Reconstruction of irregular bodies from multiple data sources

Matti Viikinkoski

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Introduction

- Goals:
  1. Construct a general $3D$ mode of the asteroid from observations
  2. Determine rotation parameters
  3. Determine asteroid surface reflectivity
- Methods for shape representation:
  1. Parametric representation
     - Simply connected surfaces
  2. Level set methods
     - Object represented as a level set of an implicit function
     - Can be represent any surface
     - Computationally demanding
  3. Statistical methods
     - Data contains unknown systematic errors

Parametric representation

- Real spherical harmonics of degree l and of order m:

$$Y_l^m = \begin{cases} P_l^m(\cos\theta)\cos m\varphi & \text{if } m > 0 \\ P_l^0(\cos\theta) & \text{if } m = 0 \\ P_l^{-m}(\cos\theta)\sin(m\varphi) & \text{if } m < 0. \end{cases}$$

  where P_l^m is a Legendre polynomial.

- The usual way to represent a general $3D$ shape is to expand each coordinate function as a linear combination of spherical harmonics:

$$x = \sum a_l^m Y_l^m \quad (1)$$

$$y = \sum b_l^m Y_l^m \quad (2)$$

$$z = \sum c_l^m Y_l^m \quad (3)$$

- However, this representation is too unstable to used in inversion
- Excessive regularization is needed to make it work

Parametric representation

- A better way is to generalize the usual representation for star-shaped objects:

$$\mathbf{x}(\theta,\varphi) = \begin{cases} x(\theta,\varphi) = e^{a(\theta,\varphi)}\sin(\theta)\cos(\varphi) \\ y(\theta,\varphi) = e^{a(\theta,\varphi)+b(\theta,\varphi)}\sin(\theta)\cos(\varphi) \\ z(\theta,\varphi) = e^{a(\theta,\varphi)+c(\theta,\varphi)}\cos(\theta), \end{cases} \tag{4}$$

where

$$\begin{aligned} a &= \sum a_l^m Y_l^m \\ b &= \sum b_l^m Y_l^m \\ c &= \sum c_l^m Y_l^m \end{aligned}$$

- Not as general as the (1), but general enough.

Regularization methods

Asteroid shape reconstruction is a typical inverse problem, regularization is needed

- Truncation of spherical parametric representation has a regulative effect
- The usual representation for starliked shapes can be obtained by setting $b = c = 0$ in the parameterization (4).
- Considering a star-shaped surface as our basic shape, the intuitively obvious measure for shape complexity is a weighted norm of coefficients $\{b_{lm}\}$ and $\{c_{lm}\}$. To this effect, we define

$$\mathcal{N} = \sum_{l,m} l \cdot (b_{lm}^2 + c_{lm}^2).$$

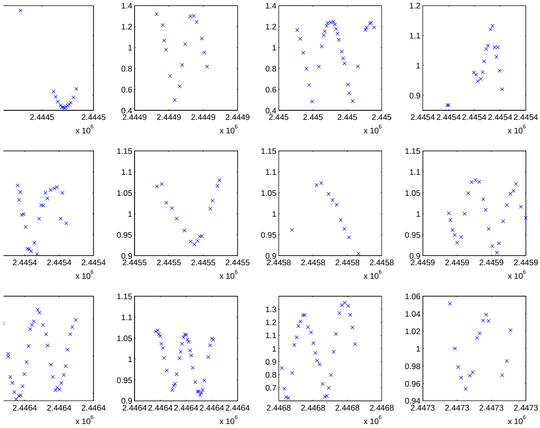
- Local smoothing by penalizing divergence from local convexity
- Physical regularization that strives to align the principal axis of maximum moment of inertia with the rotation axis of the object

Possible data sources

- Lightcurves
- Profile contours
- Interferometry
- Doppler radar images

Lightcurves

- Lightcurve is the brightness of an asteroid as a function of the time
- Depends on asteroid shape and surface reflectivity



Lightcurves

- *View direction* ω, *illumination direction* ω_0 are the directions of the sun and the earth as seen from the asteroid
- *Solar phase angle* is the angle between ω and ω_0
- A surface patch ds with a normal vector $\mathbf{n}$ is visible and contributing to the total brightness, if both $\omega \cdot \mathbf{n}$ and $\omega_0 \cdot \mathbf{n}$ are positive
- Surface scattering law is assumed to be a combination of Lommel-Seeliger and Lambert laws:

  $$S = \frac{\mu\mu_0}{\mu + \mu_0} + c\mu\mu_0,$$

  where $\mu = \omega \cdot \mathbf{n}$, $\mu_0 = \omega_0 \cdot \mathbf{n}$ and c is a constant.
- Total brightness of the asteroid is

  $$\int_{\mathcal{A}_+} S\, ds,$$

  where the integral is over the visible part of the surface.

Lightcurves

- For actual computations, a parametric surface is triangulated
- Triangulation can be easily constructed by transfering a standard triangulation on the unit sphere to the surface using the parameterization
- Visibility of each surface facet is determined by raytracing

Profile contours

- Boundary curves obtained from adaptive optics images

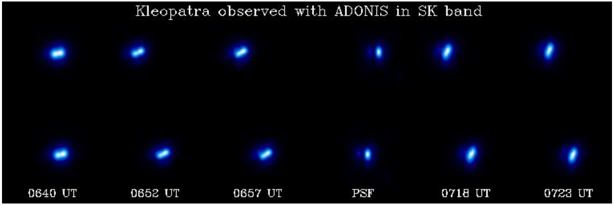
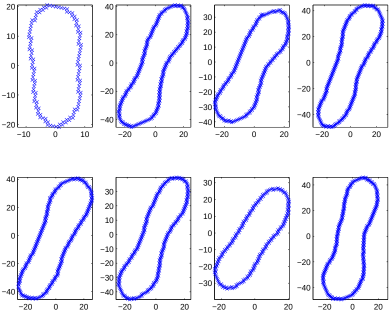


Figure: Kleopatra ao images by Hestroffer et al.

- Duo to adaptive optics artifacts, only boundary contains reliable information

Profile contours



Profile contours

- Object $\mathcal{B}$ is projected to the view plane, and the boundary curve is extracted.
- A distance $d(e, P_0)$ between a point P_0 and a line segment e with endpoints P_1 and P_2 is defined as follows: Let $d_1(e, P_0)$ be the perpendicular distance of the point P_0 from the line defined by P_1 and P_2 if its projection is inside the line segment. Letting $d_2(e, P_0)$ be the smallest of distances between the point P_0 and P_1, and between the point P_0 and P_2, we may set

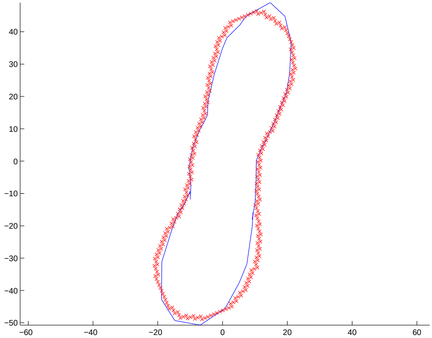
$$d(e, P_0) = \min\{d_1(e, P_0), d_2(e, P_0)\}.$$

- Goodness-of-fit measure between the model boundary $\partial\mathcal{B}$ and a set $\varkappa$ of the observed boundary points $\varkappa_i$ is defined as follows:

$$\chi^2_\partial = \sum_{e \in \partial\mathcal{B}} \min_i d(e, \varkappa_i) + \sum_i \min_{e \in \partial\mathcal{B}} d(e, \varkappa_i).$$

Profile contours

- Displacement of the profile contour with respect to the observed contour in the viewing plane is assumed to be unknown. The optimal offset parameters are determined during the inversion algorithm.



Interferometry

- Interferometric curves obtained from Hubble space telescope's fine guidance sensor(HST/FGS)
- An interferometric curve is obtained by projecting the image of the object on the plane-of-sky to one of the orthogonal HST/FGS axes
- The response function $S(x)$ of the HST/FGS can be calculated by convolving the brightness distribution $I(u,v)$ of the projected image of the object with the template transfer function $T(x)$ of the instrument:

$$S(x) = y_0 + \frac{1}{L} \int \int I(u,v) T(x_0 + x - u\cos\gamma + v\sin\gamma)\, du\, dv,$$

where

$$L = \int \int I(u,v)\, du\, dv$$

is the total brightness of the visible part of the object and γ is the angle between the image axis and the FGS axis.

Interferometry

- Parameters x_0 and y_0 are the location offset values of the object with respect to the FGS coordinates and are determined during optimization
- The template transfer function $T(x)$ cannot be written in analytical form and is thus given as a set of sampled values. To obtain a continuous function, the transfer function is linearly interpolated between the sampled points.

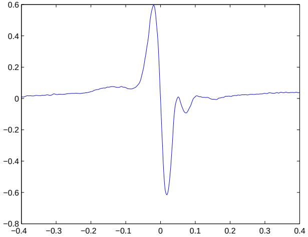


Figure: A template transfer function of HST/FGS

Interferometry

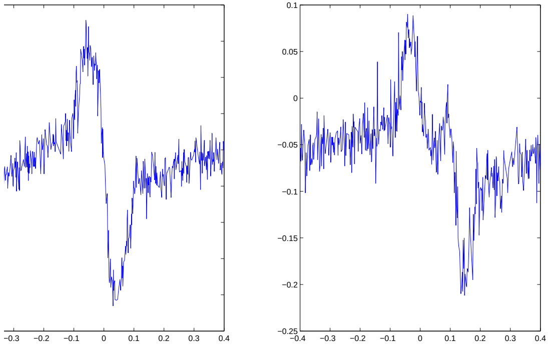


Figure: Typical s-curves obtained from the HST/FGS

Optimization

- General Goodness-of-fit measure

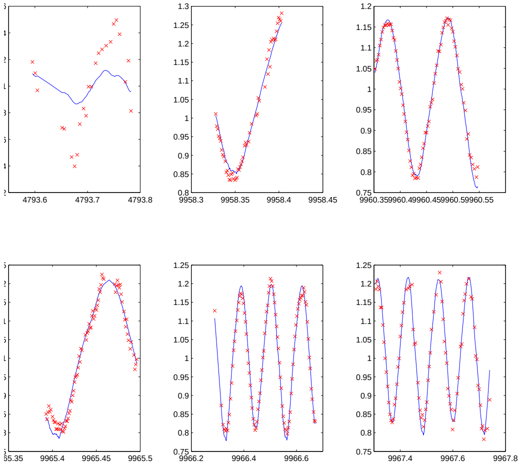
$$\chi^2 = \chi^2_{lc} + \lambda_1 \chi^2_{pc} + \lambda_2 \chi^2_{sc} + \lambda_3 \mathcal{N}^2$$

  where χ^2_{lc}, χ^2_{pc} and χ^2_{sc} are the fits obtained from the lightcurves, profile contours and S-curves, respectively.
- Analytic derivatives of χ^2 with respect to shape parameters a_{lm}, b_{lm} and c_{lm} can be calculated
- χ^2 is minimized using Levenberg-Marquart optimization algorithm

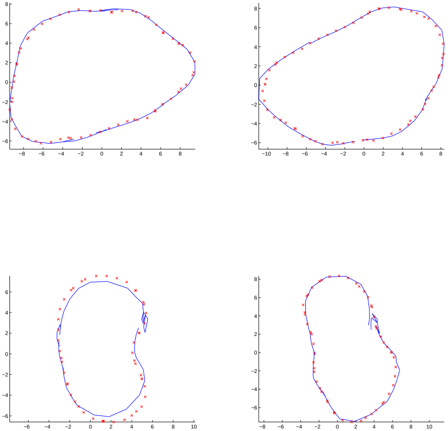
Examples:Hermione

- 41 lightcurves
- 4 boundary curves

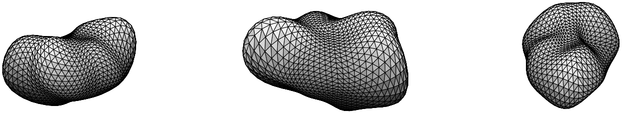
Examples:Hermione



Examples:Hermione



Examples:Hermione



Examples:Kleopatra

- Assumed to an bifurcated asteroid, a dogbone-like shape
- 18 Boundary curves
- 46 Lightcurves
- 18 Interferometric curves
- Data contains systematic errors

Examples:Kleopatra

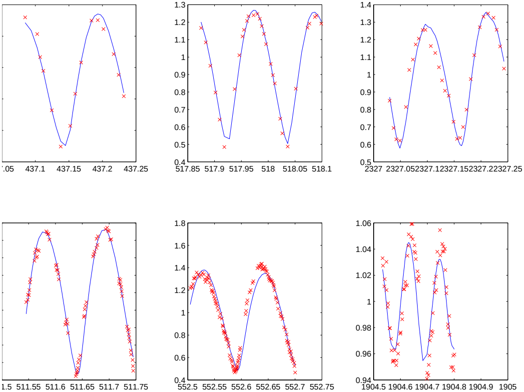
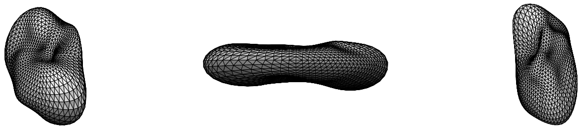


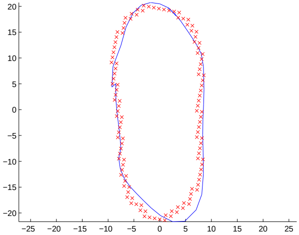
Figure: Model fit to the lightcurves

Examples:Kleopatra



A fit obtained from lightcurves and profile contours only. Note the almost convex shape.

Examples:Kleopatra



Next we will include our interferometry data.

Examples:Kleopatra

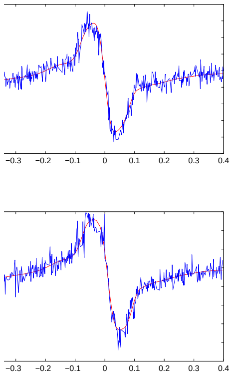


Figure: Model fit to the S-curves

Examples:Kleopatra

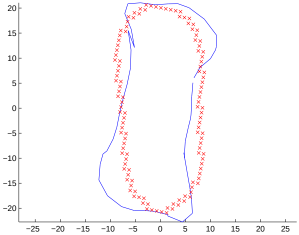
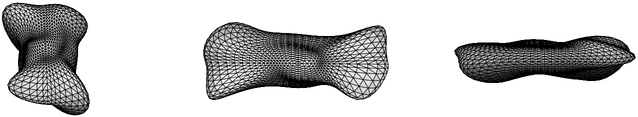


Figure: An example of discrepancy between data obtained from AO images and S-curves

Examples:Kleopatra



Conclusions

- Shape inversion of general (not necessarily star-shaped) asteroid is possible
- More data is needed for reliable reconstruction
- Systematic errors in the data make error analysis challenging