Sparse Frequency Domain Spectrum Sensing and Sharing Based on Cyclic Prex Autocorrelation

DOI
10.1109/JSAC.2016.2633058

Sparse Frequency Domain Spectrum Sensing and Sharing based on Cyclic Prefix Autocorrelation

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***Abstract*—Cognitive radio (CR) is considered an important solution to the current spectral scarcity, which is expected to be a significant issue in the next generation of wireless communication systems, namely 5G. Wideband spectrum sharing and sensing constitute highly desirable features of CR systems as they aim to increase the probability of identifying available spectral bands, which ensures a more efficient resource utilization. The present work proposes an efficient frequency-domain cyclic prefix (CP) autocorrelation based wideband spectrum sensing and sharing method that can provide accurate detection of orthogonal frequency-division multiplexing (OFDM) based primaries in wideband CR systems. Novel analytic expressions are derived for the corresponding threshold, probability of false alarm and probability of detection in the presence of noise uncertainty (NU) and frequency selectivity. The derived models are validated by extensive comparisons with respective results from computer simulations. It is demonstrated that the introduced autocorrelation based sensing method is able to counteract NU and the frequency-selective multipath channel effects in realistic wideband communication scenarios. Furthermore, the method facilitates partial band sensing, allowing the sensing of weak OFDM-type primary user (PU) signals in channels which are partly overlapped by other strong PU or CR transmissions. This is considered a crucial element in practical spectrum sharing scenarios. Since, the proposed sensing method makes use of sparsity in the spectral domain, it can be technically considered as compressed sensing method. The flexibility of this approach supports robust wideband multi-mode, multi-channel sensing with low complexity. Finally, it is shown that the offered results are particularly useful in the context of spectrum sharing as their high performance and reduced complexity can enable the co-existence of non-exhaustive yet highly efficient algorithms.**

***Index Terms*—Cognitive radio, OFDM, wideband sensing, sparsity, time and/or frequency domain CP autocorrelation compressed spectrum sensing, spectrum sharing, energy detector, frequency selective channels, and noise uncertainty.**

I. Introduction

This work was supported in part by the Academy of Finland under the project no. 251138, no. 284694, no. 284724 and no. 288670, and the Finnish Cultural Foundation. Early stage results of this paper were presented in part at IEEE VTC - Spring 2016, Nanjing, China [1].

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It is widely known that wireless communications have been growing exponentially and the scarcity of available radio frequency spectrum has become a core issue, despite intensive research and development efforts for more effective technologies. Also, cognitive radio (CR) has received increasing attention as a potential solution to the current spectrum scarcity issue for the next generation of wireless communication systems namely, 5G, with the aid of sophisticated algorithms for efficient spectrum sharing and sensing realizations [1]–[4].

It is recalled that the aim of spectrum sensing is to identify available spectral holes for opportunistic utilization by secondary users (SU). To this end, different sensing algorithms have been proposed in the literature, but none of them fully satisfies the criteria of all associated metrics, such as efficiency, implementation complexity, reliability, and secondary system throughput degradation [1]–[5]. Spectrum sharing is also a core part of CR and can be applied with different approaches, such as, opportunistic / coexistence sharing, overlaying sharing, and underlay sharing [6]–[9]. If sharing is considered as a coexistence based process, SUs are essentially invisible to the primary user (PU). Therefore, all of the complexity of sharing is a burden of the SU system. Before any SUs can be deployed, spectrum sensing must be completed to protect the PU from any harmful interference. Opportunistic sharing can be considered as an effective method that ensures adequate performance at a relatively non-exhaustive complexity [6]–[8]. This is based on the fact that the opportunistic approach forces the SU to sense the PUs spectrum holes, i.e., unused PU bands, and allows transmission on these bands only when they are unused by the PU [6]–[8].

Energy detection (ED) is commonly considered an efficient spectrum sensing technique in CR due to its simple practical realization and relatively low computational complexity [3], [4]. However, the performance of ED is sensitive to noise uncertainty (NU) effects under low signal-to-noise ratio (SNR) levels [3], [10]–[13]. Yet, it is necessary to operate under very low PU SNR in various CR scenarios due to the multipath fading and shadowing phenomena, which result in power fluctuations of received PU signals. Different sensing methods have been presented in the literature to overcome the NU issue [14]–[16]. To this end, eigenvalue-based advanced spectrum sensing methods have been proposed as an alternative sensing approach which can overcome the effects of NU [17]–[20]. However, because of the required calculation of the covariance matrix of the received signal and its eigenvalues, these techniques involve particularly high computational complexity. Alternative eigenvalue based approaches, which only require the largest eigenvalue and trace of the covariance matrix, have been presented in [21] to decrease the complexity. Yet, the computational complexity of these proposed methods is still quite high. Energy spectral density (ESD) based maximum − minimum ED sensing techniques have also been studied in the literature [22]–[26]. These approaches have been shown to provide acceptable performance at the low SNR regime under the presence of NU effects. The intuitive idea behind these approaches is that in certain scenarios, the minimum subband energy can be regarded as an estimate for the noise variance. Moreover, the presence of a PU signal introduces frequency variability of the received power spectral density (PSD), which is not critically affected by the NU [22]–[26]. Cyclostationary feature detection is also proposed as an alternative and effective sensing technique to detect PUs by exploiting the cyclostationary features of the received signal [27]–[31]. Given also that the noise is wide-sense stationary and modulated signals are cyclostationary with spectral correlation, the PU signal can be differentiated from noise. Additionally, distinguishing among different types of transmissions and primary users can be achieved by cyclostationary feature detection [27], [28]. However, also cyclostationary based approaches bring particularly high computational complexity, which is practically problematic.

It is recalled that orthogonal frequency devision multiplexing (OFDM) is widely used in current and emerging wireless communications standards, and thus PUs in future CR based communications are expected to use OFDM waveforms. Therefore, it is of paramount importance to exploit the specific features of OFDM in spectrum sensing. The presence of cyclic prefix (CP) introduces peaks in the autocorrelation of the received waveform at lags corresponding to the length of the useful symbol period. This characteristic has been typically used, e.g., for synchronization purposes and recently also for spectrum sensing operations [32]–[39]. In this context, CP autocorrelation (CP-AC) approach appears as an effective spectrum sensing method, which overcomes the NU phenomena of ED.

In terms of the bandwidth, spectrum sensing and sharing techniques can also be classified as narrowband or wideband. The main aim of the traditional narrowband spectrum sensing is to explore spectral opportunities over a relatively narrow frequency range, typically within a single PU frequency channel. However, CR is required to exploit spectral opportunities over a wide frequency range to determine more spectral opportunities and achieve effective resource allocation. Wideband spectrum sensing, operating effectively over multiple PU channels, is highly desirable to increase the probability of determining unoccupied spectrum bands. Cooperative wideband spectrum sensing under fading channels has been recently proposed in the literature [40]. This approach not only provides computation and memory savings compared to the existing wideband spectrum sensing methods, but also reduces the hardware acquisition requirements and the energy costs at CRs [40]. Some indicative applications for wideband spectrum sensing and sharing include the cognitive access to the unused portions of some specific frequency bands, such as industrial, scientific and medical (ISM) band or the TV white space (TVWS), which includes the channels that are not used by digital terrestrial television (DTT) or program making and special events (PMSE) users, and those that became available after the switch-over from analogue to digital TV broadcasting [41]–[43].

In the wideband scenarios, a major challenge is due to the striking requirements on the analog-to-digital converter (ADC) for sampling the received wideband multi-channel spectrum at the Nyquist rate. Generally, wideband spectrum sensing is typically categorized into two types: Nyquist wideband sensing [44]–[47] and sub-Nyquist wideband sensing (see [48], [49] and the references therein). In Nyquist wideband sensing processes, the received signal is sampled at or above the Nyquist rate, which practically leads to unaffordable high sampling rate and implementation complexity. Sub-Nyquist sampling based cyclic feature detection has been presented, but for only binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK) modulated signals in [50].

It is also recalled that compressed spectrum sensing (CSS) has been proposed as an effective method for reducing the processing complexity of emerging communication systems. In this context, it has been also applied in wideband communication scenarios by exploiting the sparseness of the wideband signal in the frequency domain [48], [51]–[53]. Furthermore, CSS based wideband spectrum sensing has attracted considerable attention because it uses substantially smaller number of samples [54]–[58]. Novel hybrid framework combining compressed spectrum sensing has been proposed for wideband spectrum under sub-Nyquist in [59]. A data-assisted non-iteratively re-weighted least squares based compressive spectrum sensing has been given to reduce the sampling rates and lower the computational complexities [59]. Similarly, two-phase single node and cooperative spectrum sensing algorithms for wideband spectrum sensing at sub-Nyquist sampling rates have been proposed to reduce the computational complexity and improve the robustness to channel noise recently in [60]. Mixed-signal parallel segmented compressive sensing (PSCS) has been proposed for cognitive radios in [61]. This approach for wideband spectrum, where the high-speed ADCs are avoided by carrying out an analog basis expansion in parallel before sampling. PSCS front-end is also able to sample and reconstruct analog sparse and compressive signals at sub-Nqyuist rate [61]. However, recovering the wideband spectrum from its compressed samples is typically realized by solving an optimization problem that requires high computational complexity. As a consequence, it becomes cumbersome to implement such techniques in the context of compact commodity radios with limited computational capabilities.

Motivated by the above, the core aim of this work is to construct effective spectrum sensing methods for CP-OFDM primaries in wideband scenarios, utilizing sub-Nyqyuist sampling with respect to the targeted PU signals. Hence, less complex and low power white space devices, e.g., for WiFi based accesses and Machine-to-Machine (M2M) communications could be operated based on the CR principles. Sensing techniques utilizing sparsity in frequency domain are investigated here as a promising approach in this direction.

In the present paper, a frequency domain autocorrelation

(FD-AC) based wideband sensing method is proposed, which is shown to be highly efficient in spectrum sensing and sharing based CR communications. This method performs first a subband decomposition of the received signal using fast Fourier transform (FFT) (or alternatively, using analysis filter bank) [5]; then, ACs for the relevant subband signals are determined and the overall AC is constructed from the subband ACs. During this process, the subbands corresponding to the active portion of a specific transmission channel may be collected for the overall AC. Such processes can be executed in parallel for different candidate PU channels, and possibly also for alternative signal parameterization regarding the OFDM FFT size, CP length, etc. Furthermore, it is not necessary to include the whole PU bandwidth in the AC calculation process and thus, it is possible to sense PUs which are partly overlapped by other interfering PU or SU transmissions. The computational complexity of the algorithm is mainly due to the FFT (or filter bank) processing for subband decomposition. The subband based AC calculations are not computationally heavy, and their complexity can be further reduced by reducing the number of subbands used for constructing the AC.

The proposed sensing method makes use of sparsity [62] in the spectral domain, and therefore it can be considered as compressed sensing method which acquires wideband signals using sampling rates lower than the Nyquist rate and detects potentially vacant spectral bands using these compressed measurements. In this paper we assume wideband signal sampling, but the ideas can be developed towards specific schemes for sampling the received signal to capture portions of the target signal spectrum for sensing purposes. Simple example is the use of multiple narrowband ADCs instead of a single wideband ADC.

More specifically, the contributions of the present paper are listed below:

- A compressed sensing method based on constructing the autocorrelation of the received signal from its subband sample sequences is developed for wideband spectrum sensing.
- Novel analytic expressions are derived for the threshold, probability of false alarm, P_{FA}, and probability of detection, P_D, for the proposed FD-AC based compressed sensing method in wideband scenarios under NU. The offered results are validated extensively through comparisons with respective computer simulations. These results provide meaningful insights that are useful for future design and deployments of CR communication systems and networks.
- The proposed approach is tested with practical signal models with main parameters following the OFDM based 802.11g like wireless local-area-networks (WLAN) signal model. However, the same approach can be easily applied to any generic signal models for CP based primaries, including basic OFDM and OFDM based single carrier waveforms.
- Effects of practical wireless channels on the FD-AC based compressed sensing method are investigated, considering both NU and channel frequency selectivity. The effect of stationary frequency selectivity on the proposed FD-AC methods is thoroughly quantified.
- It is demonstrated that the proposed approach is quite robust to NU challenges compared to the traditional energy detector based methods, which are rather sensitive to the uncertainties on noise variance.
- FD-AC techniques are shown to exhibit lower complexity than the traditional eigenvalue based methods, which are considered advanced sensing techniques overcoming the NU challenges.
- The proposed technique is particularly useful in the context of spectrum sharing as it allows partial band sensing focusing on the non-interfered parts of the PU spectrum. In addition, the use of sparsity in the spectral domain allows to develop wideband sensing and sharing schemes with low complexity and low energy consumption, e.g., by utilizing parallel narrowband sensing processes.

To the best of the authors' knowledge, the autocorrelation based approaches in frequency domain, with the compressed sensing element, have not been previously reported in the open technical literature.

The remainder of the paper is organized as follows: Section II revisits the general concept about time domain autocorrelation based spectrum sensing methods, including also NU in the model as a novel element. The novel FD-AC based compressed spectrum sensing method is presented in Section III. The signal and frequency selective channel models are described in Section IV along with numerical results for the corresponding sensing performance and computational complexity. Finally, closing remarks are provided in Section V.

II. Time Domain CP Autocorrelation based Spectrum Sensing under Noise Uncertainty

The main aim of the CP autocorrelation based detector is to differentiate AWGN and OFDM signal samples, which have similar statistical properties. A received signal $y(n)$ can be formulated by the following hypothesis test [3]:

$$\begin{aligned} \mathcal{H}_0 &: y[n] = w[n] \\ \mathcal{H}_1 &: y[n] = \overbrace{s[n] \otimes h[n]}^{x[n]} + w[n] \end{aligned} \tag{1}$$

where, $y[n]$ is the signal observed by the sensing receiver with $s[n]$ and $w[n]$ denoting the OFDM type PU information signal and the zero-mean complex circularly symmetric AWGN, respectively. Furthermore, $h[n]$ denotes the channel impulse response and $x[n]$ is the received PU signal with channel effects while $\otimes$ denotes linear convolution. Under hypothesis $\mathcal{H}_0$, $y[n]$ consists only of $w[n]$ in the absence of the PU whereas the PU signal $x[n]$ is present along with $w[n]$ under hypothesis $\mathcal{H}_1$.

The structure of an OFDM symbol is illustrated in Fig. 1, where useful data samples and CP length are represented by N_d and N_c, respectively. The total number of samples in an OFDM symbol N_s can be written as $N_s = N_d + N_c$. The N_d samples also determine the useful data bearing length of the block and these samples are derived from the inverse fast Fourier transform (IFFT) of the sequence of N_d complex subcarrier symbols, some of which may be zero corresponding

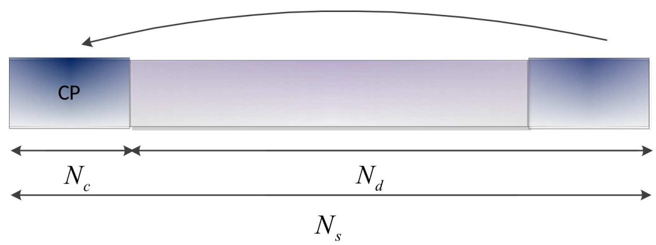


Figure 1: OFDM block structure.

to guard bands. The CP is a sequence of N_c samples that is the replication of N_c last samples of the useful part of the symbol. The main aim of the CP is to provide the orthogonality of the modulation symbols at the receiver by converting the Toeplitz convolution structure of the channel to a circulant one.

We focus on exploiting the CP-autocorrelation property of OFDM systems for the detection of primary users. The involved autocorrelation may be estimated as follows

$$R(\tau) = \frac{1}{N}\sum_{n=1}^{N} y(n)y^*(n+\tau). \tag{2}$$

It is recalled that according to the Central Limit Theorem (CLT) the sum of sufficiently many independent and identically-distributed (i.i.d.) random variables, under the assumption that the sum of the variables has a finite variance, follows Gaussian distribution. Assuming sufficiently large IFFT size for the OFDM signal, and invoking the CLT, we have $s(n) \sim \mathcal{N}_c(0, \sigma_s^2)$, $w(n) \sim \mathcal{N}_c(0, \sigma_w^2)$, $x(n) \sim \mathcal{N}_c(0, \sigma_x^2)$ and $y(n) \sim \mathcal{N}_c(0, \sigma_x^2 + \sigma_w^2)$ where it has been assumed that $s(n)$ and $w(n)$ are independent of each other and $\mathcal{N}_c(.)$ denotes the distribution of complex Gaussian random variable, σ_s^2, σ_x^2 and σ_w^2 denote the variances of the transmit signal, the received signal and the AWGN process, respectively. In practice, the noise variance can be expected to lie within the range $\sigma_w^2 \in [(1/\rho)\sigma_n^2, \rho\sigma_n^2]$, where $\rho > 1$ is a parameter that quantizes the corresponding uncertainty. It is noted here that NU is usually expressed as $\varrho = 10\log_{10}\rho$, in dB. When including NU in the analytical models, we make the worst-case assumption: The actual noise variance is $\rho\sigma_n^2$ under H_0 and σ_n^2/ρ under H_1.

It is recalled that due to the presence of the CP, the OFDM block exhibits cyclostationarity. As a result of these statistics, a peak value appears in the autocorrelation function at the lag of N_d samples, when a set of OFDM samples of minimum length $2N_d + N_c$ is considered. The nonzero autocorrelation coefficients corresponding to lags $n = \pm N_d$, are shown to be the log likelihood ratio test (LLRT) statistic [35].

Since data are unknown and cyclic prefix changes from symbol to symbol, $r = R(\tau)\,|_{\tau=\pm N_d}$ is also a random variable with mean and variance. Under $\mathcal{H}_1$, the mean of r is given by

$$E[r\,|\mathcal{H}_1] = E\left[\frac{1}{N}\sum_{n=1}^{N} y(n)y^*(n+N_d)\right] \tag{3}$$

$$= \frac{1}{N}\sum_{n=1}^{N} E\left[y(n)y^*(n+N_d)\right] \tag{4}$$

$$= \frac{1}{N}\sum_{n=1}^{N} E\left[(x(n)+w(n))(x^*(n+N_d)+w^*(n+N_d))\right] \tag{5}$$

$$= \frac{1}{N}\sum_{n=1}^{N} E\left[x(n)x^*(n+N_d)\right] + \frac{1}{N}\sum_{n=1}^{N} E\left[w(n)w^*(n+N_d)\right] \tag{6}$$

$$= \frac{1}{N}\sum_{n=1}^{N} E\left[x(n)x^*(n+N_d)\right] \tag{7}$$

$$= \mu\sigma_x^2 \tag{8}$$

where $\mu = \frac{N_c}{N_c+N_d}$.

The second moment of r under $\mathcal{H}_1$ can be calculated as

$$E[|r|^2\,|\mathcal{H}_1] = E\left[\left|\frac{1}{N}\sum_{n=1}^{N} y(n)y^*(n+N_d)\right|^2\right] \tag{9}$$

$$= \frac{1}{N^2}\sum_{n_1=1}^{N}\sum_{n_2=1}^{N} E\left[y(n_1)y^*(n_1+N_d)y^*(n_2)y(n_2+N_d)\right] \tag{10}$$

which can be equivalently expressed according to (11), at the top of the next page. If a, b, c, and d are jointly Gaussian (complex or real) random variables, it follows that

$$\begin{aligned} E\left[abcd\right] =& E[ab]E[cd] + E[ac]E[bd] + E[ad]E[bc] \\ &- 2E[a]E[b]E[c]E[d]. \end{aligned} \tag{12}$$

Based on the above expression and by using the independence of the signal samples $x(n_1)$ and $x(n_2)$ and noise samples $w(n_1)$ and $w(n_2)$, where $n_1 \neq n_2$, we can expand (9) yielding

$$E\left[|r|^2\,|\mathcal{H}_1\right] = \frac{\left(\sigma_x^2 + (1/\rho)\,\sigma_n^2\right)^2 + 3\mu_1^2}{N} \tag{13}$$

where $\mu_1 = \mu\sigma_x^2$ and ρ presents the corresponding NU parameter based on the worst case assumption. Therefore, it follows that the variance of r under $\mathcal{H}_1$ is given by

$$\mathrm{Var}\left(r\,|\mathcal{H}_1\right) = E\left[|r|^2\,|\mathcal{H}_1\right] - \left|E\left[r\,|\mathcal{H}_1\right]\right|^2 \tag{14}$$

$$= \frac{\left(\sigma_x^2 + (1/\rho)\,\sigma_n^2\right)^2 + 2\mu_1^2}{N}. \tag{15}$$

Under $\mathcal{H}_0$, the independence of $w(n)$ and $w(n+N_d)$ yields $E[r|H_0] = 0$ and thus, the variance is expressed as

$$\mathrm{Var(r}\,|\mathcal{H}_0\,) = \frac{\left(\rho\sigma_n^2\right)^2}{N}. \tag{16}$$

$$E[|r|^2 \,|\mathcal{H}_1\,] = \frac{1}{N^2} \sum_{n_1=1}^{N} \sum_{n_2=1}^{N} E[(x(n_1) + w(n_1))\,(x^*(n_2) + w^*(n_2))\,(x^*(n_1 + N_d) + w^*(n_1 + N_d))\,(x^*(n_2 + N_d) + w^*(n_2 + N_d))]. \tag{11}$$

A. Autocorrelation Magnitude as Test Statistic

One possible test statistic is the magnitude, $T_{\rho,c}$, of the complex value of autocorrelation peak at lag N_d. It can be expressed as:

$$T_{\rho,c} = \frac{\frac{1}{N}\left|\sum\limits_{n=0}^{N-1} y(n)y^*(n+N_d)\right|}{\frac{1}{N+N_d}\sum\limits_{n=0}^{N+N_d-1} |y(n)|^2}. \tag{17}$$

Under the H_0 hypothesis, the above test statistic is distributed according to:

$$\mathcal{H}_0 : T_{\rho,c} \sim \mathcal{N}_c\left(0, \frac{1}{N}\right). \tag{18}$$

It is noted here that due to the Gaussian statistics, $T_{\rho,c}$ has a probability of exceeding threshold, $\gamma_{\rho,c}$, which is given by

$$P\left(T_{\rho,c} > \gamma_{\rho,c}\right) = Q\left(\frac{\gamma_{\rho,c}}{\sigma_c}\right) \tag{19}$$

where, $Q(.)$ denotes the complementary error function and σ_c is the standard deviation of the complex signal. Based on this, it follows that

$$P_{FA} = P\left(T_{\rho,c} > \gamma_{\rho,c}\,|\mathcal{H}_0\,\right) = Q\left(\sqrt{N}\gamma_{\rho,c}\right). \tag{20}$$

With the aid of (18), the expected value of $T_{\rho,c}$ under the null hypothesis is zero. Then, the $P\left(T_{\rho,c} > \gamma_{\rho,c}\,|\mathcal{H}_0\,\right)$ can be obtained as a probability of false alarm, P_{FA}, in the context of detecting AWGN samples under the $\mathcal{H}_0$ hypothesis. Hence, given the desired P_{FA}, the threshold, $\gamma_{\rho,c}$, can be expressed as:

$$\gamma_{\rho,c} = \frac{1}{\sqrt{N}} Q^{-1}\left(P_{FA}\right) \tag{21}$$

whereas under the hypothesis $\mathcal{H}_1$, $T_{\rho,c}$ is distributed according to:

$$\mathcal{H}_1 : T_{\rho,c} \sim \mathcal{N}_c\left(\mu_1, \frac{\left(\sigma_x^2 + (1/\rho)\,\sigma_n^2\right)^2 + 2\mu_1^2}{N}\right). \tag{22}$$

Finally, the corresponding probability of detection can be determined by the following closed form representations

$$\begin{aligned} P_D &= P\left(T_{\rho,c} > \gamma_{\rho,c}\,|\mathcal{H}_1\,\right) && (23)\\ &= Q\left(\frac{\gamma_{\rho,c} - \mu\sigma_x^2}{\sqrt{\frac{(\sigma_x^2+(1/\rho)\sigma_n^2)^2+2\mu^2\sigma_x^4}{N}}}\right) && (24)\\ &= Q\left(\frac{\gamma_{\rho,c} - \left(\frac{N_c}{N_c+N_d}\right)\sigma_x^2}{\sqrt{\frac{(\sigma_x^2+(1/\rho)\sigma_n^2)^2+2\left(\frac{N_c}{N_c+N_d}\right)^2\sigma_x^4}{N}}}\right). && (25) \end{aligned}$$

B. Real Part of Autocorrelation as Test Statistic

Second possible test statistic is the real part of autocorrelation peak at lag N_d [35], [36], [39]. It can be expressed as

$$T_{\rho,r} = \frac{\frac{1}{N}\sum\limits_{n=0}^{N-1} \Re\left\{y(n)y^*(n+N_d)\right\}}{\frac{1}{N+N_d}\sum\limits_{n=0}^{N+N_d-1} |y(n)|^2} \tag{26}$$

where $\Re\{.\}$ and $\{.\}^*$ are the real part of a complex samples and the complex conjugate of the values, respectively. The total number of samples used for the autocorrelation is $N+N_d$ while as shown in [36], the $T_{\rho,r}$ under the H_0 is distributed according to:

$$\mathcal{H}_0 : T_{\rho,r} \sim \mathcal{N}_R\left(0, \frac{1}{N}\right) \tag{27}$$

where $\mathcal{N}_R$ refers to the Gaussian distribution for real valued numbers. Because of the Gaussian statistics, $T_{\rho,r}$ has a probability of exceeding threshold, $\gamma_{\rho,r}$, which is given by

$$P\left(T_{\rho,r} > \gamma_{\rho,r}\right) = \frac{1}{2} Q\left(\frac{\gamma_{\rho,r}}{\sqrt{2}\sigma_r}\right) \tag{28}$$

and therefore

$$P_{FA} = P\left(T_{\rho,r} > \gamma_{\rho,r}\,|\mathcal{H}_0\,\right) = \frac{1}{2} Q\left(\sqrt{N}\gamma_{\rho,r}\right). \tag{29}$$

With the aid of (27), the expected value of $T_{\rho,r}$ under the null hypothesis is zero. Based on this, the $P\left(T_{\rho,r} > \gamma_{\rho,r}\,|\mathcal{H}_0\,\right)$ can be obtained as a probability of false alarm, P_{FA}, in the context of detecting AWGN samples under $\mathcal{H}_0$ hypothesis. Hence, given the desired P_{FA}, the threshold, $\gamma_{\rho,r}$, can be expressed as:

$$\gamma_{\rho,r} = \frac{1}{\sqrt{N}} Q^{-1}\left(2P_{FA}\right). \tag{30}$$

Similarly, it is shown in [36] that $T_{\rho,r}$ under the $\mathcal{H}_1$ is distributed according to:

$$\mathcal{H}_1 : T_{\rho,r} \sim\ \mathcal{N}_R\left(\eta, \frac{\left(1-\eta^2\right)^2}{N}\right) \tag{31}$$

where

$$\eta = \frac{N_c}{N_d + N_c}\frac{\sigma_x^2}{\sigma_x^2 + (1/\rho)\,\sigma_n^2}. \tag{32}$$

To this effect, the corresponding probability of detection can be calculated as

$$\begin{aligned} P_D &= P\left(T_{\rho,r} > \gamma_{\rho,r}\,|\mathcal{H}_1\,\right) && (33)\\ &= \frac{1}{2} Q\left(\sqrt{N}\frac{\gamma_{\rho,r} - \eta}{1-\eta^2}\right) && (34)\\ &= \frac{1}{2} Q\left(\sqrt{N}\frac{\gamma_{\rho,r} - \frac{N_c}{N_c+N_d}\frac{\sigma_x^2}{\sigma_x^2+(1/\rho)\sigma_n^2}}{1-\left(\frac{N_c}{N_c+N_d}\frac{\sigma_x^2}{\sigma_x^2+(1/\rho)\sigma_n^2}\right)^2}\right) && (35) \end{aligned}$$

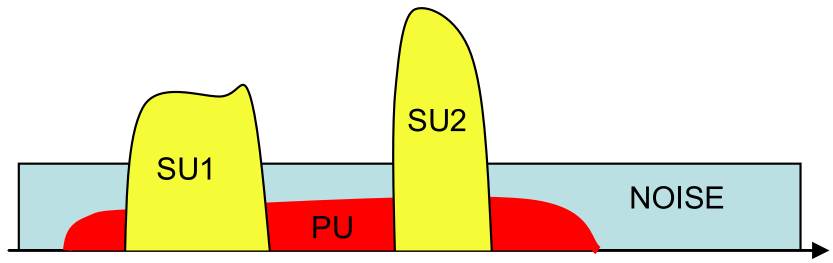


Figure 2: A typical spectrum monitoring scenario with uncoordinated secondary systems.

which has a simple algebraic representation that allows a straightforward computation.

III. Novel Sparse Frequency Domain Spectrum Sensing based on CP Autocorrelation under Noise Uncertainty

It is recalled that the autocorrelation of the received waveform is basically a time-domain operation. Usually, it is assumed that the spectrum sensing bandwidth matches the full bandwidth of the PU signal, and contains either noise only or noise plus PU signal. Considering a CR scenario where broadband PUs might be present, a PU signal might be partly overlapped by other transmissions with relatively narrow spectrum, as illustrated in Fig. 2. This could be a case of a reappearing PU before the SU system has detected it. In this situation, time-domain autocorrelation based spectrum sensing is expected to fail. One possibility is to perform the time-domain autocorrelation for the clean part(s) of the PU signal band only. Nevertheless, this would require additional highly configurable and complicated filtering in the receiver. However, frequency-domain implementation of autocorrelation can be envisioned, where only the subband signals which are clean from interfering SUs are utilized. Partial band autocorrelation computation from subcarrier samples can also be utilized in the context of compressive sensing using sparsity[1], in order to reduce the computational complexity or simplify the analog-to-digital conversion interface. Therefore, we focus on considering frequency domain spectrum sensing, where autocorrelation is performed using subband samples at the output of an FFT process. A spectrum sensing and sharing framework utilizing alternative sensing algorithms is illustrated in Fig. 3. Our wideband subband based sensing approach supports both the proposed FD-AC and traditional ED based sensing methods, thus supporting multi-channel, multi-mode sensing of different candidate primaries in the target frequency band.

To simplify the analytical models, we assume here digitization of the wideband analog signal, followed by FFT, the output of which is expressed as $y_{k,m}$, where $m = 1, \ldots, M$ is the subband sample index and $k = 1, \ldots, K$ is the subband index. Here K is the FFT size of the sensing receiver, which is considered to be independent from the IFFT size of the PU transmitter. In the context of spectrum sensing the subband signals can be expressed as follows:

$$y_{k,m} = w_{k,m} \qquad \mathcal{H}_0 \qquad (36)$$

$$y_{k,m} = x_{k,m} + w_{k,m} \qquad \mathcal{H}_1. \qquad (37)$$

Here,

$$[y_{1,m}, y_{2,m}, ..., y_{K,m}] \\ = FFT[y((m-1)K+1, y((m-1)K+2, ..., y(mK)] \qquad (38)$$

and $x_{k,m} \cong H_k s_{k,m}$ is the PU information signal at the m^{th} FFT output sample in subband k, H_k is the complex gain of subband k, and $w_{k,m}$ is the corresponding noise sample. Furthermore, it is assumed that $w_{k,m} \sim \mathcal{N}(0, \sigma^2_{w,k})$ and $x_{k,m} \sim \mathcal{N}(0, \sigma^2_{x,k})$, with $\sigma^2_{x,k}$ denoting the PU signal variance in subband k. Since FFT is used for spectrum analysis, the subband noise variances can be assumed to be the same, $\sigma^2_w/K \simeq \sigma^2_{w,k}$, while the channel noise is assumed to be white. It is recalled that the noise distribution is in the range $\sigma^2_{w,k} \in [(1/\rho)\sigma^2_{n,k}, \rho\sigma^2_{n,k}]$, where ρ presents the corresponding NU parameter. The worst-case P_{FA} and P_D scenarios are considered in both the analytical and simulated models.

A timelag of N_d can be expressed after FFT in subband domain as:

$$y_{k,m+m_\Delta} e^{j2\pi\tau k} \qquad (39)$$

where $m_\Delta = \text{round}(N_d/K)$ and $\tau = N_d/K - m_\Delta$. Here m_Δ is the coarse value of the lag as an integer number of subband samples and τ is the fractional part of the lag, which appears as a linearly frequency-dependant phase term. The normalized CP autocorrelation can then be expressed in terms of subband samples as follows:

$$C_Y(\tau) = \frac{\frac{1}{MK_{\text{comp}}} \sum_{m=1}^{M} \sum_{k\in\Omega} y_{k,m} y^*_{k,m+m_\Delta} e^{-j2\pi\tau k}}{\frac{1}{MK_{\text{comp}}} \sum_{m=1}^{M} \sum_{k\in\Omega} |y_{k,m}|^2} \qquad (40)$$

$$= \frac{\sum_{k\in\Omega} \left(\sum_{m=1}^{M} y_{k,m} y^*_{k,m+m_\Delta} \right) e^{-j2\pi\tau k}}{\sum_{m=1}^{M} \sum_{k\in\Omega} |y_{k,m}|^2} \qquad (41)$$

where Ω is the set of K_{comp} used subcarriers, and M is the integration length in FFT subband samples. The sample complexity is now $N = K(M+m_\Delta)$. In addition, the number of subcariers used in the autocorrelation calculation, K_{comp},

[1]The detailed analysis of sparsity is provided in the Appendix.

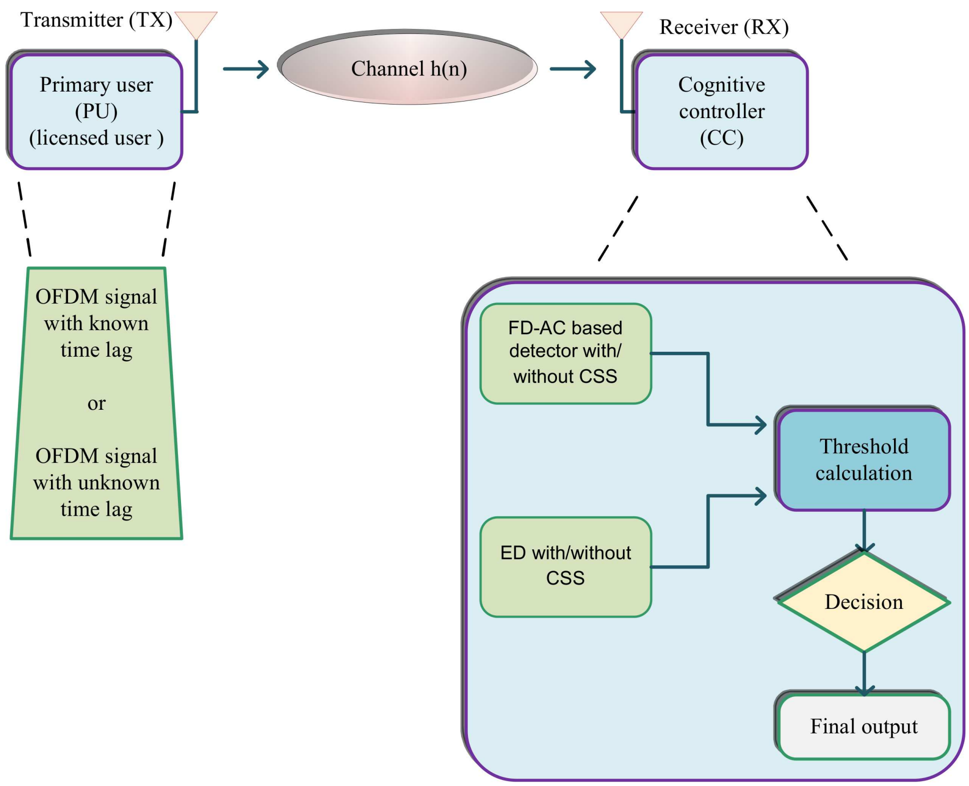


Figure 3: A spectrum sensing framework utilizing alternative spectrum sensing and spectrum sharing algorithms.

can be chosen, together with N, to take different values for a proper tradeoff between interference suppression capability, implementation complexity, and sensing performance. Importantly, (41) is computationally more efficient representation of (40). It indicates the possibility of using different subcarrier samples in the basic autocorrelation calculation, with spacing of m_{Δ}. This assists in maximizing the correlation observation for different combinations of the OFDM symbol duration and FFT based subband sample interval.

Similar to the time domain CP autocorrelation based approach, Gaussian distribution can also be considered for the test statistics in the proposed FD-AC. Based on the Gaussian approximation, $C_Y(\tau)$ under the H_0 is distributed according to:

$$\mathcal{H}_0 : C_Y(\tau) \sim \mathcal{N}_c\left(0, \frac{1}{MK_{\text{comp}}}\right) \tag{42}$$

and the corresponding probability of false alarm under compressed sensing can be expressed as

$$P_{FA} = P\left(C_Y(\tau) > \gamma_{\text{comp}}\,|\mathcal{H}_0\right) \tag{43}$$

$$= Q\left(\sqrt{MK_{\text{comp}}}\gamma_{\text{comp}}\right) \tag{44}$$

while the corresponding threshold can be expressed as

$$\gamma_{\text{comp}} = \frac{1}{\sqrt{MK_{\text{comp}}}} Q^{-1}\left(P_{FA}\right). \tag{45}$$

Likewise, $C_Y(\tau)$ under the $\mathcal{H}_1$ is distributed according to:

$$\mathcal{H}_1 : C_Y(\tau) \sim \mathcal{N}_c\left(\mu\sigma_x^2, \frac{\left(\sigma_x^2 + (1/\rho)\,\sigma_n^2\right)^2 + 2\mu_1^2}{MK_{\text{comp}}}\right) \tag{46}$$

and finally, the detection probability under compressed sensing can be given by

$$P_D = P\left(C_Y(\tau) > \gamma_{\text{comp}}\,|\mathcal{H}_1\right) \tag{47}$$

$$= Q\left(\frac{\gamma_{\text{comp}} - \mu\sigma_x^2}{\sqrt{\frac{(\sigma_x^2+(1/\rho)\sigma_n^2)^2+2\mu^2\sigma_x^4}{MK_{\text{comp}}}}}\right) \tag{48}$$

$$= Q\left(\frac{\gamma_{\text{comp}} - \left(\frac{N_c}{N_c+N_d}\right)\sigma_x^2}{\sqrt{\frac{(\sigma_x^2+(1/\rho)\sigma_n^2)^2+2\left(\frac{N_c}{N_c+N_d}\right)^2\sigma_x^4}{MK_{\text{comp}}}}}\right) \tag{49}$$

which is also expressed in a simple form.

In addition to the proposed algorithm, a similar concept can be also applied for the case of unknown time lag. If the lag values are considered with the time resolution of the FFT input, then the outer summation in (41) can be interpreted as an IFFT. Thus, while in the known lag case, only one element of the IFFT needs to be calculated, the unknown lag case requires calculation of the whole IFFT. The test statistic value $C_Y(\tau)$ is obtained from the second maximum peak (the first one is the peak at zero-lag).

The test statistic for both algorithms is taken from the magnitude of the autocorrelation function with the corresponding lag, in contrast to using the real part as in (26) and in [35], [36], [39]. In fact, we employ magnitude values because any frequency offset of the PU signal introduces phase rotation to the autocorrelation. In spectrum sensing scenarios, it cannot be assumed that the sensing station is synchronized to the PU signal.

Table I: Parameters of signal model for both traditional and proposed algorithms.

- Sampling rate: 20 MHz
- OFDM symbol duration in samples (FFT size): $N_d = 64$
- CP length: $N_c = 16$
- CP-OFDM symbol duration: $N_s = N_d + N_c = 80$
- Number of active OFDM subcarriers: 31
- FFT size in FD- AC sensing: $K = 1024$
- Number of FFTs averaged for correlation: $M = 100$
- Subcarrier sample offset: $m_\Delta = 0$
- Number of OFDM symbols in sensing: 1280
- Total number of samples in sensing $N = (M + m_\Delta)K = 102400$

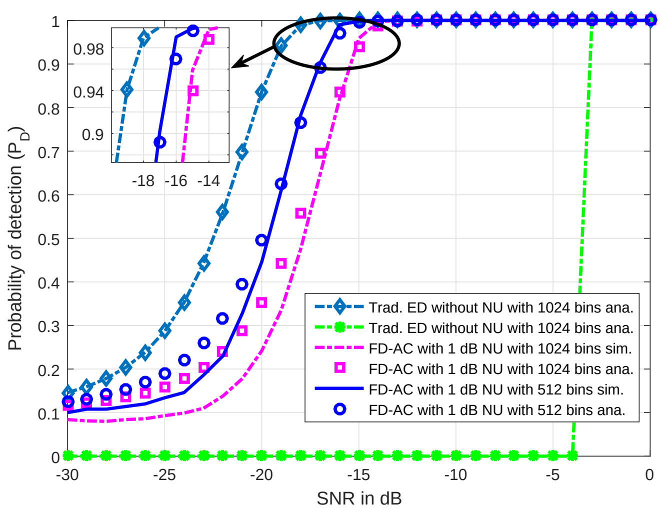


Figure 4: Analytical and simulated detection probability for both traditional ED and proposed FD-AC based sensing under AWGN channel with/without NU considering known time lag. Magnitude test statistic, full-band sensing with 1024 bins and partial band sensing with 512 bins under target $P_{FA} = 0.1$.

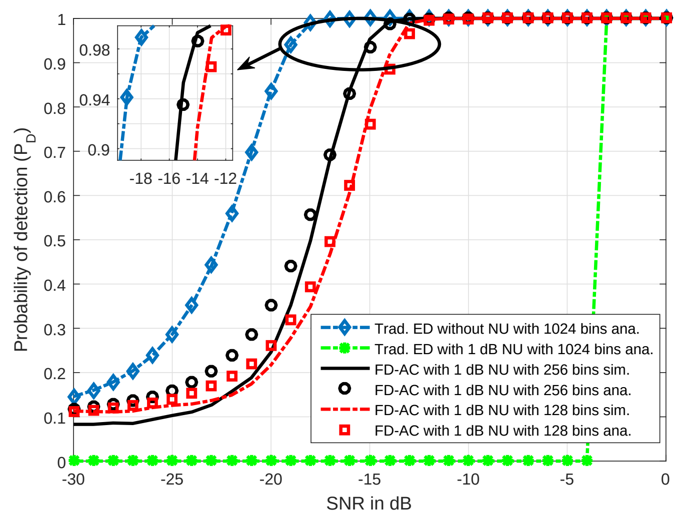


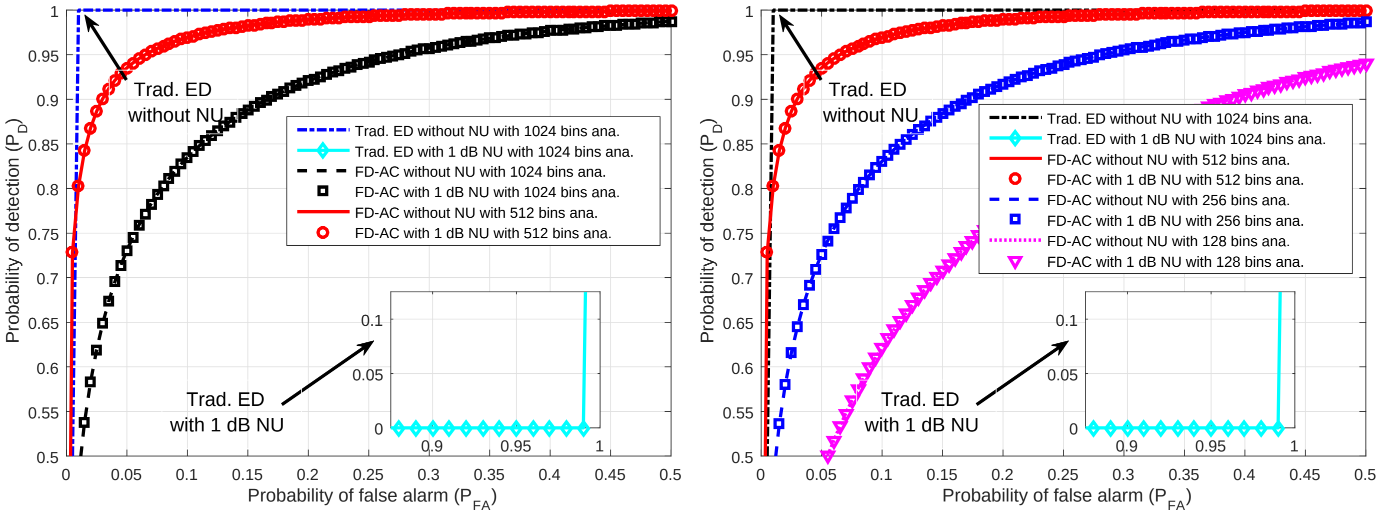
Figure 5: Analytical and simulated detection probability for both traditional ED and proposed FD-AC based sensing under AWGN channel with/without NU considering known time lag. Magnitude test statistic, partial-band sensing with 256 bins and 128 bins under target $P_{FA} = 0.1$.

IV. Numerical Results and Analysis of Computational Complexity

A. Numerical Results

This section analyzes the performance of the proposed frequency domain CP-autocorrelation (FD-AC) methods, with and without CSS element, against the traditional energy detector as a reference model. Given that CP-OFDM is the dominating multicarrier technology in the field of wireless communications, the CP-OFDM based PU signal is considered by the spectrum sensing function of CR.

The signal model parameters of the proposed algorithm are depicted in Table I for Fig. 4 – Fig. 12. It is noted that some values in Table I such as K and M have been chosen as example test values and these values significantly affect both the detection performance and computational complexity of the proposed method. Different values can be chosen in the different scenarios. The SNR is defined here assuming white noise in the whole observed bandwidth of 20 MHz. The analytical and simulated detection probabilities of the proposed FD-AC based sensing in the case of known time lag and 1 dB NU are shown in Fig. 4 and Fig. 5. Both full-band and partial band cases are included and the traditional ED with / without 1 dB NU [11]–[13] are included as reference. In these figures, the AC magnitude at the known lag is used as the test statistic and 1000 Monte Carlo simulations are used for reliable evaluation of the detection probability. It can be observed that the simulation results match adequately the corresponding theoretical results, especially in the interesting range of $P_D \geq 0.9$. However, due to the approximation of the distribution, there are some differences between the analytical and simulated results in the intermediate SNR range.



(a) (b)

Figure 6: Receiver Operating Characteristics (ROC) curves of traditional ED and proposed FD-AC based sensing under -16 dB SNR considering AWGN channel with/without NU with known time lag: (a) Magnitude test statistic, full-band sensing with 1024 bins, partial band sensing with 512 bins; (b) Magnitude test statistic, partial band sensing with 512 bins, 256 bins and 128 bins.

It should be understood that the P_D characteristics below the targeted minimum SNR have minor impact on the operation of a CR system. Since missed detections cause interference to PU operation, the sensing process should reach sufficient P_D under conditions where the CR operation might harm the PU operation. Therefore, $P_D = 0.9$ should be considered as a coarse lower limit for the interesting range. Additionally, for efficient CR operation, sufficiently low P_{FA} should be reached in a reliable way. While testing this aspect, it was found that the analytical and simulated false alarm probabilities match very well. While the analytical false alarm probabilities of FD-AC under all full-band and partial band cases are 0.1, the simulated false alarm probabilities are 0.094, 0.099, 0.097 and 0.105 for 1024, 512, 256 and 128 FFT bins, respectively.

The corresponding receiver operating characteristics (ROC) curves for the proposed FD-AC with magnitude test statistic and -16 dB SNR are illustrated in Figs. 6 (a) and 6 (b), along with ED as reference. Full-band and selected partial band cases are included for FD-AC. These results reflect a fundamental tradeoff between P_{FA} and P_D. Also while NU such as 1 dB affects the detection performance significantly in the conventional ED, it does not affect the proposed FD-AC based approach which thus, provides a robust detection performance.

Fig. 7 shows the effect of the sensing bandwidth on the sensing performance of FD-AC. This study is made for AWGN channel without NU and the known time lag model is used. The parameters are the same as in Table I, except for the usage of different numbers and different configurations of frequency bins in sensing. The sensing bandwidths of 32, 64, 128, 256, 512, 1024 bins are considered, both in contiguous manner and also in interleaved manner. In the interleaved case, sensing bands of 16 FFT bins are used in such a way that they cover

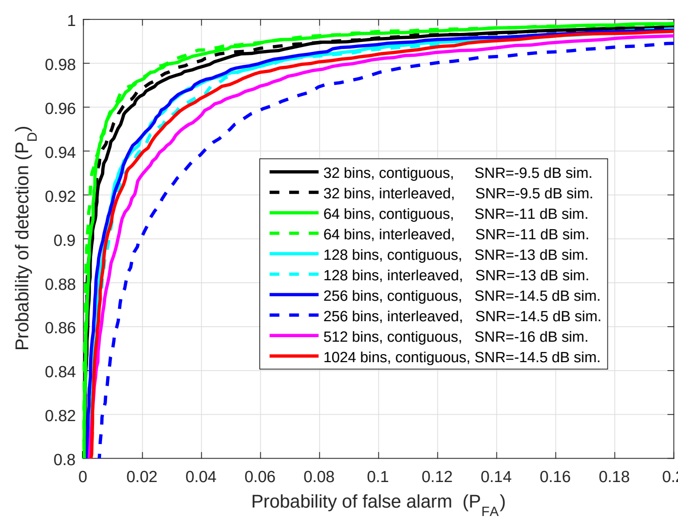


Figure 7: FD-AC based sensing performance using different numbers of sensing FFT bins in contiguous or frequency-interleaved configurations. AWGN channel model without NU and known time lag are assumed. The PU SNR is selected to reach $P_D \geq 0.97$ for P_{FA} = 0.1 in all cases.

the PU signal band. The PU SNR was selected in such a way that at least 0.97 detection probability is reached with 0.1 false alarm probability. Based on this, resulting SNR values are -9.5 dB, -11 dB, -13 dB, -14.5 dB, -16 dB, -14.5 dB for 32, 64, 128, 256, 512, 1024 FFT bins, respectively.

We can see that the sensitivity of the FD-AC process depends on the total sensing bandwidth, as expected. Doubling the sensing bandwidth gives 1.5 dB to 2 dB improvement in sensitivity. The optimum performance is reached with 512 bins, covering the PU signal band, but not including frequency

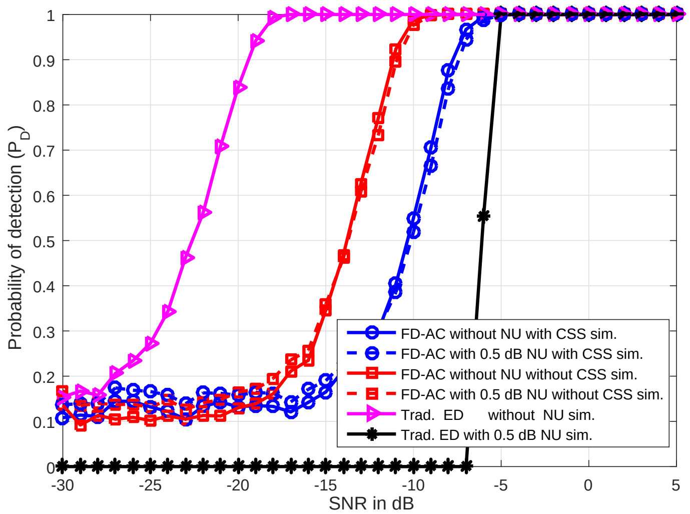


Figure 8: Detection probability of both traditional ED and proposed FD-AC based sensing in the case of known time lag without / with 0.5 dB NU and without / with compressed spectrum sensing under AWGN. The target $P_{FA} = 0.1$ and $K_{\text{comp}} = 242$.

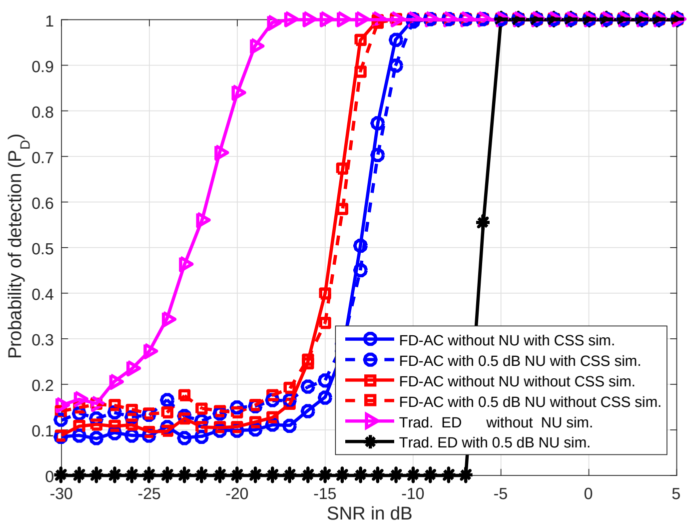


Figure 9: Detection probability of both traditional ED and proposed FD-AC based sensing in the case of unknown time lag without / with 0.5 dB NU and without / with compressed spectrum sensing under AWGN. The target $P_{FA} = 0.1$ and $K_{\text{comp}} = 242$.

bins with noise only. In the case of 1024 bins, the sensitivity is reduced by about 1.5 dB. We can also observe that the contiguous and interleaved sensing schemes have quite similar performance. However, a more detailed and extensive study about the performance in more irregular interleaved sensing subcarrier configurations remains as a topic for future study.

Fig. 8 and Fig. 9 demonstrate the detection probabilities of the FD-AC algorithms with known and unknown time lag under AWGN, respectively, against the traditional ED as the reference sensing method. In the two figures, the results are given both without and with the compressed sensing method,

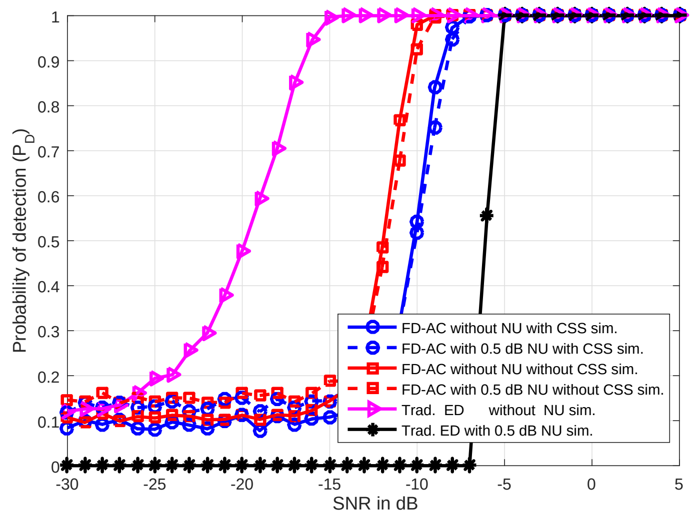


Figure 10: Detection probability of both traditional ED and proposed FD-AC based sensing in the case of unknown time lag without / with 0.5 dB NU and without / with compressed spectrum sensing under Indoor channel. The target $P_{FA} = 0.1$ and $K_{\text{comp}} = 242$.

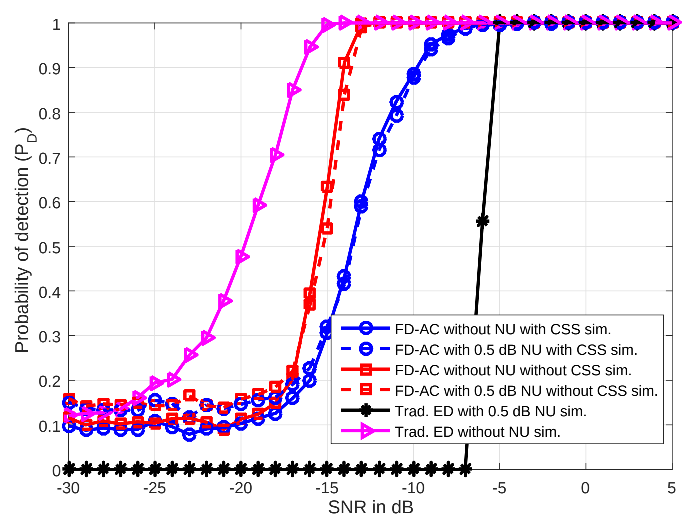


Figure 11: Detection probability of both traditional ED and proposed FD-AC based sensing in the case of unknown time lag without / with 0.5 dB NU and without / with compressed spectrum sensing under ITU-R Vehicular A channel. The target $P_{FA} = 0.1$ and $K_{\text{comp}} = 242$.

the latter one with 242 used subcarriers.

The results are given for 0 dB and 0.5 dB NU values, and the robustness of FD-AC methods can be clearly observed. Another important observation is that the unknown lag model has a clearly better sensitivity. The reason for this is that due to the channel effects and noise, the AC peak may not be precisely located at the expected location. Even in the known lag case, there is a need to search for the peak in the neighborhood of the expected lag, within a few high-rate sample intervals. The performance vs. complexity tradeoffs related to this approach remains an interesting topic of future

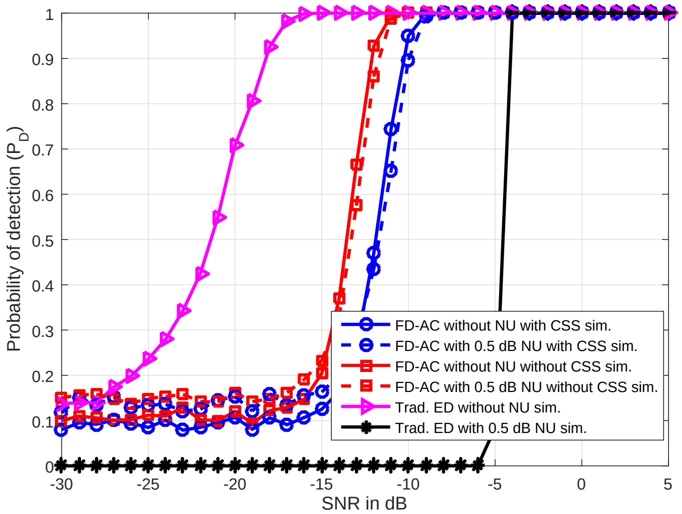


Figure 12: Detection probability of both traditional ED and proposed FD-AC based sensing in the case of unknown time lag without / with 0.5 dB NU and without / with compressed spectrum sensing under SUI 1 channel. The target $P_{FA} = 0.1$ and $K_{\text{comp}} = 242$.

research. In the rest of this section, the results are shown for unknown lag algorithm only.

The proposed methods were also tested in the context of three different frequency selective channel models namely, Indoor, International Telecommunication Union (ITU)-R Vehicular A, and Stanford University Interim (SUI)-1 [63], in order to quantify the effects of practical wireless multipath channels. The results are shown in Fig. 10 − Fig. 12. The Indoor channel model has 16 taps and 80 ns RMS delay spread whereas Vehicular A channel has 6 taps with about 2.5 μs maximum delay spread. SUI-1 channel model has 3 Ricean fading taps and 0.9 μs delay spread [63].

Fig. 10 illustrates the corresponding results in case of the frequency selective Indoor channel model with 0.5 dB NU. Also under highly frequency selective channels, both FD-AC with and without the compressed sensing element exhibit adequate robustness against NU and non-ideal channel characteristics.

Fig. 11 − Fig. 12 illustrate the corresponding results in case of the frequency selective ITU-R Vehicular A and SUI-1 channel models for the case of 0.5 dB NU value. It is also shown that under different frequency selective channels, both FD-AC with and without the compressed sensing element exhibit robustness against NU and non-ideal channel characteristics.

B. Computational Complexity

Computational complexities of the sensing algorithms are calculated in terms of the number of real operations that the methods must perform to complete the decision statistics on the spectrum occupancy. The FD-AC methods need $(M + m_\Delta)K$ samples for M correlations, and we assume the same sample complexity for the other methods included in the comparison. The comparison includes also, as a reference, an eigenvalue based method, which is well-known as a spectrum sensing method which is robust to NU [13]. The eigenvalue detector is based on maximum eigenvalue over minimum eigenvalue, and it uses the smoothing factor of $L = 8$ and overlapping factor of $M_{\text{over}} = 1$. The used smoothing and overlap factors result in relatively low complexity, which slightly compromises the sensing performance.

Table II depicts generic expressions, as well as numerical values in our example case, for different sensing methods. In our example case, it is assumed that $m_\Delta = 0$, in which case the FD-AC is obtained from the squared magnitudes of the subband samples. In FD-AC CSS cases, $K_{\text{comp}} = 242$ is assumed. Also, in all our numerical results, the sensing duration has been fixed to 102400 samples at 20MHz sampling rate. Based on the different number of bins such as 1024, 512, 256 and 242, sample rate can be considered as 20 MHz, 10 MHz, 5 MHz and 4.73 MHz, respectively. With 102400 samples, the overall computational complexity (number of real multiplications) of the eigenvalue based algorithm is 3277313, contrary to the 614400 and 772368 counterparts for the time-domain AC and the proposed FD-AC based spectrum sensing methods, respectively. Using the real part of the time-domain autocorrelation would reduce the required number of multiplications to $4N$ or 409600 in our example case. It is noted here that this number of multiplications is needed for each time lag value when searching for the actual maximum peak location around the theoretical one obtained from the OFDM parameters. This would significantly increase the complexity of time-domain AC with practical multipath channels, due to the uncertainty of the exact lag value for the correlation peak.

In time domain implementation, it would be sufficient to carry out the autocorrelation calculation over the CP length only, if the spectrum sensing device is synchronized to the PU. This would in principle reduce the complexity and the noise in the decision statistic, but such assumption is not realistic in practical spectrum sensing scenarios.

The proposed FD-AC based sensing method tends to have slightly higher complexity than the basic time-domain AC method, but the complexities of both methods are significantly smaller than those of the eigenvalue based techniques. Therefore it can be considered an effective sensing method that can also meet the requirements for adequate co-existence with efficient spectrum sharing strategies.

V. Conclusion

In this contribution, we investigated spectrum sensing / sharing methods which utilize the autocorrelation of the received signal using sparsity in frequency domain. These techniques can also be considered as compressed sensing / sharing methods due to the sparsity properties, which acquires wideband signals using the sampling rates lower than the Nyquist rate and detects the spectral opportunities using these compressed measurements. It was observed that the proposed methods are able to overcome the problem of NU under both AWGN and frequency selective channels. The detection performance was found to be better than that of the traditional energy

Table II: Computational complexity of traditional and proposed spectrum sensing algorithms.

Spectrum sensing algorithms	**Complexities (Real multiplications)**	**Numerical value for scenario**
Basic Energy Detector	$2N$	204800
Eigenvalue Detector	$4MLN + O(M^3L^3)$	3277313
Time-domain ACF based	$6N$	614400
Proposed FD-AC CSS with unknown lag	$(M+1)K(\log_2(K)-3)+2MK_{comp}$	772368

Notes: Complex multiplication assumed to take 4 real multiplications. Squared magnitude takes 2 real multiplications. Split radix algorithm used for FFT.

detector under both moderate and high NU values. Novel analytic expressions were also derived for the corresponding false alarm and detection probabilities that account for the detrimental effects of frequency selective characteristics. The offered results were validated extensively through comparisons with respective computer simulations and were subsequently employed in providing insights that can be useful in future design and deployments of CR communication systems and networks.

Specifically, it was shown that while the proposed FD-AC based sensing method has comparable complexity with that of basic time-domain AC method, its complexity is much smaller than the complexity of eigenvalue based detectors, which are often considered as the solution for the NU issue. Most importantly, the proposed approach has great flexibility for wideband multimode spectrum sensing of OFDM primary signals, possibly with different bandwidths, FFT sizes, and CP lengths. It is also applicable for cases where the OFDM signals are partly overlapped, e.g., secondary transmissions or other interfering PU transmissions, which is also important for the robustness of the spectrum sharing scheme. CP-autocorrelation based sensing methods are applicable only for CP based primaries, including basic OFDM, OFDM based single carrier waveforms and multicarrier CDMA. However, since OFDM is a very popular waveform in communication system development, these methods find important applications, e.g., in the context of TV white-space CR as well as WLANs and various other systems operating in the industrial, scientific and medical (ISM) frequency bands. Furthermore, the proposed methods can be fully combined with subband energy detection based wideband/multichannel spectrum sensing approaches [3], [5]. A wideband sensing platform could run different sensing processes in parallel for different frequency channels and different types of primaries. Finally, their high performance and relatively low complexity render it capable of providing sufficient co-existence with highly accurate spectrum sharing methods.

Appendix

If the output of the FFT is sparse, or approximately sparse, for an *output-sensitive* algorithm its runtime will depend on ψ, which is the number of computed *large* coefficients. Formally, given a complex vector a, its Fourier transform is $\hat{\mathbf{a}}$; then it is required that the algorithm for the output is an approximation $\hat{\mathbf{a}}'$ to $\hat{\mathbf{a}}$, that satisfies the following ℓ_2 guarantee:

$$\|\hat{\mathbf{a}} - \hat{\mathbf{a}}'\|_2 \leq \varphi \underbrace{\min}_{\psi\text{-sparse b}} \|\hat{\mathbf{a}} - \mathbf{b}\|_2 \tag{50}$$

where φ is an approximation factor and the minimization over ψ-sparse signals. It is important that the best ψ-sparse approximation could be obtained by setting all but the largest ψ coefficients of $\hat{\mathbf{a}}$ to 0. Such a vector can be represented using only $\mathcal{O}(\psi)$ numbers. Thus, if ψ is small, the output of the algorithm can be expressed succinctly, and one can consider that the runtime of an algorithm is *sublinear* in the signal size N.

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