3D Finite Element Model as a Tool for Analyzing the Structural Behavior of a Railway Track

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Abstract
The rising public and commercial demands on railway network increases the need to improve systems that comprise the railway network. Especially in case of mixed corridors, the demands for track smoothness and load carrying capacity increase simultaneously. From this perspective, the optimization of track design creates efficiency and reduction of life-cycle costs. Hence, there is a great need for a tool which enables designing the load-carrying capacity of a railway track structure as a whole and simultaneously evaluates the stress and/or strain levels of each track component such that the life cycle of the track structure is optimized. The main focus of this study was to create a three dimensional structural model in which the stress-strain behavior of different railway track components could be evaluated realistically. The created model is based on finite element method using PLAXIS 3D software which is specialized in geotechnical problems. Differing from most of the traditional methods, which are based on a theory of linear elasticity, Finite Element Method-based approach with the chosen tool provides a non-linear solution and a three dimensional stress state. As features, the created structural model enables variation in structural layer thickness, rail size, sleeper type (wood/concrete) and material properties of rail pad, ballast, subballast layers and subgrade.

Keywords: Railway Track, Mechanical Behavior, Modelling, 3D Finite Element Method, Load Carrying Capacity

1 Introduction

Regarding the reduction of life-cycle costs of railway infrastructure, optimizing the design of different track components hold a great relevance. In respect to the mechanical characteristics of ballast and trackbed layers, numerical models can provide better understanding on the behavior of the whole railway track structure.

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Ballasted track is the most common superstructure type used for railways. It is a rather complex ensemble on the load-carrying capacity point of view due to the properties of different components having deviant stress-strain dependencies. This leads to complicated interactions between various components. Some nonlinear behavior in granular components of the track also forms when subjected to traffic loads. The stress-strain response of a railway track can be evaluated with help of several different models. The existing models can roughly be divided to the models based on the traditional analytical methods and models based on computational analysis.

Traditional analytical models are typically formulated with help of theory of a Beam on Elastic Foundation (BOEF) firstly presented by Winkler in 1867. Skoglund (2002), among other authors, has presented a variety of different applications of BOEF-model. Previous studies defining the track response due to train loads have been carried out (Robnett et. al. 1975, Chang et. al. 1979, Chang et.al. 1980, Huang et. al. 1984, Rose & Konduri 2006, etc.). The development of different models has evolved over time simultaneously with the increase of computational capacity. Some of these models, such as GEOTRACK and KENTRACK have been in intensive use which has enabled parametric studies and preparation of diagrams describing the relations between relevant variables.

In general, the above studies consider linear elastic behavior of the structural layers of the track. The main focus of this study was to create a three dimensional structural model in which the stress-strain behavior of different railway track components could be varied and evaluated realistically. The created model is based on finite element method using PLAXIS 3D software which is specialized in geotechnical problems. Differing from most of the traditional methods, which are based on a theory of linear elasticity, Finite Element Method-based approach with chosen tool provides a non-linear solution and a three dimensional stress state. As features, the created structural model enables variation in structural layer thickness, rail size, sleeper type (wood/concrete) and material properties of base plate, ballast, subballast layers and subgrade. In addition, the ballast layer is divided into sections under the sleeper which enables studying the effects of ballast degradation on track performance.

2 3D Track Model

The model developed for the purposes of this study was created with PLAXIS 3D which is a finite element software that has been developed especially for the analysis of deformation and stability in geotechnical engineering projects.

PLAXIS uses 10-node tetrahedral elements for soil layers and 6-node plate elements. Plate elements are based on Mindlin’s plate theory (Bathe 1982). User can define the desirable refinement for elements and the program calculates the target element size based on the outer model geometry dimensions. In addition, user can affect the meshing procedure by defining the relative element size factor, polyline angle tolerance and surface angle tolerance.(Brinkreve et. al. 2012) In this project, the finest mesh at automatic meshing procedure was used, i.e. target element size was 0.5, polyline tolerance angle was 30° and surface angle tolerance 15° respectively.

The model created in this study is illustrated in figure 1 and consists of a straight and flat railway section. The length of modeled track is 17.34 meters which is equal to 29 sleepers with 0.61 m spacing. The loads are produced with point loads representing wheel loads of a 25 ton axle.
2.1 Material Models

PLAXIS contains several different material models within the software. The material models used in this track model are shortly described. The Hardening-Soil Model (HS) is an advanced model for the simulation of soil behavior. Limiting states of stresses are described by means of the friction angle, \( \varphi \), the cohesion, \( c \), and the dilatancy angle, \( \psi \). Soil stiffness is described by using three different input stiffnesses: the triaxial loading stiffness, \( E_{50} \), the triaxial unloading stiffness, \( E_{ur} \), and the oedometer loading stiffness, \( E_{oed} \). All these stiffnesses relate to a reference stress, 100 kPa in this study. The HS model was chosen for aggregate materials (ballast, subballast and frost protection layers) since the yield surface is not fixed but can expand due to plastic straining. The hardening rules can be divided into two main types of hardening, namely shear and compression hardening. Shear hardening is used to model plastic strains due to primary deviatoric loading. Compression hardening is used to model irreversible strains in oedometric and isotropic loading. Therefore, the stiffness of aggregate layers are more appropriate on both sides of the yield surface i.e. when subjected to deviatoric loading, the soil stiffness decreases simultaneously with the development of irreversible strains.

The Linear Elastic Model (LE) was chosen for other components of this track model. The modelled components were rail, rail pads, sleepers and subgrade. Justifications for selecting the LE model for these components are presented in the following chapters.

2.2 Model Components and Calculation Parameters

**Rail**

The rails were modelled as plate elements with a equal width of rail foot. Plate element has an artificial thickness in model to create the flexural rigidity. To match the properties of desired rail profiles the stiffness parameters used in calculations have to be converted with help of the desired moment of inertia for each rail type. Three most common rail types used in Finland were selected for the study and the conversion and calculation parameters are presented in table 1. The software manual provides a solution to match the bending stiffness of an actual rail in which Poisson’s Ratio is set to 0 for correct solution. This conversion is also shown in table 1.

**Rail Pads**

Rail pads were modeled as linear elastic block elements. The stiffness of a standard pad was determined similar to the commonly used rubber pad type (Vossloh Zw 900 NT) in Finland. The moduli value of a standard pad in this model is based on static compression tests. Taking the prestress
due to the fastening systems into account, the stiffness of the pad was determined between 30 and 70 kN load to provide pad stiffness to match the track conditions. The stiffness of a standard pad in this model was approximately 80 MN/mm. The actual thickness of modeled pad was 8 mm. In the first simulations the pads were modeled as 10 mm thick, 140 mm x 180 mm elements having a moduli value of 100 MPa. Due to some difficulties at the meshing procedure of software, the thickness of the pad was decupled to avoid meshing failure. Therefore also the moduli values of simulated pad stiffnesses were decupled. The moduli values in simulations were 1,000 MPa for typical rail pad, 500 MPa for soft rail pad and 2,000 MPa for stiff rail pad, respectively. Since the modeled pad material was rubber, Poisson’s Ratio is close to 0.5. A value of 0.495 (maximum available in used software) was chosen.

For wood tie simulations, a steel base plate was defined as 160 mm x 360 mm block element having stiffness of 2,100 GPa ($\nu = 0.3$).

<table>
<thead>
<tr>
<th>Rail type</th>
<th>Cross-section area A</th>
<th>Moment of inertia I</th>
<th>EA N/m</th>
<th>EI Nm²/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>K43</td>
<td>5.56 $\times 10^{-3}$</td>
<td>1.469 $\times 10^{-5}$</td>
<td>1.17 $\times 10^8$</td>
<td>3.085 $\times 10^6$</td>
</tr>
<tr>
<td>54 E1</td>
<td>6.98 $\times 10^{-3}$</td>
<td>2.338 $\times 10^{-5}$</td>
<td>1.47 $\times 10^8$</td>
<td>4.910 $\times 10^6$</td>
</tr>
<tr>
<td>60 E1</td>
<td>7.67 $\times 10^{-3}$</td>
<td>3.038 $\times 10^{-5}$</td>
<td>1.61 $\times 10^8$</td>
<td>6.380 $\times 10^6$</td>
</tr>
</tbody>
</table>

Table 1. Definition of calculation parameters for plate elements simulating the rails.

Sleepers

The definition of sleeper parameters was defined with a separate model simulating an extensive series of cyclic loading tests reported in detail in Kerokoski et al. (2012). The sleeper type modeled was pre-stressed concrete monoblock B97. A single sleeper could be modeled accurately but the tilted surfaces at sleeper and ballast interfaces caused failure in meshing procedure of the actual model. Therefore the actual sleeper used in the model was somewhat simplified as illustrated in table 2. The moment of inertia of the sleeper was matched with the actual sleeper at sleeper end, in the middle of the sleeper and at rail seat section. The size of simulated wood tie as well as calculation parameters are also shown in table 2. Moduli value used for concrete sleeper was 40 GPa ($\nu = 0$). Stiffness parameters used for wood ties were E=10 GPa and $\nu = 0.3$ respectively.

Ballast

Since ballast material is in severe loading environment and the layer consists of coarse-grained uniformly graded unbound granular media, the material is susceptible to attrition and breakage of ballast particles. While the ballast material fous the stiffness and strength properties change gradually during the life-cycle of ballast. Ballast degradation also has a marked effect on the support ballast is providing to sleepers and rails. One of the most significant aspects in this study was to create a model that can simulate the varying ballast condition. Therefore the ballast layer is divided into 33 sections under the sleeper area in the created model. The divided sections are presented in figure 2.

Figure 2. The ballast layer is divided into 33 sections under the sleeper area.
The material parameters were defined to three quality degrees of ballast material; fresh, slightly fouled and heavily fouled ballast. The stiffness parameters of ballast materials are based on triaxial test series from previous studies performed at TUT (Nurmikolu & Kolisoja 2011). Strength parameters were defined with help of literature review (Skoglund 2002, Indraratna et. al. 2011). The HS-model material parameters used for ballast are presented in table 3.

<table>
<thead>
<tr>
<th>Sleeper material</th>
<th>Length of sleeper</th>
<th>Width of sleeper</th>
<th>Heigh of sleeper</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m</td>
<td>m</td>
<td>m</td>
</tr>
<tr>
<td>Concrete</td>
<td>2.600</td>
<td>0.260</td>
<td>1)</td>
</tr>
<tr>
<td>Wood</td>
<td>2.700</td>
<td>0.240</td>
<td>0.160</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Height of B97 monoblock sleeper</th>
<th>Moment of inertia at calculated cross-section</th>
<th>Modeled height of the sleeper</th>
</tr>
</thead>
<tbody>
<tr>
<td>At sleeper end</td>
<td>1.508 * 10^4</td>
<td>0.185</td>
</tr>
<tr>
<td>At rail seat section</td>
<td>2.274 * 10^4</td>
<td>0.210</td>
</tr>
<tr>
<td>In the middle of sleeper</td>
<td>9.992 * 10^5</td>
<td>0.165</td>
</tr>
</tbody>
</table>

Table 3. Calculation parameters for ballast materials.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>c´</th>
<th>φ'</th>
<th>ψ</th>
<th>E50 ref</th>
<th>Eoed ref</th>
<th>Eur ref</th>
<th>νur</th>
<th>m</th>
<th>K0 ref</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>kPa</td>
<td>°</td>
<td>°</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td></td>
<td>-</td>
<td>kPa</td>
<td></td>
</tr>
<tr>
<td>Fresh</td>
<td>40</td>
<td>50</td>
<td>10</td>
<td>325</td>
<td>270</td>
<td>650</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.300</td>
</tr>
<tr>
<td>Slightly fouled</td>
<td>20</td>
<td>45</td>
<td>10</td>
<td>275</td>
<td>230</td>
<td>550</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.300</td>
</tr>
<tr>
<td>Heavily fouled</td>
<td>10</td>
<td>45</td>
<td>5</td>
<td>225</td>
<td>190</td>
<td>450</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.300</td>
</tr>
</tbody>
</table>

Table 2. Sleeper dimensions used in model simulations.

Subballast and Frost Protection Layers

Subballast and frost protection layers are constructed with unbound non-frost susceptible granular materials. According to Finnish instructions the layers can consist of both natural sand and gravel or crushed rock aggregates. Typical gradations of materials in Finland have been reported earlier e.g. by Kalliainen et. al. (2011). For this study, a variety of material parameters were defined with help of earlier studies (Kolisoja 1997, Vuurimies &Kolisoja 2000). The HS-model parameters used in simulations are presented in table 4. A large variety of parameters is necessary to simulate different types of tracks. Majority of Finnish railway network has been constructed before early 1900’s and often with lower quality materials compared to modern track beds.

Subgrade
The important role of subgrade stiffness on the mechanical behavior of a railway track has been observed in previous railway track modeling studies (Kalliainen & Kolisoja 2013). The importance of subgrade stiffness has also been notified by other researchers (e.g. Rose & Konduri 2006). Since the desired situation in this model was to produce a stress-strain relationship under a single loading pulse. Due to thick structural layers of the track, permanent deformation in subgrade at this stage should not exist. Therefore the linear elastic model was chosen for subgrade layers. The simulation included subgrade stiffnesses of 20, 40, 80, 160 and 320 MPa while the Poisson’s ratio was 0.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>c’</th>
<th>q’</th>
<th>ψ</th>
<th>E_{50}^{ref}</th>
<th>E_{oed}^{ref}</th>
<th>E_{ur}^{ref}</th>
<th>v_{ur}</th>
<th>m</th>
<th>p’^{ref}</th>
<th>K_{0}^{nc}</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>kPa</td>
<td>°</td>
<td>°</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td></td>
<td></td>
<td>kPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subballast</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gravel</td>
<td>10</td>
<td>42</td>
<td>5</td>
<td>200</td>
<td>190</td>
<td>400</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.347</td>
<td>0.9</td>
</tr>
<tr>
<td>Crushed rock</td>
<td>10</td>
<td>45</td>
<td>5</td>
<td>250</td>
<td>210</td>
<td>500</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.300</td>
<td>0.9</td>
</tr>
<tr>
<td>Frost protection layers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gravelly sand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer 1</td>
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<td>38</td>
<td>5</td>
<td>140</td>
<td>137</td>
<td>280</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.384</td>
<td>0.9</td>
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<tr>
<td>Layer 2</td>
<td>5</td>
<td>36</td>
<td>5</td>
<td>120</td>
<td>120</td>
<td>240</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.412</td>
<td>0.9</td>
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<tr>
<td>Layer 3</td>
<td>5</td>
<td>36</td>
<td>5</td>
<td>100</td>
<td>100</td>
<td>200</td>
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<td>0.5</td>
<td>100</td>
<td>0.412</td>
<td>0.9</td>
</tr>
<tr>
<td>Layer 4</td>
<td>5</td>
<td>36</td>
<td>5</td>
<td>80</td>
<td>80</td>
<td>160</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.412</td>
<td>0.9</td>
</tr>
<tr>
<td>Sand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Layer 1</td>
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<td>35</td>
<td>5</td>
<td>95</td>
<td>95</td>
<td>190</td>
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<td>0.5</td>
<td>100</td>
<td>0.426</td>
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<td>Layer 2</td>
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<td>3</td>
<td>80</td>
<td>80</td>
<td>160</td>
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<td>0.5</td>
<td>100</td>
<td>0.455</td>
<td>0.9</td>
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<tr>
<td>Layer 3</td>
<td>5</td>
<td>33</td>
<td>3</td>
<td>65</td>
<td>65</td>
<td>130</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.455</td>
<td>0.9</td>
</tr>
<tr>
<td>Layer 4</td>
<td>5</td>
<td>33</td>
<td>3</td>
<td>50</td>
<td>50</td>
<td>100</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.455</td>
<td>0.9</td>
</tr>
<tr>
<td>Crushed rock</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer 1</td>
<td>10</td>
<td>45</td>
<td>5</td>
<td>180</td>
<td>150</td>
<td>360</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.300</td>
<td>0.9</td>
</tr>
<tr>
<td>Layer 2</td>
<td>10</td>
<td>45</td>
<td>5</td>
<td>170</td>
<td>145</td>
<td>340</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.300</td>
<td>0.9</td>
</tr>
<tr>
<td>Layer 3</td>
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<td>45</td>
<td>5</td>
<td>160</td>
<td>135</td>
<td>320</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.300</td>
<td>0.9</td>
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<tr>
<td>Layer 4</td>
<td>10</td>
<td>45</td>
<td>5</td>
<td>150</td>
<td>130</td>
<td>300</td>
<td>0.2</td>
<td>0.5</td>
<td>100</td>
<td>0.300</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 4. Calculation parameters for subballast and frost protection materials.

3 Results

The first simulations were performed to simulate the construction of a new railway track line. In the nominal case simulation the track components were: 60 E1 type rail, normal rail pads (E’ = 100 MPa) and fresh ballast. Material for the sub-structure layers was crushed rock, thickness of the sub-structure layers was 1.5 meters and subgrade stiffness was 80 MPa. The results of simulations are summarized in table 5. The values given in table 5 are differences in percentage to the nominal case simulation. The changes in component properties are:

- Rail Type: -= 54 E1 rail (as illustrated in Table 2)
- Rail Pad Stiffness: += 200 MPa; -= 50 MPa
- Ballast Condition: -= Slightly fouled ballast (as illustrated in Table 3)
- Thickness of Sub-structure layers: += 2.1 m; -= 1.2 m
- Subgrade stiffness: += 320 MPa; -= 20 MPa
- Material of Sub-structure Layers: -= Gravel in subballast layer, Gravelly sand in frost protection layers (as illustrated in Table 4)
Table 5 reveals that subgrade stiffness and properties of sub-structure layers have a marked effect on the track performance as a whole. On soft subgrade conditions all the aggregate materials encounter greater shear strain levels, which can lead to rapid accumulation of permanent deformation. On the other hand, changing the superstructure components only seem to affect the other superstructure components.

## 4 Conclusions

The most significant outcome of the study was the model itself which enables future work leading to more detailed analysis of stress-strain dependency of different track components. The analysis performed so far compares the stresses and strains of track components caused by a change of one track component properties at a time. The obtained results indicate that the subgrade stiffness and the thickness and material properties, drainage in particular, of structural layers are the most significant factors affecting the load carrying capacity of a railway track. The simulation results will be presented later in more detail elsewhere.

One of the main focuses of the study was an effort to create a model which could be used in evaluating of the effects of ballast degradation and varying sleeper support. Based on the simulations, the model provides a credible stress-strain relationship with current model specification. However, some additional laboratory tests for different ballast materials are required to confirm the stiffness and strength parameters of ballast.

### Table 5. Comparison of the effect of varying track components properties with respect to the nominal case simulation of new railway track

<table>
<thead>
<tr>
<th></th>
<th>Rail Type</th>
<th>Rail Pad Stiffness</th>
<th>Ballast condition</th>
<th>Thickness of Sub-structure layers</th>
<th>Subgrade Stiffness</th>
<th>Material of Sub-structure layers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_z$ of the ballast layer, under the sleeper end area</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$\sigma_z$ of the ballast layer, under the sleeper end area</td>
<td>8.2</td>
<td>4.0</td>
<td>-5.5</td>
<td>-1.4</td>
<td>0.6</td>
<td>2.5</td>
</tr>
<tr>
<td>$\gamma_s$ of the ballast layer, under the sleeper end area</td>
<td>4.6</td>
<td>4.9</td>
<td>-5.5</td>
<td>28.2</td>
<td>-3.4</td>
<td>2.1</td>
</tr>
<tr>
<td>$u_z$ on top of the subballast</td>
<td>0.7</td>
<td>0.6</td>
<td>-0.2</td>
<td>0.9</td>
<td>-5.2</td>
<td>4.1</td>
</tr>
<tr>
<td>$\sigma_z$ on top of the subballast</td>
<td>2.7</td>
<td>2.7</td>
<td>-3.6</td>
<td>2.4</td>
<td>-0.1</td>
<td>1.1</td>
</tr>
<tr>
<td>$\gamma_s$ on top parts of the subballast</td>
<td>2.7</td>
<td>1.4</td>
<td>-1.9</td>
<td>-5.5</td>
<td>-9.2</td>
<td>-1.3</td>
</tr>
<tr>
<td>$u_z$ on top of the subgrade</td>
<td>0.2</td>
<td>-0.2</td>
<td>0.1</td>
<td>1.2</td>
<td>-7.6</td>
<td>9.9</td>
</tr>
<tr>
<td>$\sigma_z$ on top of the subgrade</td>
<td>-1.7</td>
<td>-0.9</td>
<td>2.1</td>
<td>3.1</td>
<td>0.4</td>
<td>11.7</td>
</tr>
<tr>
<td>$\gamma_s$ on top parts of the subgrade</td>
<td>1.0</td>
<td>0.5</td>
<td>-0.6</td>
<td>1.7</td>
<td>-25.6</td>
<td>37.5</td>
</tr>
</tbody>
</table>

$u_z = \text{vertical displacement, } \sigma_z = \text{vertical stress and } \gamma_s = \text{shear strain.}$
Acknowledgments

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References


