( \tau_{A} \) Time constant of A-side pressure dynamics [s]

\( \tau_{B} \) Time constant of B-side pressure dynamics [s]

\( \tau_{D} \) Time constant of derivative term

\( \hat{\cdot} \) Estimate of \( \cdot \)

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**References**


Figure 1. Digital valve system with five parallel connected valves. Orifices are used to adjust the flow capacity of the valves according to binary series.
Figure 2. Block diagram of the velocity control system.

Figure 3. Multiplicative modelling error of the velocity controlled system with $m = 250$-$500$ kg, $\tau_a = 1$–$6.4$ ms, $\tau_B = 1$–$6.4$ ms, and delay $d = 1.5$–$7$ ms.
Figure 4. Both sides of Equation 19 with the multiplicative modelling error of Example 1.
Figure 5. Velocity step responses with the parameter variations of Example 1.

Figure 6. Block diagram of the position control system.
Figure 7. Multiplicative modelling error of Example 3.
Figure 8. Position step responses with the parameter variations of Example 1.
Figure 9. Simulated Bode diagrams of the force controller with a 500 kg load mass.
Figure 10. Position tracking results with a 250 kg load mass.
Figure 11. Position tracking results with a 500 kg load mass.