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A simplified Solar Radiation Pressure Model for GNSS autonomous orbit prediction

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***Abstract*— We present a two-parameter Solar Radiation Pressure model for GNSS autonomous orbit prediction, in which the parameters vary with the daily and seasonal variations in the angle between the Sun and the satellite's orbital plane. The estimator for this model's parameters is described. We present the test results of orbit prediction with this model for GPS satellites using initial orbit states based on broadcast ephemeris data.**

I. Introduction

Autonomous orbit prediction for GNSS satellites is used for shortening the time to first fix (TTFF) on mobile device. The device receives one or more broadcast ephemerides (BE), then solves the equation of motion to determine satellite trajectory several days into the future, to be used by the device as assistance data [1], [2].

For orbit prediction, two factors significantly affect the accuracy: the initial position and velocity, and the forces model. Algorithms for adjusting the initial position and velocity for orbit prediction have been presented in [3], [4], [5].

Solar radiation pressure (SRP) plays a key role in the force model. The SRP model parameters can be assumed to be constant for calculation of a trajectory over a few days. However, the optimal parameters vary over time spans of weeks. Because autonomous orbit prediction is intended to run on devices whose network connection is expensive, unreliable, or nonexistent, updating the parameters of the SRP model from a server is not an option. Detailed physics-based SRP models have been developed for GNSS orbit determination. However, we prefer to use an empirical SRP model, because an extremely accurate SRP model is not essential for our implementation, and also the details of satellites' geometry and albedo properties are not easy to obtain. Building on our previous two-parameters empirical SRP model [6], in this paper we develop an algorithm for adapting the SRP parameters online, and present results showing how the algorithm improves the precision of autonomous orbit prediction for GPS satellites.

II. The CODE SRP model

The CODE SRP model and its variations are widely used for orbit determination and orbit prediction in GNSS data analysis centers. Its original version [7] includes 18 parameters:

$$\begin{aligned}
\alpha_D &= D_0 + D_{c2}\cos(2\beta) + D_{c4}\cos(4\beta)\\
\alpha_Y &= Y_0 + Y_c\cos(2\beta)\\
\alpha_B &= B_0 + B_c\cos(2\beta)\\
\alpha_{Zp} &= \{Z_0 + Z_{c2}\cos(2\beta) + Z_{s2}\sin(2\beta)\\
&\quad + Z_{c4}\cos(4\beta) + Z_{s4}\sin(4\beta)\}\sin(\mu-\mu_0)\\
\alpha_{Xp} &= \{X1_0 + X1_c\cos(2\beta) + X1_c\cos(2\beta)\}\sin(\mu-\mu_0)\\
&\quad + \{X3_0 + X3_c\cos(2\beta) + X3_s\sin(2\beta)\}\sin(3\mu-\mu_0)
\end{aligned} \tag{1}$$

Here α_D is the acceleration in the satellite-to-Sun direction, α_Y is along the spacecraft solar-panel in axis, α_B is in the direction of the cross product of Y and D, α_{Zp} is in the satellite-Earth direction, and α_{Xp} is along the spacecraft X axis, which complete the satellite body-fixed Cartesian frame. Also μ is the argument of latitude, μ_0 is the argument of latitude at midnight, and β is the Sun elevation angle.
The periodic terms α_{Xp} and α_{Zp}, vary with μ, which has a period of about 12 hours. The forces of α_D, α_B, α_{Xp}, and α_{Zp} are in B-D plane. In fact, α_{Xp} and α_{Zp} can be projected into directions D and B by multiplying with sine or cosine of ϵ (the Sun-satellite-Earth angle). The terms with β account for the variation of the area of satellite bus exposed to the Sun as the Sun-Satellite-Earth angle changes. The variation of β is slower than μ, less than 1 degree per day, so we can regard it as a constant within an orbit cycle. For GPS orbits, the effects of time-varying terms are much smaller than the constant terms, details are in table 3 of [7].

Our simplified SRP model take the value of SRP forces in the direction of D and Y for the parameters, averaged over μ, and these parameters are β angle dependent; details are given in Section IV.

III. Evolution of β angle

The Sun elevation angle β has the range $[-78^0, +78^0]$ for GPS constellation. Satellites in the same orbital plane have the same β. Fig. 1 shows the variation of β over a 5 year time period for orbital planes B and D. We can see that for orbital plane B, over nearly 5 years, β does not cover all the value range: it only sweeps $[-56^0, 57.5^0]$. But for orbital plane D, within half a year, β covers most of its possible range. This fact indicates that for developing empirical SRP model, calibrating β dependent SRP parameters could require several years' precise orbit reference data for satellites in some

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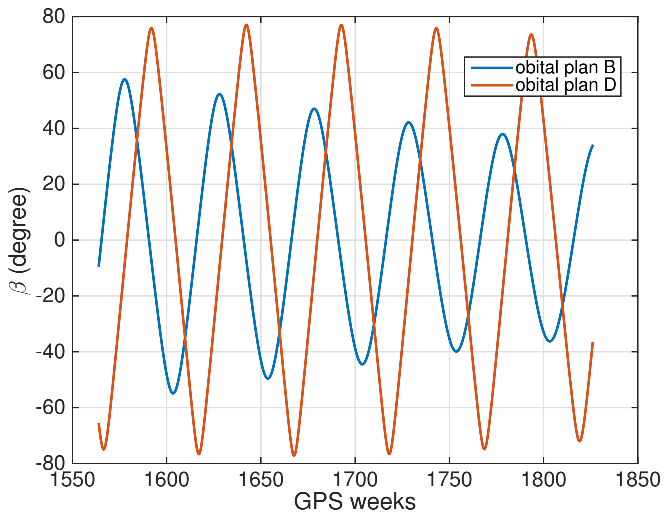

Fig. 1. Variation of solar elevation angle over 5 years for GPS satellites.

orbital planes. For newly deployed satellites, there is not so much orbit data, so the calibration of parameters of SRP model is a challenge.

In [7], it is mentioned that D_0, Y_0, B_0 in (1) are satellite-specific. Our research shows that the average value of SRP forces in D (Sun direction) and Y (Y-bias) are block type dependent, and their long-term variation is β angle dependent. The force variation as a function of β is similar between the satellites with the same block type; details are given in Section VIII. In the original CODE SRP model (1), the SRP parameters are β angle dependent for the non-periodic term.

IV. Simplified SRP model

Our previous SRP model [6] uses two constant parameters in D and Y direction from original CODE model (1). The two SRP parameters do not change during orbit prediction, and remain constant until the device uploads a software upgrade. For more precise SRP force evaluation, we can use β, the angle between Earth-Sun vector and satellite orbit plane, as an argument to describe the long-term variation trend of the parameters for SRP model.

A. Two parameter SRP model

The solar radiation pressure acceleration can be approximated as,

$$\alpha_{srp} = (\alpha_1 \cdot e_D + \alpha_2 \cdot e_y) f \tag{2}$$

where α_1 and α_2 are scalar SRP parameters, e_D is satellite-to-Sun direction vector,

$$e_D = \frac{r_{\text{Sun}} - r_{\text{Sat}}}{\|r_{\text{Sun}} - r_{\text{Sat}}\|} \tag{3}$$

where r_{sun} and r_{Sat} are Sun and satellite position in Earth initial frame, e_y is the direction vector along the satellite Y-axis,

$$e_y = \frac{r_{\text{Sat}} \times (r_{\text{Sun}} - r_{\text{Sat}})}{\|r_{\text{Sat}} \times (r_{\text{Sun}} - r_{\text{Sat}})\|} \tag{4}$$

and

$$f = \lambda \left(\frac{AU}{\|r_{\text{Sun}} - r_{\text{Sat}}\|}\right)^2 10^{-9} \tag{5}$$

where $\lambda \in [0, 1]$ indicates how much the satellite is outside Earth's shadow; AU is the average distant between Earth and Sun, the squared term describes the variation of light density as the satellite moves with Earth around the Sun, and the factor 10^{-9} is used to enlarge the display value of estimated parameters, then the unit of SRP acceleration is transfered into nano level of m/s^2.

B. The variation of SRP parameters with β angle

The solar elevation angle is

$$\beta = \frac{\pi}{2} - \arccos(e_{\text{Sun}} \cdot e_{\text{H}}) \tag{6}$$

where e_{Sun} is the unit vector of Earth-Sun, and the unit vector along angular momentum of orbit in Earth inertial frame is

$$e_{\text{H}} = \frac{r_{\text{Sat}} \times v_{\text{Sat}}}{\|r_{\text{Sat}} \times v_{\text{Sat}}\|} \tag{7}$$

For SRP parameters, polynomials of β angle can be used to fit the long-term variation. In satellite-Sun direction,

$$\alpha_1 = D_0 + a\beta^2 + b\beta^4 + c\beta^6 \tag{8}$$

in y-bias direction,

$$\alpha_2 = y_0 + d\beta^2 + e\beta^4 \tag{9}$$

In essence, α_1 and α_2 correspond to α_D and α_Y in (1). The polynomial form makes it easier to extend the model to other constellations or satellites than the trigonometric form used in the CODE model, which is developed for GPS constellation.

V. Estimator for SRP parameters

We use IGS's (International GNSS Service) final precise orbit products as observations to estimate SRP parameters. The estimation algorithm is Extended Kalman Filter (EKF) [8]. The processing is summarized below.

Given prior covariance matrix P_{k-1}, state $\hat{X}_{k-1}$, observation Y_k, and observation noise R_k.

A. Integrate from t_{k-1} to t_k

$$\begin{aligned} \dot{X}^* &= F(X^*, t) \\ \dot{\phi}(t, t_{k-1}) &= A(t)\phi(t, t_{k-1}) \end{aligned} \tag{10}$$

with the initial condition

$$\begin{aligned} X^*(t_{k-1}) &= \hat{X}(t_{k-1}) \\ \phi(t_{k-1}, t_{k-1}) &= I \end{aligned} \tag{11}$$

Here X^* is the nominal states, in our case it is an 8 component vector, $[position \quad velocity \quad \alpha_1 \quad \alpha_2]$. $\hat{X}_{k-1}$ is the estimated state from previous time step (19). For $\hat{X}_0$, the position is directly read from IGS's precise orbit, but IGS precise orbit have no velocity information. We use 4 position values, 2 before and 2 after t_0, to compute a velocity value, the equation of velocity at t_0 is

$$v_0 = \frac{-r_{t0-2} + 8r_{t0-1} - 8r_{t0+1} + r_{t0+2}}{12h} \quad (12)$$

where h is the time interval of precise orbit, and r_{t0-2}, r_{t0-1}, r_{t0+1}, r_{t0+2} are the 4 position vectors around t_0. As the sequential processing proceeds, velocity and estimated parameters will converge towards to true value.

The state derivative with respect to time, $\dot{X}^*$, is also an 8 component vector, $[velocity \quad acceleration \quad 0 \quad 0]$. $acceleration$ represents our forces model; it's a summation of several force items: Earth gravity up to 8 degrees and 8 orders, solar gravity, lunar gravity, and SRP force (2).

In (10), $A(t) = [\partial F(t)/\partial X(t)]^* = \partial \dot{X}^*(t)/\partial X^*(t)$ is the partial derivative matrix, $*$ means estimated on the nominal trajectory, and its form is

$$A(t) = \begin{bmatrix} 0_{3\times3} & I_{3\times3} & 0_{3\times2} \\ \partial a/\partial X^* & & \\ 0_{2\times8} & & \end{bmatrix} \quad (13)$$

where $\partial a/\partial X^*$ is a summation of each force item's partial derivative with respect to state X^*. The details of the matrices of these partial derivatives for Earth gravity, solar gravity and lunar gravity are in [9]. For calculating Earth gravity, and its partial derivative, the Earth Orientation Parameters (EOP) are involved. We use the final product of EOP from Earth Rotation and Reference Systems Service (IERS) [10]. The SRP's partial derivative with respect to X^* is

$$\partial a_{\text{srp}}/\partial X^* = [0_{3\times6} \quad e_D \quad e_y]\, f \quad (14)$$

where the direction vector e_D and e_y are defined by (3), (4); and f is given by (5).

The state derivative $\dot{X}^*$, and $\dot{\phi}$ are integrated one time step forward with the initial condition (11). The step size is 15 minutes, the same as the time interval of precise orbit from IGS, and the integrator is Dormand-Prince 8.

B. Covariance matrix prediction

$$\bar{P}_k = \phi(t_k, t_{k-1}) P_{k-1} \phi^T(t_k, t_{k-1}) \quad (15)$$

Our EKF's covariance prediction has no process noise involved, since we assumes the SRP parameters are constant. The process noise being set to be zero eventually cause the EKF to stop, which means the observations will have no effects to the estimation outputs at some point. So this EKF can't follow the variation of SRP parameters. The β angle's variation is small, only several degree within our estimation curve, so our β dependent SRP parameters are assumed as constant within one estimation. Further research might need to introduce process noise into our EKF for parameters estimation.

C. Observation, observation matrix, gain matrix

$$y_k = Y_k - G(X_k^*, t_k) \quad (16)$$
$$\tilde{H}_k = \partial G(X_k^*, t_k)/\partial X_k \quad (17)$$
$$K_k = \bar{P}_k \tilde{H}_k^T (\tilde{H}_k \bar{P}_k \tilde{H}_k^T + R_k)^{-1} \quad (18)$$

The measurement residual is y_k, and K_k is the gain of EKF. The function G take the position part of X_k^*, and transfer it from Earth inertial frame into Earth fix frame, ITRF, which is used by IGS's precise orbit products. $\tilde{H}_k = [W(t)PN(t)R(t) \quad O_{3\times5}]$, is the observation matrix, where $W(t)$, $PN(t)$, $R(t)$ are the Earth Rotation, Precession with Nutation, and Polar motion Matrix; these matrices are formed by EOP parameters [9].

D. Measurement and reference state update

$$\hat{X}_k = X_k^* + K_k y_k \quad (19)$$
$$P_k = [I - K_k \tilde{H}_k] \bar{P}_k \quad (20)$$

$\hat{X}_k$ is the output of EKF, and P_k is the updated covariance matrix. We take a 7 days precise orbit as observation trajectory for each estimation. Since the prior of velocity is a rough value, our estimator needs about 3 days to get stable state output for SRP parameters. We use the last two or three days' estimated curve of SRP parameters, and take the median value from this curve for each estimation.

VI. SRP PARAMETERS ESTIMATION AND FITTING

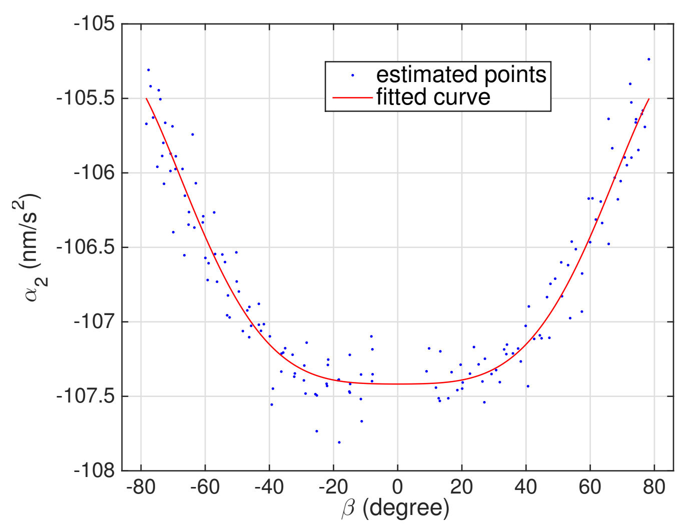

Fig. 2. SRP parameter in Satellite-Sun direction of satellite PRN 1

Fig. 2 and 3 show the variation of α_1 and α_2 against β angle for GPS PRN1 (Block Type IIF). The blue dots are estimated points, and the red line is the fitted curve by a least square algorithm with the model described in equations (8) (9). The estimator mentioned in section V process precise orbit data of

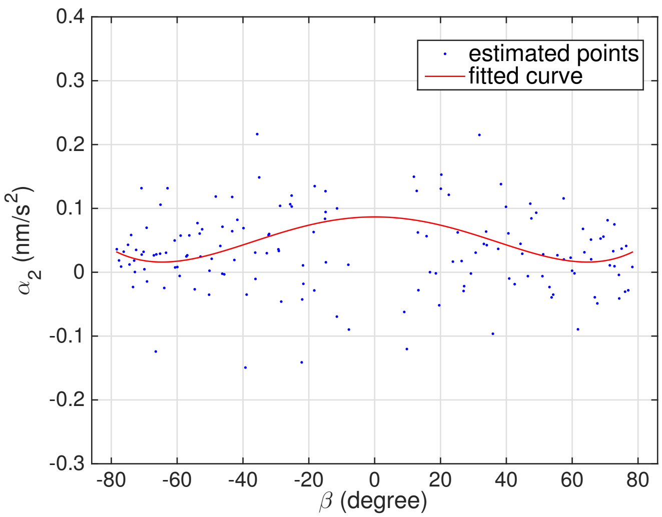

Fig. 3. SRP parameter of Y-bias of satellite PRN 1

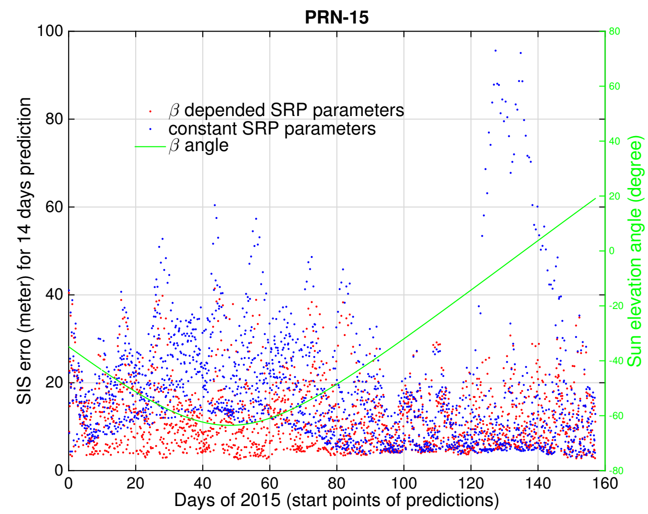

Fig. 4. 14 days orbit prediction for satellite PRN 15

IGS from Sep 2011 to Dec 2014. The input orbit curves for reference are arranged as 7 days long pieces. Each output is 2 SRP parameters labeled with β value. The points for which the β angle is smaller than 6.5° are excluded, since they suffer deep eclipse. The outliers of the estimation points are also removed. For example, the value of $\alpha 1$ for GPS IIF satellite is between -108 and -105, any points for the satellites of this block with $\alpha 1$ outside the range should be removed; Similar process for $\alpha 2$. Then the left several outliers can be manually deleted.

VII. Orbit Prediction Results

For evaluating the performance of orbit prediction, the metric of orbit-only SIS error is introduced below. GNSS receivers have different UREs (User Range Error) with different location. For GPS's average rms SIS URE is given in [11] by

$$\text{rms} \quad URE = \sqrt{(0.98R - C)^2 + (T^2 + N^2)/49} \tag{21}$$

where R, T, and N are the error in radial, tangential and normal direction, and C is the clock error. Ignoring the clock error, we have the orbit-only SIS UREo [11],

$$\text{rms} \quad UREo = \sqrt{(0.98R)^2 + (T^2 + N^2)/49} \tag{22}$$

In Fig. 4, the maximum of orbit-only SIS UREs for 14 days orbit prediction are shown for GPS PRN15. The red dots are the results with the the SRP parameters varying as β angle changing; the parameters D_0, a, b, c, y_0, d, e are generated by fitting the precise orbit for 4 years, until the end of 2014. And for the blue dots are the results of same orbit prediction, but the two SRP parameters, α_1, α_2 keep constant, it's the average values of estimations from several years precise orbit of IGS. The initial points of prediction are taken from every BE from Jan to June of 2015, and the unhealthy BEs are excluded. The orbit information of initial points are corrected by an EKF [4], then predict forward 14 days with our propagator, and finally the results are subtracted from IGS's precise orbit product to get errors in R, T, N direction; for the orbit-only SIS error we use equation (22). The force items for orbit prediction are the same as our estimator mentioned in V-A.

From the Fig. 4, we can say that with the β dependent SRP parameters, the performance of orbit prediction became better than using constant SRP parameters, especially the peak values of errors are smaller.

TABLE I

14-days prediction results of PRN15

SIS UREo %\(m)	<5	<10	<15	<20
constant α_1, α_2	8.39	30.41	51.30	68.14
β dependent α_1, α_2	15.82	49.65	72.31	87.17

Table I summarizes the results of the orbit predictions shown in Fig 4. The percentage of the numbers that maximum orbit-only SIS error is smaller than 5, 10, 15, and 20 meters for 14 days prediction is listed.

For the whole GPS constellation during the time span mentioned above, the 14-days orbit predictions with BEs, using the β angle dependent SRP model, the 68% quantile of orbit-only SIS error for is within 16 meters; with the constant SRP parameters, this quantile is between 17 to 20 meters, depends on the quality of estimated parameters.

VIII. Discussion

An empirical SRP model normally needs long-term orbit data to adjust SRP parameters. In [7], it is mentioned that CODE SRP model used 5.6 years of data to tune the parameters. But for newly lunched satellites, the data for precise orbit curves is lacking, we still have to get some acceptable SRP parameters. We found that for the same block type of GPS satellites, the shape of SRP curve with β angle are similar. The value ranges of SRP parameters for the satellites within the same block type are also similar. This fact allow us use relative short period of precise orbit data to estimate SRP parameters for new deployed satellite. One could even replace the curve of SRP parameter with β by other satellites' within the same block type, just through adding a small constant value to D_0 or y_0, and validate them by orbit prediction results.

A new block IIF satellite was deployed into orbit in May 2014, designated as PRN6. We use half a year's precise orbit data from IGS, Jun 2014 to Dec 2014, to estimate α_1 and α_2. The curves with β angle are shown in Fig. 5 and Fig. 6. Using the SRP parameters from these curves, we predict from every BE of half years during 2015 forward 14 days. The maximum orbit-only SIS errors are shown in Fig. 7, 79.8% of errors are smaller than 20 meters.

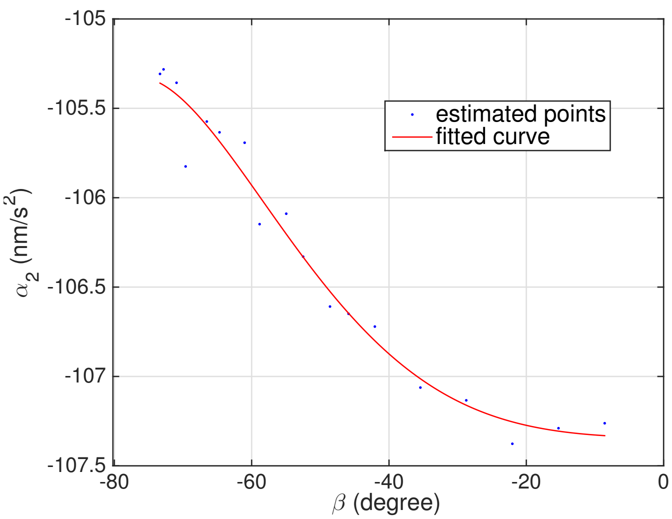

Fig. 5. SRP parameter in Satellite-Sun direction of satellite PRN 6

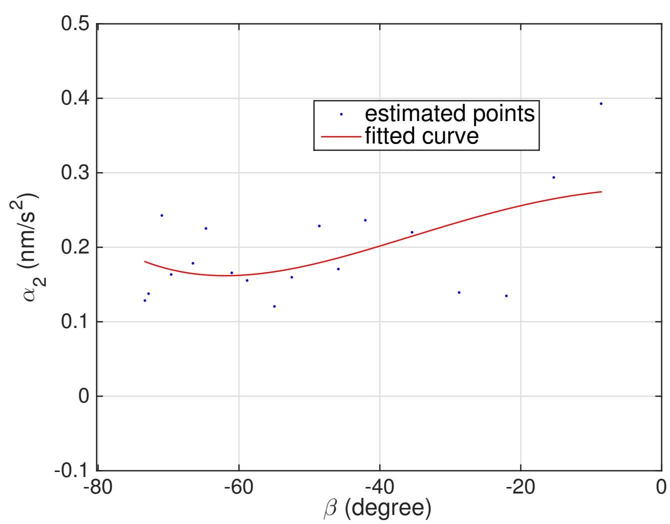

Fig. 6. SRP parameter of Y-bias of satellite PRN 6

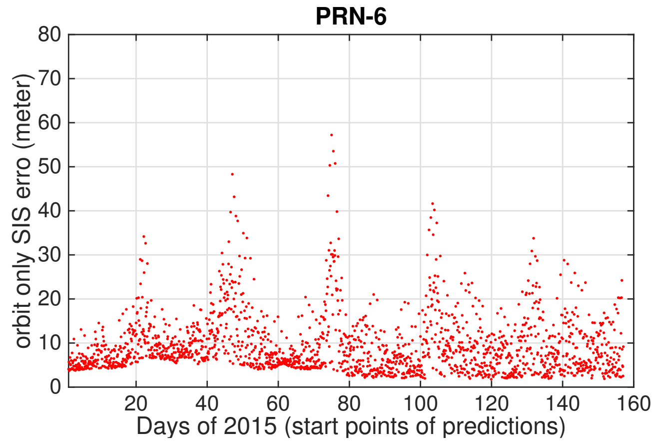

Fig. 7. Results of orbit prediction for satellite PRN 6

From Fig. 7, the prediction results for the first 80 days are a little worse than the last 80 days'. It indicates that the curve of SRP parameters with β angle might need small adjusting. Other possible reasons are that we don't include any processing in our SRP model for attitude maneuver for satellites during solar eclipse, nor detect shadow border for integrator. We use an multiple-steps integrator with 900 seconds step size. When satellite cross shadow border, the discontinues force of SRP may worse the results of orbit propagation.

The performance of orbit predictions may have some effects during Earth eclipse season, we can see from Fig. 7, 30 day to 60 day.

We can find that there are error peaks about every 28 days in Fig. 7. These might be due to the fact that our force model does not include solid tides and ocean tides; further investigation for these two items' effects to orbit prediction are needed.

In further development for SRP model, we will consider introducing force term in B direction, and also add short-period terms in D and B direction, which account the SRP force variation per orbit evolution.

The typical value of SRP parameters for three block type GPS satellites are listed in Table II

TABLE II
COEFFICIENTS OF SRP PARAMETERS

Block	PRN	D_0	a	b	c	y_0	d	e
IIRM	5	-98.81	-0.61	2.95	-0.94	-0.63	0.093	-0.050
IIRM	12	-99.83	-0.87	3.72	-1.16	-0.66	0.098	-0.011
IIF	1	-107.4	0.097	1.07	-0.31	0.087	-0.11	0.044
IIF	6	-107.3	0.38	1.47	-0.59	0.28	-0.20	0.086
IIR	19	-102.2	-0.20	2.281	-0.56	-0.50	0.13	-0.037
IIR	20	-101.7	-0.41	2.64	-0.76	-0.91	0.15	-0.086

From the table, we notice that for the GPS satellites have the same bock type, the SRP coefficients are in the same value domain, especially for D_0 and y_0. The differences of theses coefficients for the same block type satellites may be because of their mass changing after orbit maneuvers, or surfaces properties varying as the satellite ages.

Currently, the SRP parameters still need to be generated offline, we use precise orbit data for parameters estimation. So for a further development of completely autonomous orbit prediction, we will consider to store a prior values for SRP curves, and using BEs to adjust SRP curve online.

IX. Conclusion

A simplified SRP model is presented, whose parameters depend on β angle. The procedure of estimation for SRP parameters, and its curve fitting were described in detail. As an example, the results of orbit prediction for PRN15 with this SRP model were shown. We also gave an example of estimating the curve of SRP parameters with a smaller number of estimation points.

The accuracy of GPS orbit prediction use BEs with the presented SRP model is much smoother than the one with constant SRP parameters. The update demands of SRP parameters with the presented model are small.

Acknowledgment

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