Performance Considerations for Positioning with Signals of Opportunity

Pedro Figueiredo e Silva and Elena Simona-Lohan
Department of Electronics and Communications Engineering, Tampere University of Technology, Finland
{pedro.silva, elena-simona.lohan}@tut.fi

Abstract—This paper presents a study on the benefit of observing several signals of opportunity for positioning purposes. Several static emitters are placed over a defined area where an user is moving and acquiring measurements to each of these emitters. The simulation considers that the user is capable of acquiring time of arrival measurements from several wireless protocols, such as WCDMA, 802.11b, 802.11g and 802.11ac. The variance in the measurements is modelled through the Cr ´amer-Rao bound and a propagation model for each technology. As conclusions, this paper discusses the benefits of using multiple signals of opportunity in the context of positioning and how much the positioning performance is affected by considering different measurements combinations from several wireless technologies.

Index Terms—Cram ´er-Rao Lower Bound, signals of opportunity, approximate maximum likelihood.

I. INTRODUCTION

Location based services have pushed the need to localize user’s in any environment, either in urban canyons or indoor facilities, such as office buildings, hospitals, schools among others [1], [2]. While global navigation services are commonly relied on for providing the location of an user, these services are meant to be used in obstruction-less environments and a clear view of the sky. For that reason, positioning with signals of opportunity, any signal designed for something else than positioning, aims to be an alternative to complement the existing positioning services [3], [4].

The proliferation of Wi-Fi networks has contributed to the appearance of several techniques for estimating the location of an user. Fingerprinting is one of the most widely used approaches [5], whose popularity arises from the fact that the required infrastructure is already in place and no significant investments are required [6], [7]. However, one of its disadvantages is the requirement of a prior training phase, which can be expensive and difficult to deal with.

Therefore, relying on one stage estimators, such as those that employ angle of arrival, time of arrival and time difference of arrival measurements is more desirable. This is the motivation for the study, which focuses on time of arrival measurements to obtain the location of a mobile receiver [8].

The goal of the study is to provide a bound for the performance of a positioning system, which is assumed to rely on time of arrival measurements of several widely available wireless protocols, such as Wi-Fi and UMTS signals. This work is of interest, for example, for future microlocation for the Internet of things [9] or for energy-efficient cooperative opportunistic positioning systems [10].

II. RELATED WORK

Related works can be found for example in [11]–[14]. In [11], a similar problem of hybrid localization with heterogeneous networks is addressed. The authors combine cellular and WiFi signals with TOA, AOA and RSS and the focus is only on the overall performance, rather than on the incremental performance of adding one additional system or emitter at a time, as done here.

In [12] the authors compare the Wi-Fi-based positioning with UMTS-based positioning by using RSS measurements, but the two systems are not considered together. They conclude that similar indoor accuracies can be achieved with Wi-Fi and UMTS when RSS measurements are used.

The work in [14] looks into positioning with a 3GPP-LTE signal and what is the gain obtained by considering several signals of opportunity, such as digital television and Wi-Fi. When aided by signals of opportunity, the gain in accuracy was seen to be 40 % to 70% better than standalone positioning with 3GPP-LTE. These gains were observed for scenarios with more than 40 user equipments and 1 to 4 additional signals of opportunity, respectively.

III. SIMULATION

In this study, the simulation model assumes the existence of several Wi-Fi signals, based on the standards IEEE 802.11ac/b/g (simply refered as 802.11ac/b/g from now on) and WCDMA signals, based on UMTS signals. Table I summarises a few key parameters of each technology, including the signal structure type, OFDM and CDMA and bandwidth. The simulation assumes an environment where several emitters, from each of these technologies, are randomly distributed inside a defined area.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>SIGNALS UNDER CONSIDERATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal</td>
<td>Type</td>
</tr>
<tr>
<td>802.11ac</td>
<td>OFDM</td>
</tr>
<tr>
<td>802.11g</td>
<td>OFDM</td>
</tr>
<tr>
<td>802.11b</td>
<td>CDMA</td>
</tr>
<tr>
<td>WCDMA</td>
<td>CDMA</td>
</tr>
</tbody>
</table>

For the given area, the user movement is modeled through a random walk in a two dimensional space [15], with a fixed step length of one meter. Each new position, \( X(t) \), at simulation time, \( t \), was obtained by summing a movement vector, \( M(s) \),
to the previous position. The movement vector is randomly chosen by drawing the step decision variable \( s \), from a random integer generator. Hence, the movement model is defined by,

\[
X(t) = X(t - 1) + M(s), \quad \text{where} \quad s = \{1, 2, 3, 4\},
\]

and,

\[
M(s) = \begin{cases} 
(0, -1) & \text{if } s = 1, \\
(1, 0) & \text{if } s = 2, \\
(-1, 0) & \text{if } s = 3, \\
(0, 1) & \text{if } s = 4.
\end{cases}
\]

(2)

On each new location, the timing measurements, \( L_n \), to each \( n \)-th emitter are computed by assuming their location known as well as the variance in the measurement error. Hence, \( L_n \) is obtained by

\[
L_n = R_n + \epsilon_n,
\]

where \( R_n \) is the geometrical distance to the emitter and \( \epsilon_n \) is the measurement error. \( R_n \) is obtained by,

\[
R_n = \sqrt{\left(x_{\text{emitter}}^{(i)} - x_{\text{user}}\right)^2 + \left(y_{\text{emitter}}^{(i)} - y_{\text{user}}\right)^2}
\]

where \((x_{\text{user}}, y_{\text{user}})\) are the coordinates of the user at a given time and \((x_{\text{emitter}}^{(i)}, y_{\text{emitter}}^{(i)})\) the position for the \( i \)-th emitter.

The measurement noise, \( \epsilon_n \), is modelled through a normal distributed distribution, with its variance set according to the Crâmer-Rao lower and the expected carrier to noise ratio \( (C/N_0) \) at the receiver’s location. Regarding the variance, the Crâmer-Rao lower bounds are computed using the result in [16], where the variance for an unbiased estimator for range measurements is given as,

\[
\text{var}(\hat{\tau}_0) \geq \frac{1}{\epsilon N_0/2 F^2} \iff \text{var}(\hat{\tau}_0) \geq \frac{1}{C N_0/2 T F^2},
\]

where, \( \epsilon \) is the signal energy, \( N_0 \) the noise spectral density, \( T \) the observation interval and \( F^2 \) the mean square bandwidth of the signal, given as,

\[
F^2 = \frac{\int_{-\infty}^{\infty} (2\pi F)^2 |S(F)|^2 dF}{\int_{-\infty}^{\infty} |S(F)|^2 dF}.
\]

Fig.1 shows the expected accuracy for the signals under consideration, WCDMA, 802.11b, 802.11g and 802.11ac, plotted against the signal’s carrier to noise ratio \( (C/N_0) \). As expected, the WCDMA is the signal showing the worst performance for timing estimates, since it is the one with the smallest bandwidth. A narrow band signal in the frequency domain equates to a larger signal in the time domain, which is undesirable for positioning purposes. Since the receiver relies on the correlation of the incoming signal with a locally generated replica to obtain a time of arrival (TOA) measurement, the larger or flatter this area is, the worse the timing estimate will be.

Therefore, besides considering each signal design individually, regarding its transmission power and bandwidth, the simulator also relies on the ITU-R propagation model to describe the expected \( C/N_0 \) at the receiver [17]. The ITU-R model is derived from the Friis equation and given as,

\[
P_r(d) = P_t + L - 20 \log_{10} \left( \frac{4\pi f}{c} \right) - 20 \log_{10} (d) + \nu,
\]

where the RSS at a distance of \( d \) (meters) is given by \( P_r(d) \), the device’s transmission power by \( P_t \), the operating frequency in Hertz as \( f \), the propagation speed, considered as the speed of light in vacuum, as \( c \), losses in the path of the signal are translated into \( \eta \) while \( L \) are other system losses. The model considers a slow fading phenomenon, described by the log-normal distributed random variable \( \nu \sim N(0, \sigma^2) \).

While the ITU-R model is used for the propagation loss, the noise component is modelled as thermal noise [17]. Fig.2 presents a diagram with the steps taken by the simulator in order to provide a measurement for the given location of the user. Afterwards, this measurement is used to estimate the user’s location.

![Fig. 1. Crâmer-Rao lower bounds for WCDMA and 802.11 ac/g/b](image-url)

Since the goal of the simulation is to infer the accuracy in a best case scenario, the network is assumed to be synchronized, meaning that no clock bias or offset is modelled and added to the measurement. Therefore, one should keep in mind that in a real system, these constrains would not hold. Nevertheless, they can give a clear image of the relative performance of the different considered approaches.

**IV. Estimation**

By using the measurements acquired at each point the user moves to (Fig.2), the simulation estimates the location of the user, \((x, y)\), through an approximate maximum likelihood (AML) [18], [19]. Hence, assume each of these measurements, as in (3), define the measurement vector, \( \mathbf{r} \), given as

\[
\mathbf{r} = [L_1, L_2, \ldots, L_n],
\]

(9)

and,

\[
\mathbf{X}(t) = \mathbf{X}(t - 1) + \mathbf{M}(s), \quad \text{where} \quad s = \{1, 2, 3, 4\},
\]
minimizes the Jacobian $J$.

**Approximate Maximum Likelihood**

The maximum likelihood (ML) estimate is the $\Theta$ that minimizes the Jacobian $J$ in the probability density function of $T$ given $\Theta$.

$$f(T/\Theta) = (2\pi)^{\frac{N}{2}} (\det Q)^{-\frac{1}{2}} \exp \left(-\frac{J}{2}\right).$$

Setting the gradient of $J$ with respect to $\Theta$ to zero, gives the two ML equations

$$\sum_{i=1}^{n} \frac{(r_i - \delta_i)(x - x_i)}{r_i} = 0,$$

$$\sum_{i=1}^{n} \frac{(r_i - \delta_i)(y - y_i)}{r_i} = 0.$$

Due to the non linearity of (16), the AML solution, as presented in [18], in matrix form can be represented as,

$$2 \sum_{i=1}^{n} g_i x_i \sum_{i=1}^{n} g_i h_i \begin{bmatrix} x \\ y \end{bmatrix} = \left[\sum_{i=1}^{n} g_i(s + \delta_i)^2\right],$$

where,

$$g_i = \frac{x - x_i}{r_i(r_i + \delta_i)},$$

$$h_i = \frac{y - y_i}{r_i(r_i + \delta_i)}.$$

The AML treats (17) as a set of linear equations. Starting from an initial $(x, y)$, it first computes $g_i, h_i$, and the least squares for $(x, y)$ from (17), in terms of $s$. Putting them into

$$s = x^2 + y^2,$$

leads to a quadratic in $s$. Therefore, the correct root needs to be chosen. For that to happen the AML acts differently on three scenarios, one root is positive, both roots are positive and both roots are either negative or imaginary. For the first case, the root with a positive value is taken as the value to replace $s$ in the least squares solution of (17). For the second case, the favored root is the one providing a smaller $J$. On the third case, it takes the absolute values of the real parts.

After $k$ iterations, the AML will have $k$ values of $J$ and in the end, the one that provides the smallest value of $J$ [18], [19].

**V. RESULTS**

This section covers a set of illustrative results obtained through the simulator. The first results show a direct consequence from the fact that narrow band signals provide an overall lower accuracy regarding timing estimates. This is seen through Fig.3 where the root mean square error is plotted against the number of emitters available for a given technology. As expected, the lowest RMSE is obtained by using 802.11ac emitters and the biggest RMSE when only WCDMA emitters are present.

Since the study sets out to understand the benefit of observing and exploiting several technologies, Fig.4 - 6 illustrate the benefit of obtaining measurements from additional emitters. In each figure, the thicker line with a circle marker represents the RMSE in meter obtained using only several WCDMA emitters, while the remaining lines represent the RMSE obtained when merging WCDMA with $N$ other emitters of a different technology. As an example, WCDMA + 3b means that $N$ WCDMA emitters are available (read from the x axis) as well as 3 other 802.11b emitters.
Fig. 3. Positioning using a single technology

Fig. 4 shows the benefit in the performance of a system that uses primarily WCDMA and when available uses either 802.11g, 802.11ac or both. As it can be seen from the plot, the curve is obtained when both 802.11ac and 802.11g are combined together with WCDMA. Furthermore, one can also observe that when using WCDMA with 10 emitters of 802.11ac. There is no necessity of using more emitters of 802.11g, since the achievable performance is the same. However, using 10 802.11g emitters with WCDMA achieves the same performance when 3 emitters of 802.11ac and 802.11g are available. Even though 802.11ac provides more accurate measurements, it does not offset the fact that with 10 emitters, the system still has more 4 distinct measurements. Moreover, when merging WCDMA with a single other technology, regardless of the one that is picked, going from 3 to 5 emitters results in a significant improvement in performance. On the contrary, when WCDMA is merged with the other two technologies, the increase in the number of emitters has little impact on the overall performance of the system. Bottom line, the main conclusions to draw from this plot are the fact that increasing the number of observables is desirable in general, but the cost of adding and managing those does not translate to a significant increase improvement on the overall performance.

Fig. 5 follows a similar approach, but now WCDMA is merged with the other technologies in this simulation with higher variance, 802.11b and 802.11g. As expected, the results also show better performance when the full number of emitters is used. It also shows the combination with 802.11g is less accurate than the one with 802.11b. This difference is particularly noticeable when 3 emitters of each technology are available, with the difference fading as the number increases. As for the best achievable performance, this seems to be attainable when using WCDMA in addition to 10 other 802.11g. The combination of the three technologies seems to fare equally well. Overall, the addition of 802.11b and 802.11g improves the performance of the system, but in some circumstances 802.11b provides the best performance.

Fig. 6 presents WCDMA measurements, being merged with the next less accurate measurement, 802.11b and with the more accurate timing measurements from 802.11ac. The best performance is achieved when the three technologies are all merged together. From the plot one can see that adding 3 emitter from either 802.11b or 802.11ac seems to provide a similar performance. This means the WCDMA is setting a
In the end, all these plots, Fig.4 - 6, show that increasing the number of observables leads to an increase in performance. However, for some combinations of technologies, this benefit might not be worth the added complexity in terms of power consumption or processing power, due to the higher bandwidth of the signals, such as preferring 5 additional measurement from a 802.11g source rather than 3 from a 802.11ac one (Fig.6). Besides that, the signal structure should also be taken into account, for example, OFDM signals are prone to phane noise and frequency offset.

VI. CONCLUSIONS

This paper has presented a study on the impact of merging several TOA measurements for different signals of opportunity, WCDMA, 802.11b, 802.11g and 802.11ac. The measurements were acquired from a simulator which derives the timing estimates from the Crámer-Rao lower bounds for each signal. In addition to that, the simulator uses a propagation model to match the received signal power to the distance the user is from the receiver. Furthermore, the simulator assumes all the systems to be synchronised, which, in reality, would be difficult to achieve. Therefore, the results provide an insight on the best case scenario that a user could experience.

As main conclusions, while adding more emitters is often desirable, the benefit in the overall accuracy is small and in some situations less accurate systems might lead to the same or comparable results. In particular, the paper shows that when observing 5 emitters of 802.11b, the overall accuracy is equivalent to the one when 10 emitters of 802.11ac are available.

Overall, for a practical system relying on signals of opportunity, some combinations, pointed out in the paper, might not be worth pursuing since it will require more resources from the user device for little added benefit in the system’s performance.

It is therefore of utmost importance to first perform a theoretical analysis, as the one illustrated here in order to pre-evaluate the possible positioning gain by using multiple emitters from heterogeneous systems. Only if the gain is large enough, the hybridization of signals from heterogeneous networks should be employed, otherwise a single system may still bring enough benefit with a lower complexity.

ACKNOWLEDGMENT

This work was financially supported by EU FP7 Marie Curie Initial Training Network MULTI-POS (Multi-technology Positioning Professionals) under grant nr. 316528. The authors also express their warm thanks to the Academy of Finland (project 250266) for its financial support.

REFERENCES


